Aspects of direct georeferencing in photogrammetry

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Abstract – **Reliable and high-precision GNSS systems and inertial measurement units are playing increasingly important role in aerial surveying. As a result, direct georeferencing is receiving increasing attention in photogrammetry. In this case, exterior orientation elements of the images are available without measuring ground control points. Using this method, fast and near real-time photogrammetric processing can be implemented. Using exterior orientation parameters provided by inertial navigation systems, the direct georeferencing task can be fulfilled by solving an overdetermined system of equations, therefore we get an approximate solution. The system of equations can be solved by analytical method, but also by numerical method using iteration. This paper reviews the characteristics of direct georeferencing, typical measurement errors, and compares solution methods, analyzing their accuracy and computational needs using MATLAB.**

Keyword – **photogrammetry, direct georeferencing, MATLAB**

I. Introduction

Direct georeferencing in photogrammetry is the process where exterior orientation parameters are measured directly. It means that image projection centers (X, Y and Z coordinates) and the rotation angles are measured at moment of taking photo. Position of projection centers can be measure by GNSS and rotation angles produces by inertial measurement unit (IMU). As a result of direct georeferencing the measurement of GCPs is not required.

Using this method photogrammetric workflow is getting simplified. One of advantages of direct georeferencing is the more economical workflow. In addition, there is no need for expensive and resource-consuming surveying of GCPs. Direct georeferencing enables fast and real-time or near real-time photogrammetric processing. Sensitive point of direct georeferencing is the accuracy of measurement of exterior parameters of images.

The realization of direct georeferencing was made possible by development of inertial navigation, including gyroscopes. Nowadays light weight satellite navigation systems with correction services (e.g. RTK) allow geodesy level accuracy and the same is true for gyroscopes too. Gyroscopes used in the early inertial measurement units were large, expensive and complex equipment, but above all they were inaccurate that made direct georeferencing possible only with large errors. Development of inertial navigation over the last years has resulted the accuracy of inertial instrument and significant reduction of their price and size.

Introduction of microelectromechanical systems (MEMS) in gyro technology resulted low-cost, light gyroscopes, while for professional application laser gyroscopes are available at an affordable price and in compact form. This meant that reliable and accurate inertial navigation systems can be installed also on the board of UAVs. A contemporary inertial navigation system consists of GNSS with correction system and inertial measurement unit, including gyroscope and magnetometer. However, it should be taken in account, that even today the errors of gyroscope (mainly its drift) provide more inaccuracy in direct georeferencing than accuracy of triangulation method using GCPs.

II. Theoretical model

In Fig. 1. the theoretical model of direct orientation can be seen. The task is to determine the position of the P point in the geographical coordinate system by the way of measuring the position of the projection center, the camera orientation and measuring points in images.

The GNSS instrument installed on the aircraft provides position in geographical coordinate system ($\overline{r}^{GNSS}$). The IMU provides the orientation of the aerial vehicle (indicated by $\mathbf{R}^{IMU}$ rotation matrix). It should take in account the fact that the GNSS instrument and the IMU are usually located in different part of the aircraft.

