

Derivation of the Permanent Magnet Synchronous Machine Model step by step from the three phase model to the d-q model

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Abstract: In education we often use simplifications and analogies during the presentation of a specific topic, but this has the disadvantage of the oversimplification of the reality. In long terms it can cause a gap between the course materials and the scientific publications that can hold back the early phases of student researches. To eliminate this, I created a – step by step complicated – model structure in Matlab/Simulink [1] that illustratively and transparently demonstrates the operating principles of the cylindrical and the salient pole synchronous machines [2]. It presents the detailed structure of the three phase model [3], the alpha-beta model [4] and the d-q model [5] and also helps to understand the drive control functions [6] belonging to the models. My goal is to help the students to join the researches of this topic as early as they can without reading hundreds of pages of scientific literature.

Keywords: PMSM; modelling; Matlab/Simulink; Three Phase Model, Alpha-Beta Model, d-q Model

1. Modelling the PMSM in different reference frames

1.1 Motivation

The Matlab/Simulink environment already has built in models for the PMSM in different toolboxes, which are making extra costs for projects to buy and use them. Some compatibility problems can occur with these blocks between different software versions which makes them difficult to use. Another case is when a blockset behaves differently in comparison to the physical model. Testing new control algorithms gave the demand to build the PMSM in all three reference frames.

1.2. Reference coordinate system

The simulation was build up by using the first coordinate-system at [11] for both the cylindrical and the Salient Pole Permanent Magnet Synchronous Motors: Beta is leading to the Alpha (90° before Alpha) and q is leading d. (The angular position “phi” in equations is denoted as “theta” in simulink model.)

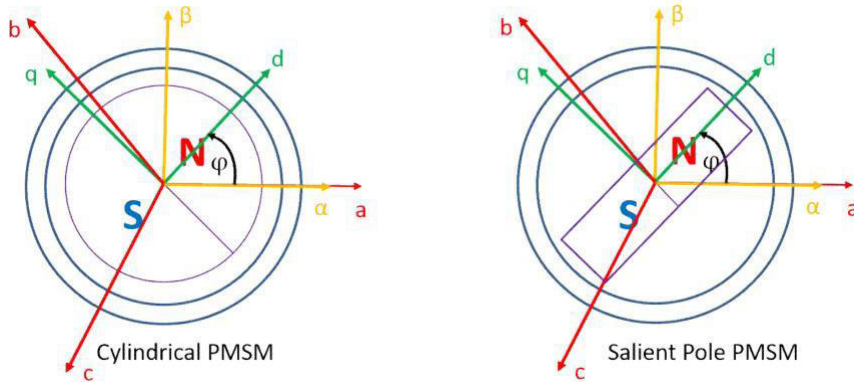


Figure 1
Cylindrical and Salient pole PMSM with reference frame coordinata-systems

1.3 Motor equations

1.3.1. Three phase reference frame, Cylindrical PMSM

The three phase reference frame is the best model to simulate asimmetry of the system, for example a short circuit between two turns in a phase winding or manufacturing discrepancies of the winding connections.

$$\begin{bmatrix} U_a \\ U_b \\ U_c \end{bmatrix} = \begin{bmatrix} R_a & 0 & 0 \\ 0 & R_b & 0 \\ 0 & 0 & R_c \end{bmatrix} * \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L_a & 0 & 0 \\ 0 & L_b & 0 \\ 0 & 0 & L_c \end{bmatrix} * \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \omega \Psi_m \begin{bmatrix} \sin(\varphi) \\ \sin(\varphi - 2\pi/3) \\ \sin(\varphi + 2\pi/3) \end{bmatrix} \quad (1)$$

To build up the Simulink model, the equations for each phases have to be solved for the derivative of the currents, because in this case, one integrator is enough to create the phase current outputs.

The expression below is for phase a, but based on Equation 1, for phase b and c can also be derived easily.

$$i_a = \frac{1}{L_a} \int (U_a - R_a i_a + \omega_e \Psi_m \sin(\varphi)) dt \quad (2)$$

The figure below shows the Simulink implementation of the integral formula for all the three phases and an additional torque producing part.

