Mérnöki Szimpózium a Bánkiban
(ESB 2020)
http://bgk.uni-obuda.hu/esb/2020

Evolution of loading surface in the ultrasonic field

Andrew Rusinko, Ali H. Alhilfi
Obuda University, Nepszinhaz St. 8, Budapest, Hungary, H-1081, ruszinko.endre@uni-obuda.hu
Misan University, Iraq, alihameed@uomisan.edu.iq

Abstract

This study discusses the evolution of loading surface associated with the phenomena of ultrasonic temporary softening and ultrasonic residual hardening and residual softening registered in the experiments for plastic deformation of aluminum and titanium in the ultrasonic field. The aim is to model these phenomena in terms of the synthetic theory of irrecoverable deformation. We extend the flow rule relationships by two terms based on microstructural processes occurring in ultrasound-assisted deformation.
Keywords: loading surface, ultrasound, plastic deformation, residual hardening, residual softening.

1 Introduction

The present paper aims to study the evolution of the loading surface in terms of the synthetic theory (ST) for plastic deformation in the ultrasonic field.
Many studies relating the effect of ultrasound upon metals' deformation properties can be resumed by Zhou et al. [1], whose results are shown in Figs. 1.
Two portions can be identified:

(i) Ultrasound-assisted deformation (acoustoplasticity). The plastic flow of both aluminum and titanium takes place at stresses less than during ordinary loading (Fig. 1, US-on). This phenomenon is referred to as ultrasonic temporary softening.

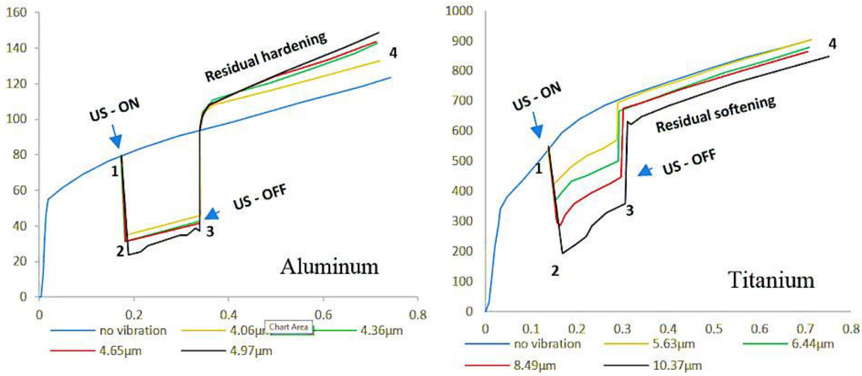
(ii) Deformation in post-sonicated period (Fig. 1, US-off). High-frequency vibration can permanently change metals' mechanical properties, which results in so-called ultrasonic residual effects.

Yao et al. [2] interpret the residual hardening effect for aluminum by the ultrasound-assisted multiplication of the crystalline lattice's defects. Zhou et al. [1] derived the softening mechanism for titanium [1] from the study of the fraction of twinning boundaries, abundant in many metals with low stacking fault energy (SFE) such as titanium, copper, gold, etc.
From the short review of experimental results above, the conclusion cannot be escaped that, at least in terms of experiments considered, the metals with high SFE, such as, e.g., aluminum, are inclined to ultrasonic residual hardening. In contrast, metals with low SFE (titanium, copper, gold) manifest the phenomenon of ultrasonic residual softening.

2 Synthetic theory

In terms of this theory [3], plastic deformation at a point of the body is determined via deformations at the microlevel of material, i.e., as a sum of plastic shifts in active slips systems where the resolved shear stress exceeds the material yield strength (the Schmidt law):

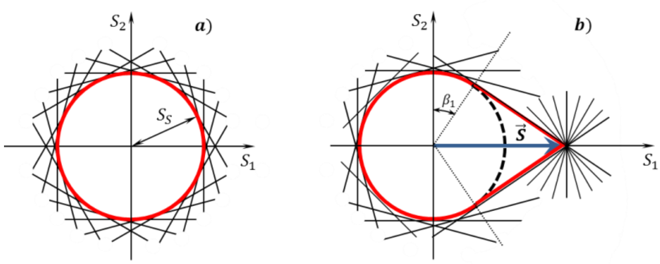
$$\vec{e} = \iiint_V \varphi_N \vec{N} dV, \quad (1)$$



1. Figure Stress-Strain diagrams of aluminum and titanium in the ultrasonic field of various oscillation amplitudes [1].

where φ_N – plastic strain intensity – is an average measure of plastic deformation within one slip system.