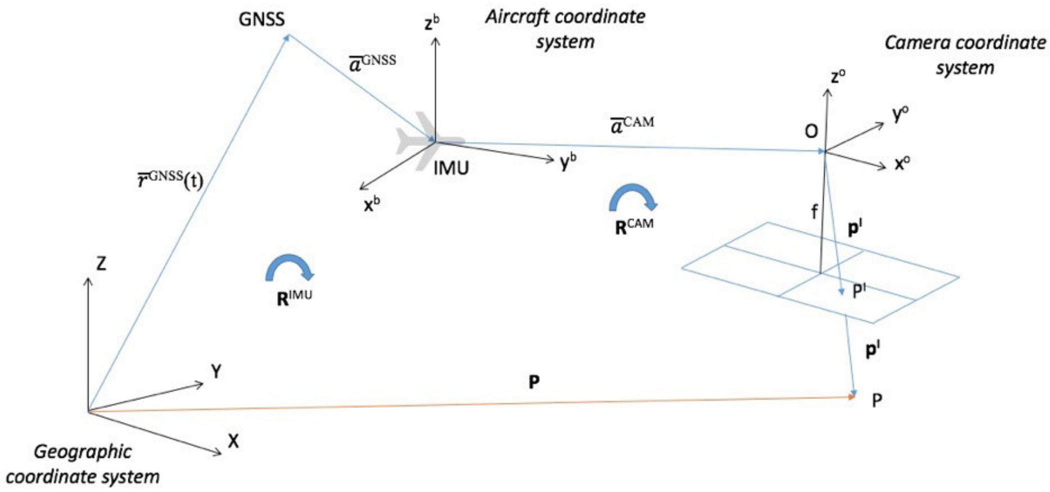

Figure 1. The model of direct georeferencing

The orientation of the camera relative to the aircraft ($\mathbf{R}^{CAM}$ rotation matrix) may also be vary. Such a case if the camera installed on gimbal. For simplification, these two rotation matrices are handled in one rotation matrix and all exterior orientation parameters transformed to the projection center.

III. Errors in direct georeferencing

Resulting errors of direct georeferencing come from interior and exterior orientation of images. In conventional photogrammetric processing using aerial triangulation errors occurred in measuring GCPs transmitted to the exterior orientation parameters by error propagation during the calculation process.

In direct georeferencing, besides the errors of interior orientation, errors of sensors can be detected directly in exterior orientation parameters. These errors coma from errors of GNSS unit and errors of IMU, mainly from gyroscope. In terms of accuracy the gyroscope is the most critical part of the inertial system.

Errors of interior orientation can also be different:

- *focal length error,* caused by distortion of the lens due to changes in pressure and temperature,
- *error of principal point,* that can be resulted by geometry error of lens caused by defect in manufacturing the lens,
- *distortion,* caused by geometry and material inhomogeneity of lens,
- *pixel error*, caused by measurement error of image point.

Fortunately, majority of interior errors can be eliminated or reduced by calibration.

IV. Solutions

In direct georeferencing determination of external orientation parameters is realized at the same time as exposing the photo by measuring data from external sensors (GNSS and IMU). In this way the photogrammetric process is considerable simplified, essentially the main task is the restoration of spatial data of terrain points (or point cloud), which is usually the last step in the conventional photogrammetric process using triangulation and measurement of GCPs.

Spatial data for point is restored by processing stereo image pairs. However, one terrain point can be represented on multiple images, so multiple stereo image pairs can be processed. It means, that for a terrestrial object point multiple coordinates are calculated. The result should be calculated by performing least square adjustment. In the following it the paper deals with only the calculation of stereo image pairs.

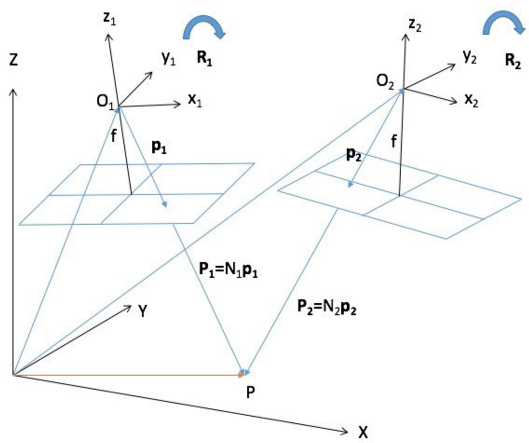

Figure 2. Theoretical model of direct georeferencing

A. Solving by geometrical method

Knowing exterior orientation data position of terrestrial point can be calculated by using spatial intersection from two projection centers. Due to the collinear criterion, the P position vector to the P terrestrial point can be calculated by the following (1) formula:

$$\boldsymbol{P} = \boldsymbol{P_o} + N\boldsymbol{p} \tag{1}$$

Where $\boldsymbol{P}$ *is a position vector,* $\boldsymbol{P_o}$ ais a position vector of projection center, *N is the scale factor,* $\boldsymbol{p}$ is the position vector of the terrestrial point represented on the image, transformed into the geographic coordinate system.

