The new algorithm for handwritten characters recognition based on the digital image conversion into planar graph

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***Abstract*—The paper presents the new algorithm for handwritten characters recognition. The algorithm is based on the procedure for converting an image into a planar graph, where the lines in the image correspond to paths in a flat graph, that is, a sequence of graph vertices connected by edges. A special function is used to determine the shape of a simple smooth curve without self-intersections. This function is invariant with respect to such transformations of the plane as rotation and scaling, which makes it very convenient for recognizing handwritten characters. The mathematical construction of the function of the angular characteristics of the curve can easily be transferred to simple broken lines. The window vector Fourier transform coefficients are used as feature vectors to identify polylines corresponding to paths in a flat graph.**

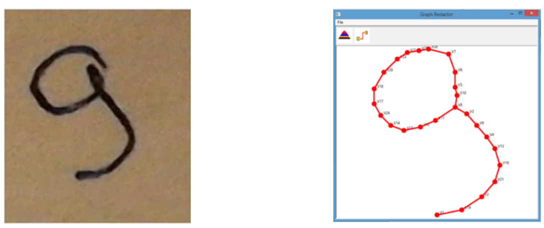
Keywords— line thin, line segment, gradient, image analysis, handwritten characters recognition, planar graph

INTRODUCTION

At present, the theory of computer vision is a rapidly developing interdisciplinary field of research. The high practical relevance of research in this area is primarily associated with its applications to robotics and the problems of automation of production. At the same time, solving problems in this area find important applications in other areas of human activity, in particular in medicine and education. Character recognition is one of the most important tasks of the theory of computer vision, for many years attracting specialists in the field of applied mathematics, artificial intelligence and computer science. The general task of character recognition is usually divided into the task of recognizing printed and handwritten characters. Due to the great practical importance (primarily for the automation of office work) and comparative simplicity, the recognition of printed characters has been well studied, while the recognition of handwritten characters has been considered one of the most interesting and complex image analysis tasks in recent years [1-5]. For the recognition of handwritten characters, as well as for solving the more complex and general task of handwriting recognition, various authors have proposed many methods (a fairly complete review of the literature on this subject can be found in [1-5]), but it is recognized that so far none of the proposed methods can not be compared either in reliability or in the speed of recognition with a human.

RESULTS AND DISCUSSION

Purposed method of handwritten symbols recognition is based on digital image to planar graph conversion procedure. According to the definition of a planar graph, there is a one-to-one mapping of the set of vertices of the graph to the set of points on the plane. Accordingly, each graph edge corresponds to a line segment in the plane, and the segments corresponding to different edges can intersect only at the end points. Each path in the graph corresponds to a simple polyline on the image plane. Suppose we have a digital photo of handwritten symbols drawn with thin lines (for example, written in pen on paper). After applying the conversion procedure to this image, each isolated symbol will correspond to a connected component of the resulting planar graph. The curves forming the images of the symbol will be approximated bypolylines (simple or closed) corresponding to the paths and cycles in the graph (see Fig.1). We have developed two algorithms for converting a raster image to a planar graph. We developed software that implements these algorithms and used it for experiments with character recognition of decimal digits on real photos. Immediately, we note that the input data for character recognition procedures, including the feature extraction procedure, are the connected components of the planar graph. Thus, the planar graph obtained as a result of applying the conversion procedure contains all the necessary information for recognition, and after converting the image into a planar graph, the original image is not used in the recognition procedures.



a b

Fig. 1. A fragment of a digital photo (a) and a screenshot of the "GraphRedactor" window (b) with the image of the planar graph of the digit symbol "9" obtained when converting an image into a graph.