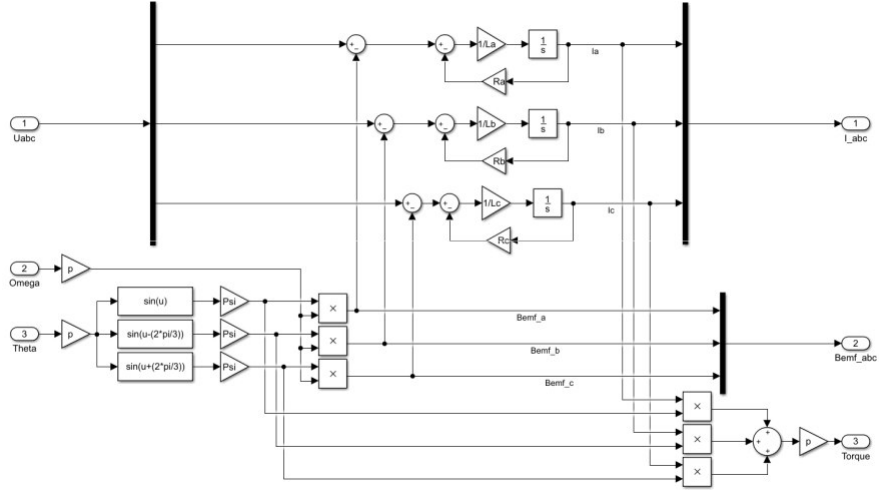


Figure 2

The electrical part of the three phase cylindrical PMSM model in three phase reference frame.

1.3.2 Three phase reference frame, Salient Pole PMSM

To consider the magnetic anisotropy of the Salient Pole Motor, the d and q directional inductance were introduced. In d-q reference frame, it can be easily imagined, but in three phase frame, the anisotropy has to be considered as a constant and a variable part of the phase inductances can be expressed with d and q directional components in function of the rotor angular position [3][4]

$$L_a = \frac{L_d + L_q}{2} - \frac{L_d - L_q}{2} \cos(2\varphi) \quad (3)$$

$$L_b = \frac{L_d + L_q}{2} - \frac{L_d - L_q}{2} \cos(2\varphi + 2\pi/3) \quad (4)$$

$$L_c = \frac{L_d + L_q}{2} - \frac{L_d - L_q}{2} \cos(2\varphi - 2\pi/3) \quad (5)$$

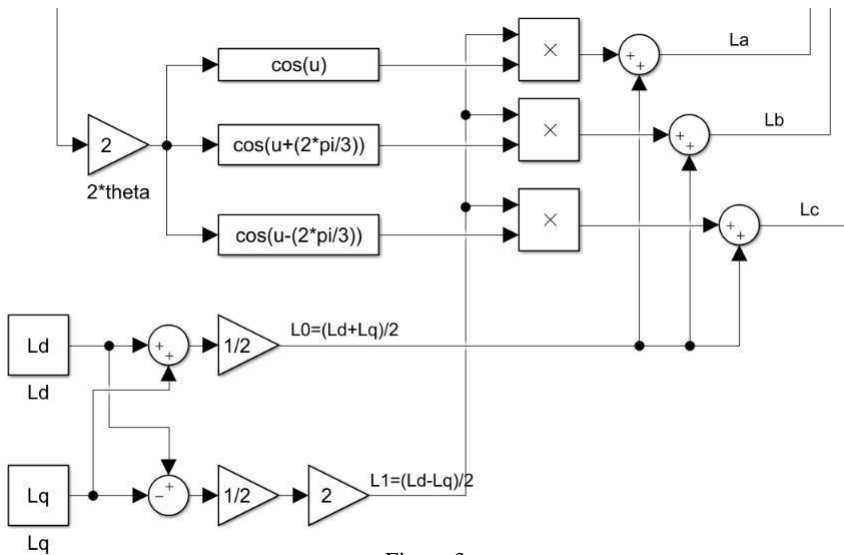


Figure 3

Implementation of the constant and variable inductances with dq components for phase inductances

Adding the expressed constant and variable inductances to the cylindrical model, the simplified three phase model of the Salient Pole Motor is complete:

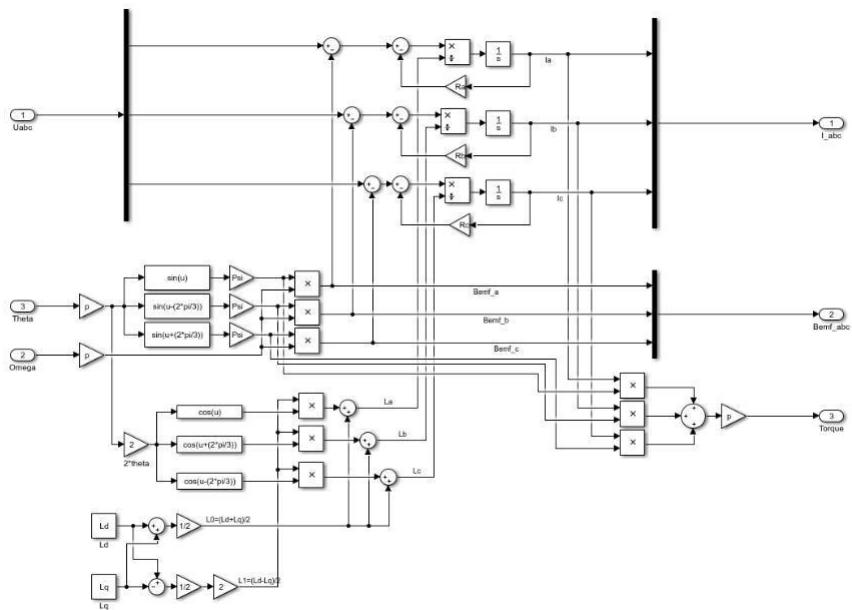


Figure 4

Implementation of the Salient Pole PMSM in three phase reference frame

1.3.3 Alpha-Beta reference frame, Cylindrical PMSM

The stationary cartesian coordinate system, so called Alpha-Beta reference frame gives an equal description of the physical system like the three phase reference frame, it is created using the Clarke transformation [8][10].

$$\begin{bmatrix} U_\alpha \\ U_\beta \end{bmatrix} = \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix} * \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \begin{bmatrix} L_s & 0 \\ 0 & L_s \end{bmatrix} * \frac{d}{dt} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \omega \Psi_m \begin{bmatrix} -\sin(\varphi) \\ +\cos(\varphi) \end{bmatrix} \quad (6)$$

From the voltage equations, it is possible to express the derivatives of the current components, to integrate:

$$i_\alpha = \frac{1}{L_\alpha} \int (U_\alpha - R_\alpha i_\alpha + \omega_e \Psi_m \sin(\varphi)) dt \quad (7)$$

$$i_\beta = \frac{1}{L_\beta} \int (U_\beta - R_\beta i_\beta - \omega_e \Psi_m \cos(\varphi)) dt \quad (8)$$

Based on this two equations, the model can be easily implemented in Simulink, the equations are extended with the torque producing part.

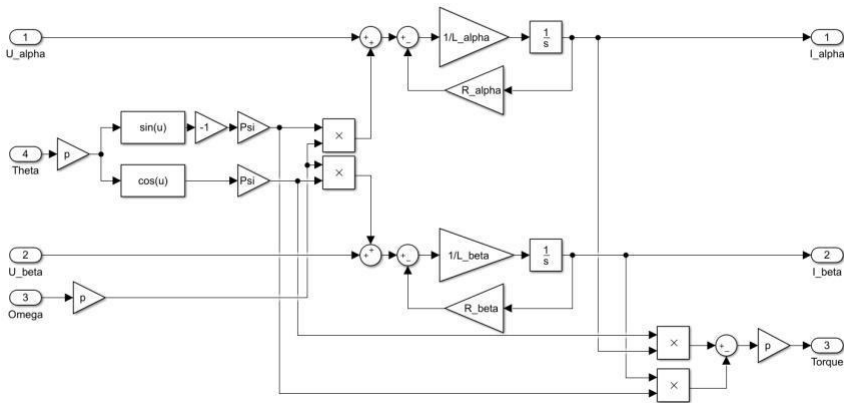


Figure 5

The electrical part of Cylindrical PMSM model in Alpha-Beta reference frame.