The transformation between the geographic coordinate system and the image coordinate system is provided by the rotation matrix. Image coordinates of the object *(*ξ, η*)* can be measured from images, and the focal length *(f)* is also known. Exterior orientation parameters measured by sensors. The above formula (1) in matrix form is the following:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_{O1} \\ Y_{O1} \\ Z_{O1} \end{bmatrix} + N_1 \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \eta_1 \\ -f \end{bmatrix}$$
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_{O2} \\ Y_{O2} \\ Z_{O2} \end{bmatrix} + N_2 \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} \xi_2 \\ \eta_2 \\ -f \end{bmatrix} \tag{2}$$

In the equation system consisting of six equations there are five variables: X, Y, and Z represent the coordinates of P terrestrial point, N_1 and N_2 represent scale factors for images. It means that the system of equation is overdetermined and having solved the result is an approximation. The r_{mn} coefficients are known and calculated from rotation angles consisting of trigonometric expressions.

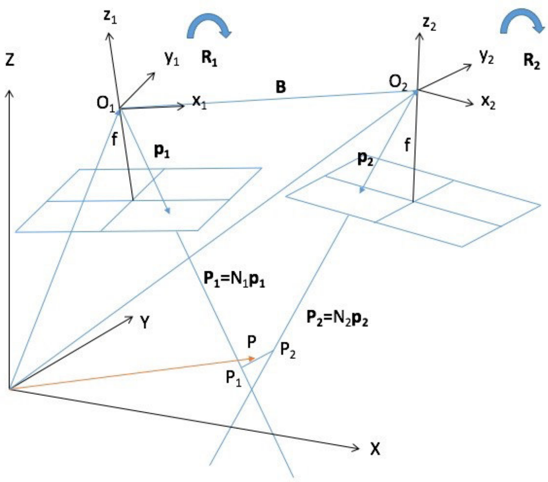

Figure 3. The real model in case of non-intersecting projection rays

It should also take in account that due to measurement errors (in exterior and interior orientation) the sights of projection do not intersect is reality (see Fig. 3.). The aim is to find solution where the point is as close as possible to the two projection lines.

In this paper there are two possible way of solution is presented. The firs one is the most commons method known from professional sources [1]. It is based on geometric approach.

According to the collinearity criterion, vector of the image point and vector of the terrestrial point are aligned is a straight line (projection ray). It means that cross product of these two vectors is zero.

Having converted the formula, the N scale factor can be expressed by the following way:

$$\begin{gathered} \boldsymbol{p_2} \times \boldsymbol{P_2} = 0 \\ \boldsymbol{p_2} \times (N_1\boldsymbol{p_1} - \boldsymbol{B}) = 0 \\ \boldsymbol{p_2} \times N_1\boldsymbol{p_1} - \boldsymbol{p_2} \times \boldsymbol{B} = 0 \\ N_1 = \frac{|\boldsymbol{p_2} \times \boldsymbol{B}|}{|\boldsymbol{p_2} \times \boldsymbol{p_1}|} \end{gathered} \tag{3}$$

Three N_1–N_2 pairs can be calculated by this method. In the case of $X=X_1=X_2$, $Z=Z_1=Z_2$ (in the XZ plain) N scale factors resulted by expressing cross product by coordinates:

$$N_1 = \frac{B_x z_1 - B_z x_2}{x_1 z_2 - z_1 x_2},\ N_2 = \frac{B_x z_1 - B_z x_1}{x_1 z_2 - z_1 x_2} \tag{4}$$

Having resulted N_1 and N_2 scale factors coordinates of object point (X, Y, Z) are calculated:

$$\begin{gathered} X = X_1 = X_2 = X_{O1} + N_1 x_1 = X_{O2} + N_2 x_2 \\ Z = Z_1 = Z_2 = Y_{O1} + N_1 z_1 = Y_{O2} + N_2 z_2 \\ Y_1 = Y_{O1} + N_1 y_1,\ Y_2 = Y_{O2} + N_2 y_2 \end{gathered} \tag{5}$$

In the real practice only one N1–N2 pair is usually calculated. In this recent case the Y coordinate is resulted by averaging the calculated Y_1 and Y_2 coordinates:

$$Y = \frac{Y_1 + Y_2}{2} \tag{6}$$

Accuracy can be improved by calculating multiple N-N pairs and averaging the coordinate values. But for computational economy reasons, this step is mostly ignored.

In visual approach the above means that task is solved in the XY coordinate plain where projected images of spatial projection rays intersect each other. In this case the problem has been simplified to a two-dimensional problem.