Another key idea of the proposed handwriting recognition method is the use of a special function to identify the shape of a simple smooth curve without self-intersections. We called this function the "curve angular characteristic function." This characteristic is invariant with respect to such transformations of the plane as rotation and zooming, which makes it very convenient for use in recognizing handwritten characters. The mathematical construction of the function of the angular characteristics of the curve is easily transferred to simple broken lines. We use the vector coefficients of the window Fourier transform as a feature vector for identifying the polylines corresponding to the paths in the planar graph. The concept of the function of the angular characteristic of a curve is discussed in detail in the section “Function of the angular characteristic of a curve”. Purposed optical handwritten symbol recognition method is divided into five phases which are digital image preprocessing, image to planar graph conversion, graph analysis, feature extraction and classification (see Fig. 2). In this paper, we will focus primarily on the feature extraction procedure, which includes the analysis of a planar graph

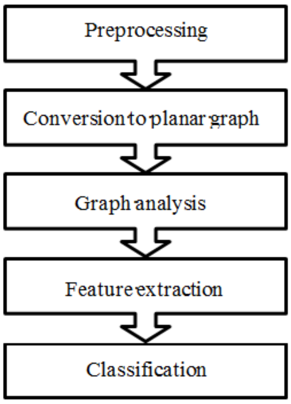


Fig. 2. Block Diagram of handwritten symbol recognition.

A. *Graph analysis. Main concepts*

There are two types of geometric primitives that we can directly classify using the methods proposed in this paper, namely, curves without self-intersections with two ends and closed curves without self-intersections. (Note that the first of these types of objects is topologically equivalent (homeomorphic) to the D^1 , and the second type is homeomorphic to S^1.). In the planar graph of the symbol, the objects of the first type correspond to the paths between the vertices of the graph, and the objects of the second type correspond to the cycles of the graph. Some symbols of decimal digits (for a specific spelling) are objects of the first group (for example, numbers 1 2 3 5), and their planar graphs are trees with two leaves. Obviously, if the planar graph is a tree whose vertices v_1 and v_2 are leaves and the graph has no other leaves, then a broken line corresponding to the path in the graph connecting the vertices v_1 and v_2 will be the most suitable object for classifying. But even if the connected component of a planar graph is a tree, then in more complex cases, the choice of paths between the vertices to classify this component becomes a task that allows for many solutions. For example, planar graphs of decimal digits 4 and 7 (in some cases of spelling) are a ‘cross’, i.e. a tree with 4 leaves, only one vertex of which A has a degree greater than 2 (more precisely $deg(A) = 4$) (see Fig. 3). In any case, it is necessary to apply some combinatorial procedures to select a classification scheme for such objects. For example, we can choose two unordered pairs from the set of the leaves of this tree, and use paths between pairs of vertices (B, D) and (E, C) for recognition. But it is clear that even if these paths are recognized as elements of the digit "4", this does not at all mean that we can classify this planar graph as the symbol of the digit "4". It is necessary to carry out additional procedures, based on the results of the first stage of classification, namely, to classify paths between the vertices (B, E) and (D, C).

Thus, the results of the graph analysis stage determine the rules by which the classification procedure will be conducted. Graph analysis can be called a topological analysis of a planar graph. In some cases, the information obtained as a result of this analysis in combination with the results of the classification of geometric primitives may not be sufficient to classify a symbol as a whole.