The main feature of the synthetic theory is the formulation of yield criterion and strain hardening rule in terms of three-dimensional stress deviator space $\mathcal{S}^3$. We do not deal with a yield surface itself, but with its tangent planes, i.e., the yield surface is considered an inner envelope of the tangent planes. The location of planes is defined by their distances (H_N) and unit normal vectors ($\vec{\boldsymbol{N}}$). For a virgin state, the synthetic theory works with the von-Mises yield criterion, which results in the set of equidistant planes in all directions (Fig. 2a).



2. Figure Yield and loading surface in terms of the synthetic theory in S_1-S_2 coordinate plane.

In terms of ST, a stress vector ($\vec{\boldsymbol{S}}$) represents loading, whose coordinates are composed of the stress deviator tensor components [3]. During the loading, the stress vector shifts at its endpoint a set of planes from their initial positions (Fig. 2b). The planes' movements at the endpoint of the stress vector are translational, i.e., the plane orientations keep changeless. Planes, which are not located at the endpoint of the vector $\vec{\boldsymbol{S}}$, remain stationary. The plane's displacement at the stress vector's endpoint represents plastic flow within the corresponding slip system.

We define a plastic strain intensity – the plastic flow rule on the micro-level of material – as

$$r\varphi_N = H_N^2 - S_S^2 = \begin{cases} \left(\vec{\boldsymbol{S}} \cdot \vec{\boldsymbol{N}}\right)^2 - S_S^2 & for\ planes\ reached\ by\ \vec{\boldsymbol{S}}\colon H_N = \vec{\boldsymbol{S}} \cdot \vec{\boldsymbol{N}} \\ 0 & for\ planes\ not\ reached\ by\ \vec{\boldsymbol{S}}\colon H_N > \vec{\boldsymbol{S}} \cdot \vec{\boldsymbol{N}} \end{cases}. \qquad (2)$$

The scalar product $\vec{\boldsymbol{S}} \cdot \vec{\boldsymbol{N}}$ determines the resolved shear stress acting in one slip system. It is evident that the plane distance H_N represents the hardening of material because the greater H_N is, the greater $\vec{\boldsymbol{S}}$ is needed to reach the plane. In Eq. (2), $S_S = \sqrt{2/3}\,\sigma_S$, where σ_S is the yield strength of the material; r is the model constant determining the slope of stress~strain curves; $[r] = \mathrm{Mpa}^2$.
In a uniaxial stress state, the stress vector $\vec{\boldsymbol{S}}\left(\sqrt{2/3}\,\sigma, 0{,}0\right)$ extends along the S_1 axis, and $H_N = S_1 N_1 = \sqrt{2/3}\,\sigma \sin\beta \cos\lambda$. Eqs. (1) and (2) have the following form [13]

$$\varphi_N = \frac{2}{3r}\left[(\sigma \sin\beta \cos\lambda)^2 - \sigma_S^2\right], \tag{3}$$

$$e = \frac{4\pi}{3r}\int_{\beta_1}^{\pi/2}\int_0^{\lambda_1}\left[(\sigma \sin\beta \cos\lambda)^2 - \sigma_S^2\right] \sin\beta \cos\lambda \cos\beta \, d\lambda d\beta = a_0 \Phi(b), \tag{4}$$

where

$$a_0 = \frac{\pi \sigma_S^2}{9 r_0}, \quad \Phi(b) = \frac{1}{b^2}\left[2\sqrt{1-b^2} - 5b^2\sqrt{1-b^2} + 3b^4 \ln\frac{1+\sqrt{1-b^2}}{b}\right]. \tag{5}$$

The integration boundaries in (4) are obtained from (3) by letting $\varphi_N = 0$ and $\lambda = 0$:

$$\sin\beta_1 = \frac{\sigma_S}{\sigma} \equiv b, \quad \cos\lambda_1 = \frac{\sigma_S}{\sigma \sin\beta}. \tag{6}$$