1.3.4 Alpha-Beta reference frame, Salient Pole PMSM

Modelling the magnetic anisotropy in Alpha-Beta reference frame is more difficult than in three phase or d-q frame, due to the additional component in the resistance and inductance metrices resulted from the flux derivatives [4].

$$\begin{bmatrix} U_\alpha \\ U_\beta \end{bmatrix} = \begin{bmatrix} R_\alpha - 2\omega L_{\alpha\beta} & \omega(L_\alpha - L_\beta) \\ \omega(L_\alpha - L_\beta) & R_\alpha + 2\omega L_{\alpha\beta} \end{bmatrix} * \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \begin{bmatrix} L_\alpha & L_{\alpha\beta} \\ L_{\alpha\beta} & L_\beta \end{bmatrix} * \frac{d}{dt} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \omega \Psi_m \begin{bmatrix} -\sin(\varphi) \\ +\cos(\varphi) \end{bmatrix} \quad (9)$$

The components of the inductance matrix (also used in the resistance matrix) can be expressed with the d-q components:

$$L_\alpha = \frac{L_d + L_q}{2} + \frac{L_d - L_q}{2} \cos(2\varphi) \quad (20)$$

$$L_\beta = \frac{L_d + L_q}{2} - \frac{L_d - L_q}{2} \cos(2\varphi) \quad (31)$$

$$L_{\alpha\beta} = \frac{L_d - L_q}{2} \sin(2\varphi) \quad (42)$$

In Simulink, the usage of “From” and “Goto” blocks for the inductance components can increase the transparency of the model, otherwise the high number of signal wires hide the main blocks:

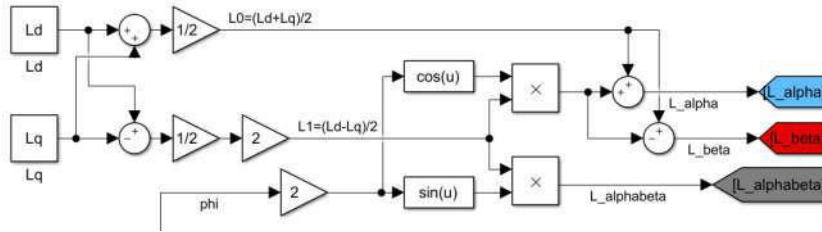


Figure 6

Implementation of constant and variable inductances with d-q components for Alpha-Beta components

With the expression of inductances, the voltage equations can be easily transformed to the integral formula of the current components:

$$i_\alpha = \frac{1}{L_\alpha} \int (U_\alpha - R_\alpha i_\alpha + \omega_e \Psi_m \sin(\varphi) + 2\omega_e L_{\alpha\beta} i_\alpha - \omega_e L_\alpha i_\beta + \omega_e L_\beta i_\beta - L_{\alpha\beta} \frac{di_\beta}{dt}) dt \quad (53)$$

$$i_\beta = \frac{1}{L_\beta} \int (U_\beta - R_\beta i_\beta - \omega_e \Psi_m \cos(\varphi) - 2\omega_e L_{\alpha\beta} i_\alpha - \omega_e L_\beta i_\alpha + \omega_e L_\alpha i_\alpha - L_{\alpha\beta} \frac{di_\alpha}{dt}) dt \quad (64)$$

Based on the expressed inductance components and the current equations, an implementation can be made:

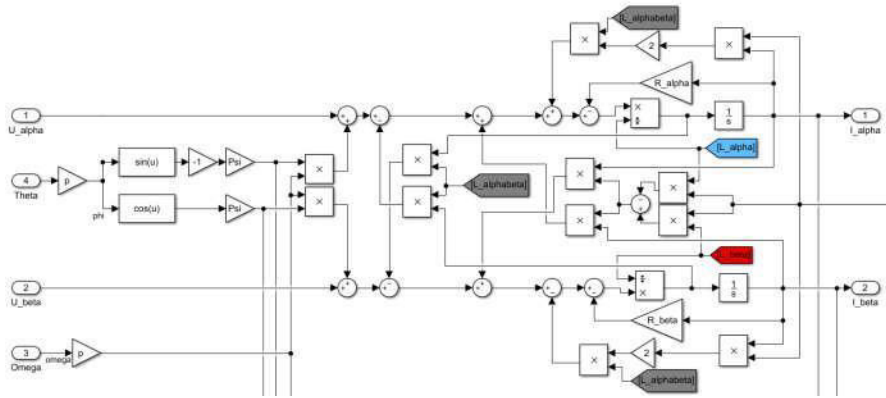


Figure 7

The electrical part of Salient Pole PMSM in Alpha-Beta reference frame.