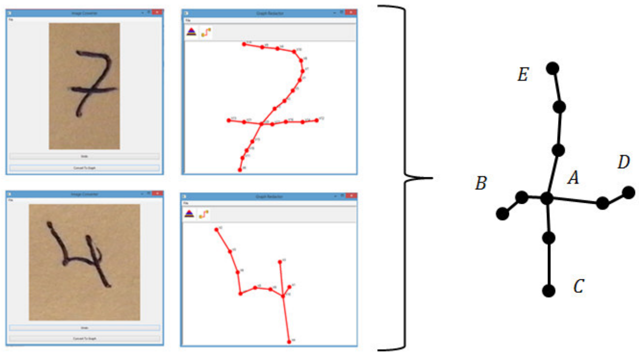


Fig. 3. Symbols of decimal digits ‘7’ and ‘4’ has same planar graph topology

In such cases, the application of ‘metric’ algorithms of computational geometry, such as calculating areas of polygons, calculating the angles between lines, etc., are required. But in most cases (when recognizing digit characters), the role of such methods of computational geometry is insignificant, compared to the value of graph analysis methods and the recognition of geometric primitives. Thus, the fundamental role of the graph analysis procedure is to choose the recognition scheme for the connected component of the planar graph. But during the analysis of the graph, the procedures for eliminating artifacts that arise when converting an image into a planar graph are also performed. As mentioned above, we assume that each symbol corresponds to a connected component of a planar graph. Of course, the first stage of image graph analysis is to find the connected components of this graph, and further graph analysis procedures are applied to each component. Below we describe the procedures for constructing the spanning tree of the connected component of the graph and generating cycles. In the course of these procedures, the elimination of artifacts is carried out and it is determined whether the component is a tree, or a graph having cycles. Below we describe the procedures for constructing the spanning tree of the connected component of the graph and generating cycles. These procedures are the first stage of analyzing a connected component of a planar graph. During these procedures, the elimination of artifacts is carried out and it is determined whether the component is a tree, or a graph that has cycles. Further graph analysis procedures related to the choice of a scheme for recognizing the corresponding connected component are rather complicated and require a lot of space for their detailed description. At the same time, the development of these procedures can be considered a technical task, since quite elementary and well-known methods are used to solve it. In this regard, we do not consider it appropriate to describe these procedures in this article

II. ANGULAR CHARACTERISTIC FUNCTION

A. *Angular characteristic function:introduction*

The function of the angular characteristics of the curve completely determines the geometric shape of this curve. More precisely, this function can be viewed as a mapping of the set of regular curves to the functional space $D \subset L^2$of functions defined on the interval $I = [0,1]$. As shown in the “Properties of angular characteristics function” section, this mapping is invariant with to plane similarity transformations. The concept of the function of the angular characteristics is easily transferred to broken lines. The functions of the angular characteristics of a broken line can be put in accordance with the feature vector (the procedure for extracting features is described in detail in the “feature extraction” section), which allows to classify it. As mentioned above, the paths in the planar graph correspond to broken lines on the plane, which explains the key role of the concept of the function of the angular characteristics in the proposed method of recognizing handwritten characters.

B. *Definitions*

We will denote the set of vectors of unit length by E, so $E = \{\vec{e} \in \mathbb{R}^2 |\ |\vec{e}| = 1\}$ .The oriented area of the parallelogram spanned by vectors $\vec{a}, \vec{b} \in \mathbb{R}^2$, we will denote by$S(\vec{a}, \vec{b})$. For an arbitrary Cartesian system on the plane, mapping $S: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$can be determined analytically by the formula (1)

$$S(\vec{a}, \vec{b}) = a_x \cdot b_y - a_y \cdot b_x \tag{1}$$

, where $\vec{a} = (a_x, a_y)$,$\vec{b} = (b_x, b_y)$. If $\vec{e}_1, \vec{e}_2 \in E$, then

$$S(\vec{e}_1, \vec{e}_2) = \sin(\varphi) \tag{2}$$

, where φ is the angle to turn the vector $\vec{e}_1$ counterclockwise so that it coincides with the vector $\vec{e}_2$. We will denote mapping $E \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, defined by (3) as$\angle(\vec{e}_1, \vec{e}_2)$.

$$\angle(\vec{e}_1, \vec{e}_2) = arcsin(S(\vec{e}_1, \vec{e}_2)) \tag{3}$$

In view of the equality $\sin(\alpha) = \sin(\pi - \alpha)$, it would be incorrect in the general case to consider $\angle(\vec{e}_1, \vec{e}_2)$ as the angle between the vectors $\vec{e}_1$and $\vec{e}_2$.Below we provide the exact definition of the concept of ‘angle between vectors’ used in this article.For two nonzero vectors $\vec{a}, \vec{b} \in \mathbb{R}^2$ we determine the value δ by formula (4)

$$\delta = arccos(\vec{e}_a * \vec{e}_b) \tag{4}$$

where $\vec{e}_a$ is normalized vector $\vec{a}$ (i.e. $\vec{e}_a = \frac{\vec{a}}{|\vec{a}|}$) and $\vec{e}_b$ is normalized vector $\vec{b}$ (i.e. $\vec{e}_b = \frac{\vec{b}}{|\vec{a}|}$). Then we define the function Ang as (5)

$$Ang(\vec{a}, \vec{b}) = \begin{cases} \delta, if S(\vec{a}, \vec{b}) \geq 0 \\ -\delta, if S(\vec{a}, \vec{b}) < 0 \end{cases} \tag{5}$$