3 Extension of the Synthetic Theory to the case of plastic straining in the presence of ultrasound

To model the effects of ultrasound on the plastic strain of metals, we extend Eq. (2) by two terms, U_t and U_r:

$$r\varphi_{NU} = H_N^2 + U_t^2 + f(\gamma) U_r^2 - S_S^2. \tag{7}$$

U_t represents the temporary softening action of ultrasound:

$$U_t = A_1 {\sigma_m}^{A_2}(2 - e^{-pt})\left(\vec{\boldsymbol{u}} \cdot \vec{\boldsymbol{N}}\right), \quad t \in [0, \tau] \tag{8}$$

where σ_m is vibrating stress amplitude (MPa), $\vec{\boldsymbol{u}}$ is a unit vector indicating the vibration mode. For longitudinal sonication, $\vec{\boldsymbol{u}}$ vector has $(1{,}0{,}0)$ coordinates in $\mathcal{S}^3$. Further, τ is the sonication duration, and p and A_k $(k = 1{,}2)$ are model constants. If to denote through $\vec{\boldsymbol{\mathcal{U}}}$ the vector $A_1 {\sigma_m}^{A_2}(2 - e^{-pt})\vec{\boldsymbol{u}}$, Eq. (8) becomes $U_t = \vec{\boldsymbol{\mathcal{U}}} \cdot \vec{\boldsymbol{N}}$, i.e.
It is the power function $2A_1 {\sigma_m}^{A_2}$ that links the stress amplitude to the temporary softening effect. As a result, the term $A_1 {\sigma_m}^{A_2} e^{-pt}$ correlates with the temporary multiplication of ultrasound-induced defects (ψ_{NU}) proposed by Rusinko [4].
Therefore, Eq. (8) is dual. On the one hand, the ultrasound defects harden the material, but, on the other hand, they become centers of softening processes. As evident from (8), since the term $(2 - e^{-pt})$ is always positive; the net effect is a prevalence of softening mechanisms during

unidirectional and oscillating load simultaneous action.
The term $f(\gamma)U_r^2$ in Eq. (7) models the deformation properties of metals after the ultrasound is off, i.e., residual effects. Its effect depends on the sign of $f(\gamma)$ function, where γ is stacking fault energy. We define U_r as stress and time-dependent function:

$$U_r = h(\varepsilon - U) \times A_3 \int_0^{\tau} {\sigma_m}^{A_4} dt, \tag{9}$$

where h is the Heaviside step function, ε is any positive infinitesimally small number so that ultrasound of any intensity results in a negative value of $\varepsilon - \sigma_m$ difference. The presence of $h(\varepsilon - \sigma_m)$ function means that the U_r takes effect only after the ultrasound is off. Again, we propose a power function to express the dependence of ultrasonic residual hardening upon the ultrasound intensity with model constants A_3 and A_4. Simultaneously, the intensity of sonication is not the only parameter governing the magnitude of the hardening effect. Namely, the duration of sonication plays a vital role as well. Summarizing, U_r reflects a post-sonicated-defect-pattern leading to the change in material characteristics/response after the acoustoplasticity.

4 Loading surface in the uniaxial stress state

4.1 Acoustoplasticity ($\sigma > 0 \wedge U > 0$)**.** For vibrating-assisted deformation, $t \in [0, \tau]$, U_t increases in the way prescribed by (8). At the same time, due to $h = 0$ in (9), we have $U_r = 0$ during the action of ultrasound.
So, Eq. (7) takes the following form

$$r\varphi_{NU} = H_N^2 + U_t^2 - S_S^2 = \left(\vec{\boldsymbol{S}} \cdot \vec{\boldsymbol{N}}\right)^2 + \left[A_1 {\sigma_m}^{A_2}(2 - e^{-pt})\left(\vec{\boldsymbol{u}} \cdot \vec{\boldsymbol{N}}\right)\right]^2 - S_S^2 =$$

$$= \frac{2}{3}\left[(\sigma_U \sin\beta \cos\lambda)^2 + \frac{3}{2}[A_1 {\sigma_m}^{A_2}(2 - e^{-pt}) \sin\beta \cos\lambda]^2 - \sigma_S^2\right]. \tag{10}$$