1.3.5 d-q reference frame, Cylindrical PMSM

The usage of the d-q reference frame, so called synchronously rotating reference frame gives a simple model of the PMSM, the clearest understanding of the separated torque producing (iq) and the magnetizing (id) current components.[6]

$$\begin{bmatrix} U_d \\ U_q \end{bmatrix} = \begin{bmatrix} R_d & -\omega L_q \\ \omega L_d & R_q \end{bmatrix} * \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} * \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \omega \Psi_m \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (75)$$

Expressing the current components gives the integral formula which contains less components than the Alpha-Beta model due to the synchronously rotating coordinate-system:

$$i_{sd} = \frac{1}{L_s} \int (U_d - R_s i_{sd} + \omega_e L_s i_{sq}) dt \quad (86)$$

$$i_{sq} = \frac{1}{L_s} \int (U_q - R_s i_{sq} - \omega_e L_s i_{sd} - \omega_e \Psi_m) dt \quad (97)$$

Implementation in Simulink:

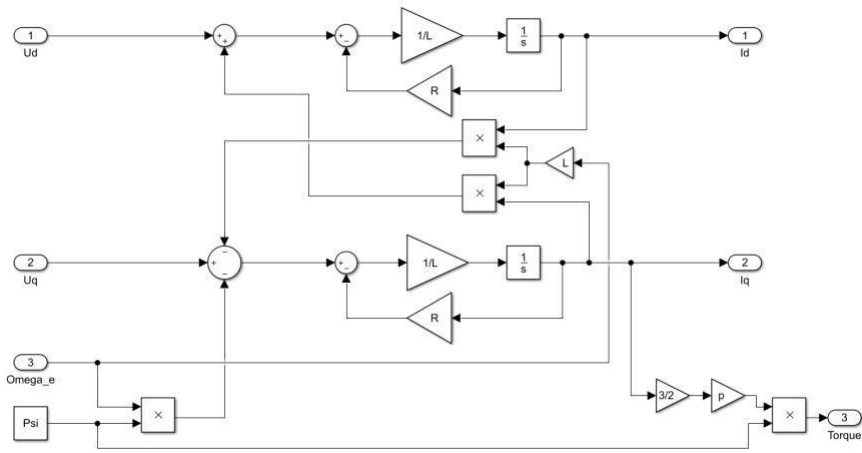


Figure 8

The electrical part of Cylindrical PMSM in d-q reference frame

Note: The “s” index in the equations notes the similar resistance and inductance values of the Cylindrical PMSM, it is not mentioned in the Simulink model, used for differentiate the voltage equations of the cylindrycal model from the salient pole model.[7]

1.3.6 d-q reference frame, Salient Pole PMSM

The synchronously rotating frame is directly oriented to the magnetic main direction, that is the main reason of the simple equations of the Salient Pole PMSM in d-q frame [2][3]:

$$\begin{bmatrix} U_d \\ U_q \end{bmatrix} = \begin{bmatrix} R_s & -\omega L_s \\ \omega L_s & R_s \end{bmatrix} * \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} L_s & 0 \\ 0 & L_s \end{bmatrix} * \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \omega \Psi_m \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (108)$$

Expressing the currents:

$$i_d = \frac{1}{L_d} \int (U_d - R_d i_d + \omega_e L_q i_q) dt \quad (119)$$

$$i_q = \frac{1}{L_q} \int (U_q - R_q i_q - \omega_e L_d i_d - \omega_e \Psi_m) dt \quad (20)$$

The implementation of the equations in Simulink only differs in a multiplication at the feedback part, and the torque calculation [9]:

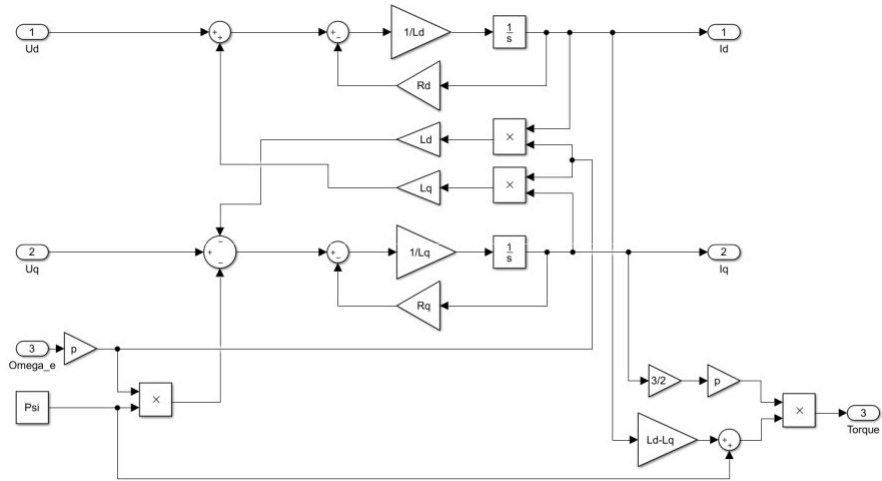


Figure 9

The electrical part of Salient Pole PMSM in d-q reference frame

1.4 Mechanical model

The mechanical equations of a motor is based on the equations of a rotating mass (J , moment of inertia) with a friction component (FF), here the dynamical torque is the difference of the electrically produced and the load torque:

$$T_d = T_e - T_l = J * \frac{d\omega}{dt} + FF * \omega \quad (212)$$

This can be also transformed to the integral formula:

$$\omega = \frac{1}{J} \int (T_d - FF * \omega) dt \quad (22)$$

$$\varphi = \int (\omega) dt \quad (23)$$

These equations are implemented to the mechanical model of the motor. Note, the equation “phi” is mentioned as “theta” in Simulink due to easily differ it from “psi” in the electrical models. The angular position information is used in the control blocks as a feedback. Due to the integration its value can infinitely increase, to limit this, an externally triggered integrator is added to the model. With this solution the angular position value will stay between 0 and 2π .

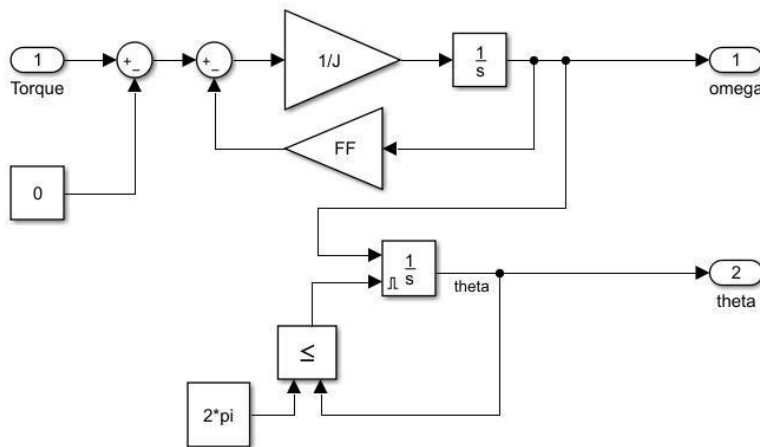


Figure 10

Mechanical model of the motor (a rotating mass)

1.5 Control model

The current controller can be implemented in d-q reference frame, and fitted to the other reference frames by Clarke-Park and its inverse transformations, and also can be implemented in the similar reference frame with the motor model.

1.5.1 Solution I. similar control model in d-q reference frame

In the first solution, we implement the current controller in d-q reference frame based on Equation 18:

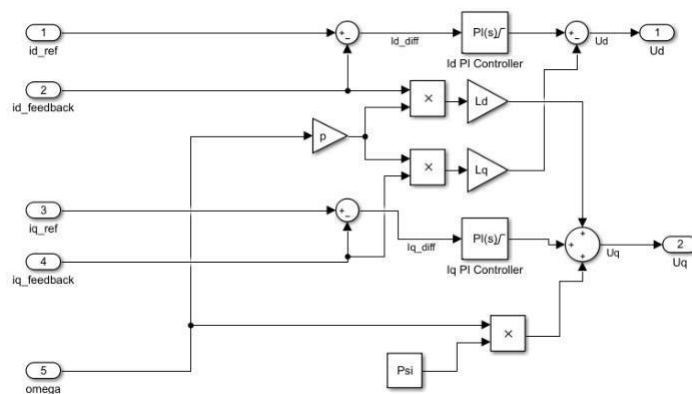


Figure 11

Current controller in d-q reference frame

And a simple speed PI controller:

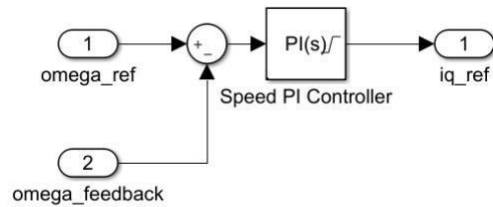


Figure 12
Speed controller

In this case, voltages were transformed to three phase and Alpha-Beta reference frame and currents were transformed back to d-q frame:

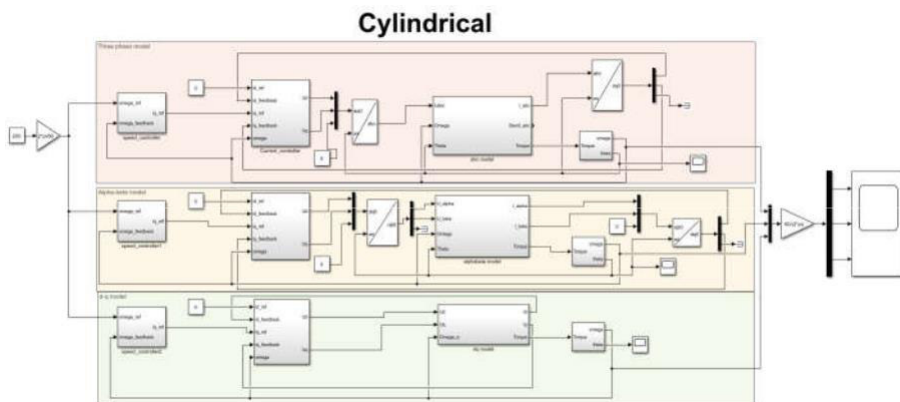


Figure 13

Combined model, containing three reference frame models of Cylindrical PMSM

1.5.2 Solution II. control in similar reference frame with the motor model

In the second solution, current controllers were implemented in Three Phase and Alpha-Beta reference frames, speed controller is similar to Figure 12 model.

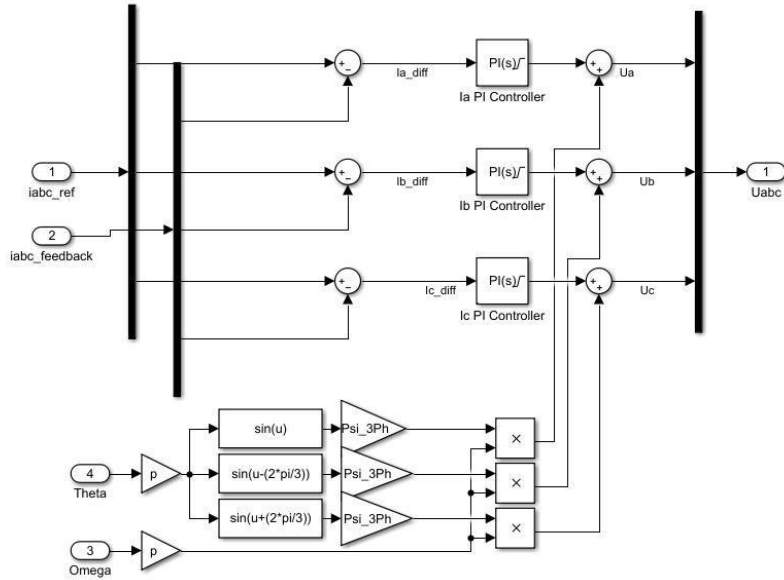


Figure 14
Current controller in Three Phase reference frame

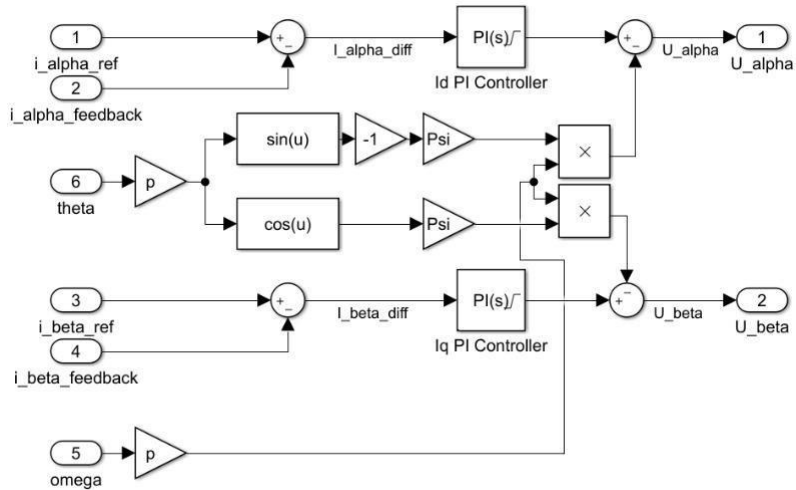


Figure 15
Current controller in Alpha-Beta reference frame

In this case, voltage and current signals of the controllers are fitting with the motor models, only i_q torque producing current reference signals have to be transformed to the d-q reference frame.