C. *Function of the angular characteristic of a smooth regular curve.*

Let γ be a regular curve with end points A and B. If we choose one of the end points (for example, A), then we can parameterize γ as follows: For an arbitrary point $M \in \gamma$, we define the corresponding value of the parameter s as the length of the curve segment AM . Thus, we define the mapping $[0, L] \to \gamma$, where L is the total length of the curve γ. For a given Cartesian coordinate system, this mapping defines two functions $x(s)$and $y(s)$. In the case of a smooth regular curve under consideration, both functions $x(s)$and $y(s)$ are differentiable. Let the mapping $[0, L] \to \mathbb{R}^2$ be given by formula (6)

$$\vec{v}(s_0) = \left(\left(\frac{dx}{ds}\right)_{s=s_0}, \left(\frac{dy}{ds}\right)_{s=s_0}\right) \tag{6}$$

As is known,for arbitrary $s \in [0, L]$ vector $\vec{v}(s) \in E$ is a tangent vector of the curve γ. It should be noted that the vector $\vec{v}(s)$ can be represented as (7)

$$\vec{v}(s) = (\cos(\alpha), \sin(\alpha)) \tag{7}$$

where α is the angle between the tangent line and the X axis (see Fig. 4).

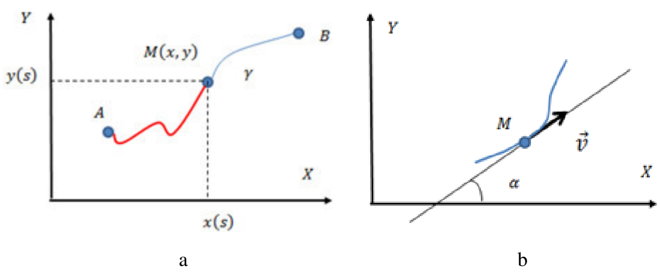


Fig. 4. Natural parameterization of regular curve. Parameter s is a length of the segment AM(left image), and tangent line (right image).

For an arbitrary point $M(s_0) \in \gamma$, we define $k(s_0)$ as (8):

$$k(s_0) = lim_{\Delta s \to 0} \frac{\angle(\vec{e}(s_0 + \Delta s), \vec{e}(s_0))}{\Delta s} \tag{8}$$

It should be noted that $|k(s_0)| = \frac{1}{R}$, where R is the radius of the tangent circle, is the curvature of the curve γat point $M(s_0)$. We define the function $\alpha(s)$ by the formula (9):

$$\alpha(s) = \int_0^s k(\tau)\, d\tau \quad (9)$$

Assuming some uncertainty, we can give the following clear geometric definition of the function $\alpha(s)$:Let $\vec{v}_0 = \vec{v}(0)$ be a tangent vector to γat endpoint A. Then $\alpha(s)$ is the angle between the vectors $\vec{v}_0$ and $\vec{v}(s)$ (see Fig.2).The function $\alpha(s)$is defined on the interval$[0, L]$, where L is the length of the curveγ. We define on the interval $I = [0,1]$ the function $\theta(x)$:

$$\theta(x) = \alpha(x \cdot L) \quad (10)$$

We will call $\theta(x)$ the function of the angular characteristic of the curve γ (or simply the angular characteristic of γ). As will be shown below, this function has properties that allow it to be effectively used in symbol recognition tasks.

D. Properties of an angular characteristic function.

1) Invariance to similarity transformations of the plane.