Plastic deformation in acoustoplasticity (e_U) is calculated by Eq. (1) with the integrand from (10). As a result,

$$e_U = a_0 \Phi(b_U), \qquad b_U = \frac{\sigma_S}{\sqrt{\sigma_U^2 + \frac{3}{2}\left(A_1 {\sigma_m}^{A_2}(2 - e^{-pt})\right)^2}}. \tag{11}$$

Eqs. (11) and (5)-(6) show that the deforming of material in the ultrasonic field develops at less stress σ_U comparing to that under ordinary loading. Consequently, Eqs. (11) and (12) describe the phenomenon of temporary ultrasonic softening analytically. To ensure the stress drop at the constant value of deformation, we demand that $\varphi_{NU} = \varphi_N$ at the same set of planes where the strain intensity is positive (compare Figs. 2A and 2B). As seen from Fig. 2B, the loading surface preserves its shape due to the compensation element $\vec{\boldsymbol{u}}$, which means that less stress can keep the deformation as the ultrasonics vibration starts.

4.2 Post-sonicated deformation ($\sigma > 0 \wedge U = 0$). While $U_t = 0$ for $t > \tau$, the integral from (9) gives nonzero value ($h(\varepsilon) = 1 \Rightarrow U_r > 0$).
Compared to (10), the plastic strain intensity loses the term U_t, which facilitated the strain intensity, but includes U_r:

$$r\varphi_{Nr} = H_N^2 + f(\gamma)U_r^2 - S_S^2. \tag{12}$$

Since, so far, there is no enough experimental data on the effect of SFE upon the post-sonicated deformation for a wide range of metals, we propose a linear relationship for $f(\gamma)$ related to the value of SFE for aluminum:

$$f(\gamma) = k(\gamma_{\mathrm{Al}} - \gamma) - 1, \tag{13}$$

where $k \geq 1$ is a model constant. Taking into account the range of SFE of metals considered, we obtain that $f(\gamma_{\mathrm{Al}}) = -1$, and $f(\gamma_{\mathrm{Ti}})$ takes a positive value because $\gamma_{\mathrm{Al}} > \gamma_{\mathrm{Ti}}$.
Now, Eq. (12) takes the following shapes for aluminum and titanium, respectively:

$$r\varphi_{Nr} = \frac{2}{3}\left[(\sigma \sin\beta \cos\lambda)^2 - \frac{3}{2}[A_3 {\sigma_m}^{A_4}\tau]^2 - \sigma_S^2\right], \tag{14}$$

$$r\varphi_{Nr} = \frac{2}{3}\left[(\sigma \sin\beta \cos\lambda)^2 + \frac{3}{2}f(\gamma_{\mathrm{Ti}})[A_3 {\sigma_m}^{A_4}\tau]^2 - \sigma_S^2\right]. \tag{15}$$

The deformation for both aluminum and titanium specimen is calculated via formula (1), with the difference being that the strain intensities φ_N are governed by Eq. (14) and Eq. (15), respectively. Fig. 3B describes the phenomenon of a decrease in the stress required to induce plastic deformation (temporary softening), which is due to the ultrasound. The loading surface in Fig. 3C, which corresponds to the end of sonication, clearly demonstrates that the sum of static ($\vec{\boldsymbol{S}}$) and acoustic ($\vec{\boldsymbol{\mathcal{U}}}$) vectors reach the loading point (corner point).
Comparing formula (3) to (14), it is evident that $\varphi_{Nr} < \varphi_N$, i.e., more significant stress is needed to keep the same strain development than for ordinary loading. Therefore, formula (14) models the phenomenon of ultrasonic residual hardening, which is observed for aluminum. Utilizing Eq. (7) for $t > \tau$, we get for aluminum that