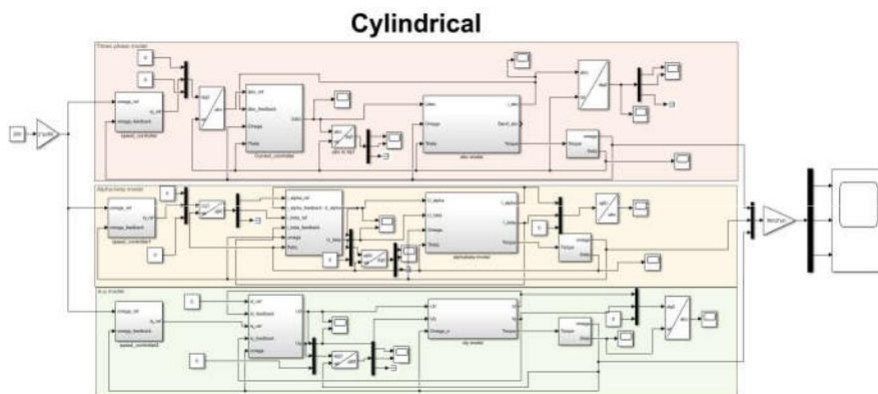


Figure 16

Current controller in Three Phase, Alpha-beta and d-q reference frames

2. Comparison of the models

As it was visible from Figure 13 and 16, a common scope were connected to the three different reference frames to make a comparison easily. The rotational speed was plotted in RPM values, the reference value were 200 RPM:

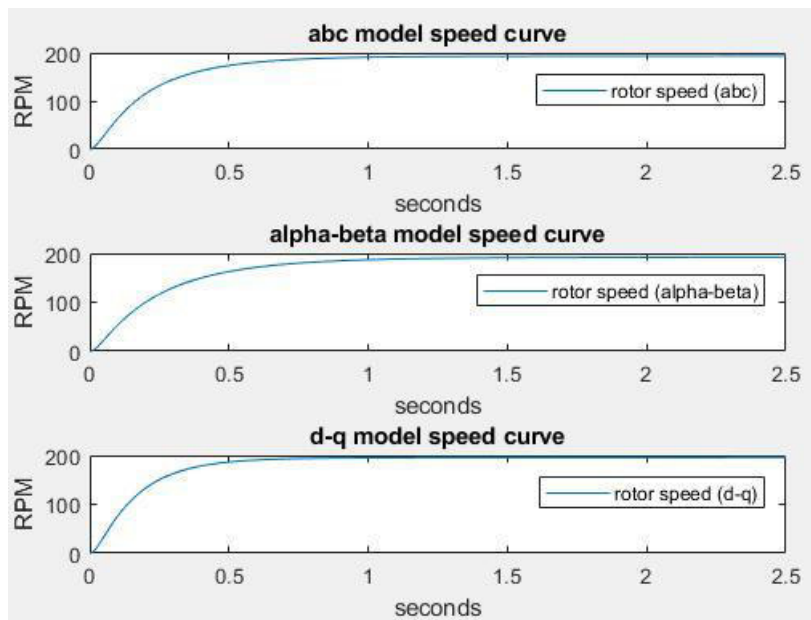


Figure 17

Comparison of the models in three different reference frames

The curves are running simultaneously, following the reference signal with a similar shape. Note: the curves are similar in both Cylindrical and Salient pole case with the same dataset at $L_d=L_q$ case.

3. Conclusions

Due to the fitting curves, the implementation of the PMSM model in different frames were successful in this level for the Cylindrical and the Salient Pole PMSM. The models can be replaced with each other for specific goals for example simulating asymmetry or for sensorless solutions. But for remark, there are still weak points of the model, it does not handle the leakages and the zero order components. In case of the Three Phase model, an extension is needed to handle the antisymmetrical part of the impedance matrices.

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