The most important property of the function of angular characteristics is its invariance to similarity transformations of the plane. Below we provide the necessary explanations: A similarity is a transformation of Euclidean plane which maps lines to lines and preserves the sizes of angels. The set of all similarities is the similarity group $S(2)$. Each similarity transformation can be viewed as a composition of translation, rotation, and uniform scaling. Let γ be a regular curve with endpoints A and B . Applying an arbitrary transformation $f \in S(2)$ to the plane transforms γ into a regular curve γ' with endpoints $f(A)$and $f(B)$. Thus, if θ_1 is the function of the angular characteristic of γ, then it corresponds to the function of the angular characteristic θ_2 of the curve γ'. The statement about the invariance of the function of the angular characteristic to similarity transformations means that equality (11) holds.

$$\forall x \in [0,1]\theta_1(x) = \theta_2(x) \quad (11)$$

2) If the curve γ is a line segment, then its function of the angular characteristic $\theta$of the γ is defined by (12)

$$\forall x \in [0,1]\theta(x) = 0 \quad (12)$$

3) If the curve is an arc of a circle cut off by an angle φ, then regardless of the radius of the circle, the function of the angular characteristics of this curve will be a linear function (13):

$$\theta(x) = \varphi \cdot x \quad (13)$$

It should be noted that the curves forming the symbol in the image can be approximated by curves made up of segments and arcs of a circle. The last two properties of the angular characteristics function make it easy to imagine the angular characteristics of these curves. In addition, the idea of approximating curves by joints of segments and arcs suggests an idea of using segmentation methods to recognize curves.

E. Function of angular characteristic of broken line.

We denote a broken line by a sequence of vertices $\{A_0, A_1, \dots, A_n\}$. Let (x_i, y_i) denote Cartesian coordinates of the i -th vertex. Then the broken line corresponds to a sequence of n vectors $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ given by equalities (14).

$$\vec{v}_i = (x_i - x_{i-1}, y_i - y_{i-1}) \quad (14)$$

We define the sequences $\{l_1, l_2, \dots, l_n\}$ and $\{\varphi_1, \varphi_2, \dots, \varphi_n\}$. by equalities (15) - (16).

$$l_i = |\vec{v}_i| \quad (15)$$

$$\varphi_i = \begin{cases} 0, if\ i = 1 \\ Ang(\vec{v}_i, \vec{v}_{i-1}), if\ i \neq 0 \end{cases} \quad (16)$$

Let $x_0, x_1, \dots, x_n$be a sequence of points on the X axis given by the equations (17)

$$x_i = \begin{cases} 0, if\ i = 0 \\ \sum_{k=1}^{i} l_k\, , if\ i \neq 0 \end{cases} \quad (17)$$

So, the interval $[0, L]$, where $L = \sum_{i=1}^{n} l_i$ is the length of broken line $\{A_0, A_1, \dots, A_n\}$, contains all points $x_0, x_1, \dots, x_n$. Now we define on the interval $[0, L]$ the step function α by (18)

$$\alpha(x) = \sum_{i=1}^{n} \varphi_i \cdot \chi_{D_i}(x) \quad (18)$$

,where $\chi_{D_i}(x)$ is the indicator function of interval D_i (19),and sequence of intervals $\{D_1, D_2, \dots, D_n\}$ is defined by equations (20).

$$\chi_{D_i}(x) = \begin{cases} 1\ if\ x \in D_i \\ 0\ if\ x \notin D_i \end{cases} \quad (19)$$

$$D_i = \begin{cases} [x_{i-1}, x_i)\ if\ 1 \leq i < n \\ (x_{i-1}, x_i]if\ i = n \end{cases} \quad (20)$$

The function of the angular characteristic θ of a broken line will be determined (as for a regular smooth curve, see (10)) by equation $\theta(x) = \alpha(x \cdot L)$.An example of a broken line (strictly speaking, a planar graph representing a tree with two leaves) and the corresponding function of the angular characteristic is shown below (see Fig. 5)