$$H_N^2 = r\varphi_{NU} + U_r^2 + S_S^2, \tag{16}$$

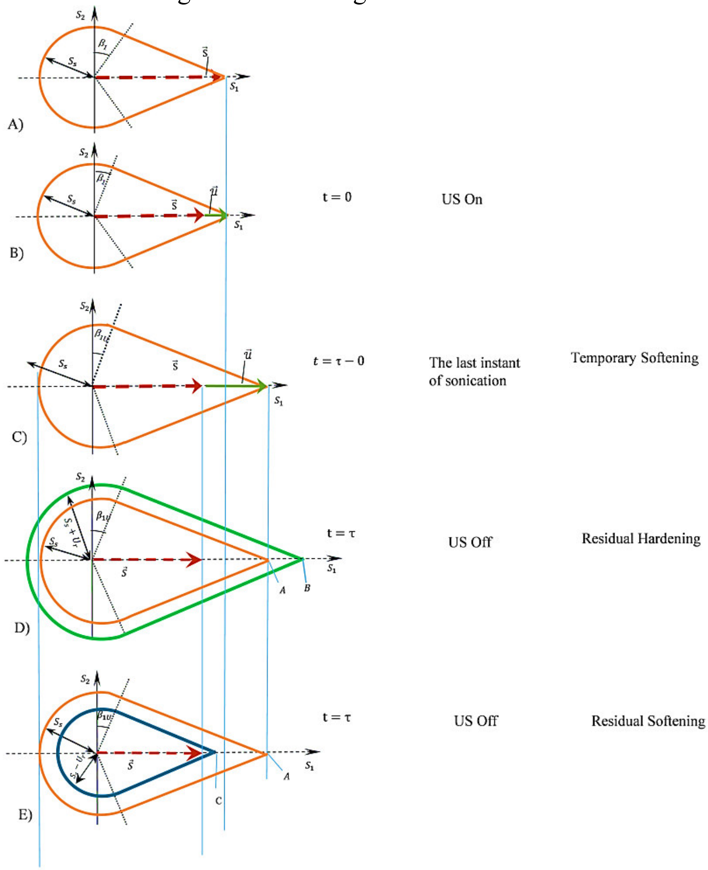
where φ_{NU} is Eq. (10) at $t = \tau$. Eq. (16) says that the plane distances yield an increment of an equal amount U_r in all directions Fig. 3D, i.e., the loading surface preserves its shape. As a result, for example, point A moves to a new position, point B. By comparing Fig. 3D to Fig. 3A, one can see that a more extended stress vector can regain the plastic deformation after the sonication. This fact means the manifestation of the ultrasonic residual hardening.
With titanium, it is clear from (3) and (15) that $\varphi_{Nr} > \varphi_N$ i.e., plastic deformation develops at less stress compared to static loading. Eq, 7 gives

$$H_N^2 = r\varphi_{NU} - f(\gamma_{\mathrm{Ti}})U_r^2 + S_S^2. \tag{17}$$

In contrast to (16), we obtain the case when the tangent planes move toward the origin of coordinates by the same amount in all directions (Fig. 3E). Now, it is clear from comparing the positions of the loading points in Figs. 3A and 3E (point C) that the plastic deformation of titanium for post-sonicated period starts and develops at less value of stress. Such a situation corresponds to the phenomenon of ultrasonic residual softening.

5 Conclusion

In this study, three phenomena observed during plastic deformation in the ultrasonic field are modeled – ultrasonic temporary softening, ultrasonic residual hardening, and residual softening. We utilize the synthetic theory of irrecoverable deformation to predict aluminum and titanium's deformation properties in the ultrasound-assisted compression. For this purpose, we extended the flow rule by two terms which govern the plastic deformation of the material in the ultrasonic field and after the ultrasound is off, respectively. The first term symbolizes how ultrasound facilitates plastic deformation development by activating blocked dislocations, localized heating, and dynamic softening. The second one reflects the effect of the defect structure formed during the sonication on metals' plastic properties for the post-sonicated period. The analytical manipulations proposed are analyzed in the context of the changes in the loading surface.



3. Figure Evolution of loading surface during and after the sonication (tangent planes are not shown)

6 References

[1] H. Zhou, H. Cui, and Q. H. Qin, Influence of ultrasonic vibration on the plasticity of metals during compression process, J. Mater. Process. Technol., 251, 146-159, 2018.

[2] Yao, Z., Kim, G. Y., Wang, Z., Faidley, L., Zou, Q., Mei, D., & Chen, Z. Acoustic softening and residual hardening in aluminum: modeling and experiments. International Journal of Plasticity, 39, 75-87, 2012.

[3] Rusinko, A., & Rusinko, K. Plasticity and Creep of Metals, Springer Science & Business Media, 2011.

[4] Rusinko, A. Analytical description of ultrasonic hardening and softening, Ultrasonics, 51, 709–714, 2011.