Suppose that the best solution using geometric approach if transverse between two projection rays is calculated. The ideal theoretical intersection of non-intersecting spatial lines lies at half of the transverse.

B. Solving by numerical method

Another solution is to define an equation system with the introduction of a residual as an unknown then using iteration while minimizing the residual.

To fulfill the numerical solution a similar method was chosen, taking advantages of MATLAB numerical computing environment. Adjusting slightly equations from collinear condition a system of linear equations is resulted. This is a more formal solution and well-managed by MATLAB. It also was a test to use MATLAB for solution photogrammetric tasks.

MATLAB as a first step to solve the system of linear equations is trying to solve it as a definite equation system. If neither the Cholesky nor the Gaussian method (decomposition of coefficient matrix) has been successful, MATLAB will solve equation system by using QR decomposition and iteration, minimizing residual vector. It is possible that MATLAB provides a solution even is case if it cannot be solved, therefore it is recommended to check the magnitude of residual vector.

Multiplying rotation matrix and image point position vector (see Eq. 2.) a new coefficient vector has been introduced containing c_{mn} coefficient, the following formula is resulted:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - N_1 \begin{bmatrix} c_{11} \\ c_{12} \\ c_{13} \end{bmatrix} = \begin{bmatrix} X_{O1} \\ Y_{O1} \\ Z_{O1} \end{bmatrix}$$
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - N_2 \begin{bmatrix} c_{21} \\ c_{21} \\ c_{23} \end{bmatrix} = \begin{bmatrix} X_{O2} \\ Y_{O2} \\ Z_{O2} \end{bmatrix} \quad (7)$$

Equations are written separately:

$$\begin{aligned} X - c_{11}N_1 &= X_{O1} \\ Y - c_{12}N_1 &= Y_{O1} \\ Z - c_{13}N_1 &= Z_{O1} \\ X - c_{21}N_2 &= X_{O2} \\ Y - c_{21}N_2 &= X_{O2} \\ Z - c_{21}N_2 &= X_{O2} \end{aligned} \quad (8)$$

Equation system is expressed in general $Ax = b$ form:

$$\begin{bmatrix} 1 & 0 & 0 & -c_{11} & 0 \\ 0 & 1 & 0 & -c_{12} & 0 \\ 0 & 0 & 1 & -c_{13} & 0 \\ 1 & 0 & 0 & 0 & -c_{21} \\ 0 & 1 & 0 & 0 & -c_{22} \\ 0 & 0 & 1 & 0 & -c_{23} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ N_1 \\ N_2 \end{bmatrix} = \begin{bmatrix} X_{O1} \\ Y_{O1} \\ Z_{O1} \\ X_{O2} \\ Y_{O2} \\ Z_{O2} \end{bmatrix} \quad (9)$$

The solution of $Ax = b$ system of linear equations in MATLAB is performed by using the command $x = A \setminus b$. The residual error can also be calculated: $r = Ax - b$

V. Calculation example

To compare the two methods, sample date was used taken from paper published on the internet [2]. Parameters of images of pair:

Projection center (UTM coordinates in meter)

$$\mathbf{O_1} = \begin{bmatrix} 432588.64254 \\ 4921230.83708 \\ 1550.10375 \end{bmatrix}$$

$$\mathbf{O_2} = \begin{bmatrix} 433038.78510 \\ 4921222.37288 \\ 1550.44516 \end{bmatrix}$$

Rotation angles in radians

$\Omega_1 = -0.000716$
$\Phi_1 = -0.000678$
$K_1 = -0.027806$

$\Omega_2 = -0.001074$
$\Phi_2 = -0.000923$
$K_2 = -0.013588$

Focal length

f = 100 mm

Image coordinates

$$\mathbf{p_1} = \begin{bmatrix} 4018.444\text{e}-006 \\ 4018.444\text{e}-006 \end{bmatrix}$$

$$\mathbf{p_2} = \begin{bmatrix} -31650.000\text{e}-006 \\ 76907.020\text{e}-006 \end{bmatrix}$$

Coordinates of point P_1, P_2 and P (location vectors) calculated by analytical method:

$$\mathbf{P_1} = \begin{bmatrix} 432664.847020694 \\ 4922168.468291757 \\ 323.811866059 \end{bmatrix}$$

$$\mathbf{P_2} = \begin{bmatrix} 432664.847020694 \\ 4.922168.836508479 \\ 323.811866059 \end{bmatrix}$$

$$\text{P} = \begin{bmatrix} 432664.847020694 \\ 4922168.652400119 \\ 323.811866059 \end{bmatrix}$$

Difference vector between point P_1 and P_2:

$$P_1—P_2 = \begin{bmatrix} 0 \\ -0.368216722272336 \\ 0 \end{bmatrix}$$

It can be seen, that there is a difference only in the Y coordinates, since in the calculation we started from the conditions of $X=X_1= X_2$ and $Z=Z_1=Z_2$. This means a difference of 36,8 cm in the Y coordinate.

The coordinate values of the P point calculated by the numerical solution of system linear equations:

$$P_{num} = \begin{bmatrix} 432664.857654643 \\ 4922168.687976455 \\ 323.765144622 \end{bmatrix}$$

The residual error vector is also acceptable:

$$r = \begin{bmatrix} 0.002209184225649 \\ 0.116025147959590 \\ 0.088850910886549 \\ -0.002209184400272 \\ -0.116025149822235 \\ -0.088850910887686 \end{bmatrix}$$

The minimum distance between the two projection lines was also calculated using coordinate geometry method. It is a transverse between two lines, perpendicular to both spatial lines. The best solution for spatial intersection is the transverse halfway point (P_{TR}). Calculating the coordinates, the location vector of the P_{TR} point is the following:

$$\mathbf{P_{TR}} = \begin{bmatrix} 432664.857654643 \\ 4922168.687976455 \\ 323.765144622 \end{bmatrix}$$

In the case of analytical solution, the difference of point P from PTR:

$$\mathbf{P—P_{TR}} = \begin{bmatrix} -0.010633948782925 \\ -0.035576337017119 \\ 0.046721436621510 \end{bmatrix},$$

the magnitude of this vector is 0,05967 m.

In the case of numerical solution, the difference of point P_{num} from point P_{TR}:

$$\mathbf{P_{num}—P_{TR}} = \begin{bmatrix} -0.058207660913467 \\ -0.931322574615478 \\ -0.012732925824821 \end{bmatrix} \cdot 10^{-9}$$

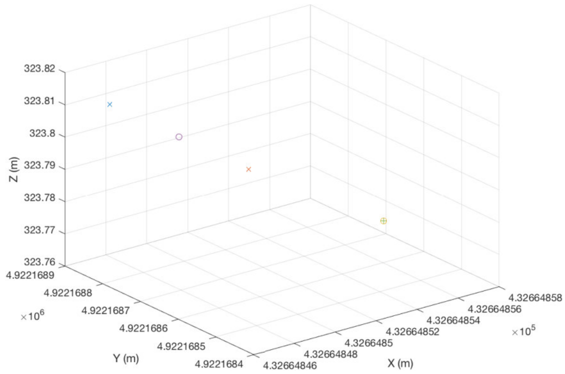

Figure 4. Graphical representation of calculated object point. From left to right: P_1, P, P_2, P_{TR} and P_{num} shown together.

VI. Conclusions

Direct georeferencing is a cost-effective and fast method in photogrammetry that is increasingly used in aerial survey. Its accuracy nowadays already is acceptable for aerial mapping or 3D point cloud generation. Solving the problem results in an overdetermined system of equations, so we only get an approximate solution to the point of intersection of the non-intersecting projection rays.

There are two methods were presented. One is an analytical solution based on geometric similarity. The other is to solve the system of equations as a system of liner equations by numerical method. The midpoint of the transverse of the two projecting lines is considered the solution. Deviation of the analytical solution from this optimal point is only 6 cm. Given that the solution of the linear system of equations also involves error minimization at the iteration, as expected, the result is practically the same as the best theoretical solution. This means that both solutions provide acceptable accuracy in practice. According to the preliminary assumption, the linear system of equations can be suitable for the simultaneous calculation of several images, but the solution of the system of equations requires further investigation.

To solve this problem, MATLAB was used, whose basic data format is the matrix. Experience shows that MATLAB is a very flexible and efficient tool for solving photogrammetric problems.

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