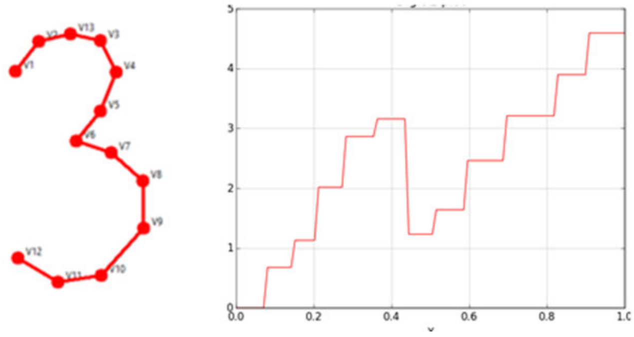


Fig. 5. Broken line (planar graph of real image) and plot of its angular characteristic function.

As Ψ is a piecewise continuous bounded function it is possible to define its absolute value $\|\Psi\|$ by (22)

$$\|\Psi\| = \sqrt{\int_{-1}^{1} \Psi^2(x)\, dx} \tag{22}$$

Let $\Omega = L^2([-1,1])$be a L^2 space of functions, defined on interval $[-1,1]$. Then ,$\Psi \in \Omega$, due to the fact that the norm $\|\Psi\| < \infty$ is defined by (2). As Ω is a L^2 space, $\Omega$is a Hilbert space, where inner product of $f_1, f_2 \in \Omega$ is defined by (23):

$$\langle f_1, f_2 \rangle = \int_{-1}^{1} f_1(x) \cdot f_2(x) dx \tag{23}$$

Each even function $g \in \Omega$ can be represented as a Fourier series in form (24)

$$g(x) = \sum_{i=1}^{\infty} c_i \cdot \varphi_i(x) \tag{24}$$

where functions φ_i is defined by equations (25)

$$\varphi_i(x) = \begin{cases} \frac{1}{\sqrt{2}}\ if\ i = 1 \\ \cos(\pi \cdot i \cdot x)\ if\ 1 < i \end{cases} \tag{25}$$

The system of functions (25) is orthonormal, which means that (26) holds true.

$$\forall i, j\ \langle \varphi_i, \varphi_j \rangle = \delta_{ij} \tag{26}$$

The coefficients c_1, c_2 ... of Fourier series expansion (24) are defined as the projections (27) of the vector $g \in \Omega$ onto the vectors of the orthonormal basisφ_1, φ_2 ...:

$$c_i = \langle g, \varphi_i \rangle = \int_{-1}^{1} g(x) \cdot \varphi_i(x) dx \tag{27}$$

F. Feature extraction.

An arbitrary function$f$ defined on an interval$[a, b]$ can be considered as a periodic function $\forall x \in \mathbb{R}\, f(x + T) = f(x)$with a period $T = b - a$.By definition, the function of the angular characteristics θ of arbitrary broken line is defined on the unit interval $I = [0,1]$and $\theta(0) = 0$, but in the general case $\theta(1) \neq \theta(0)$ .Consequently, a periodic function θ in the general case will have jump discontinuities. Due to Gibbs phenomenon, the partial sum of the Fourier series will be poorly approximated to the original function θ .To solve this problem, we use the following method: For a given function of angular characteristic θ, we define an even function Ψ defined on the interval $[-1,1]$ by equation (21)

$$\Psi(x) = \theta(1 - |x|) \tag{21}$$

For a given N, the N-th partial sum of the Fourier series (28) will approximate the original function.

$$g(x) \approx \sum_{i=1}^{N} c_i \cdot \varphi_i(x) \tag{28}$$

We will consider the real numbers $c_1, c_2 \ldots c_N$, as components of the vector $\vec{g} \in \mathbb{R}^N$,so in some orthonormal basis of $\mathbb{R}^N \vec{g} = (c_1, c_2 \ldots c_N)$. Let designate by $g \to \vec{g}$ correspondence between even function $g \in \Omega$ and vector $\vec{g} \in \mathbb{R}^N$ of Fourier series coefficients. Then, for arbitrary even functions $f_1, f_2 \in \Omega$ approximations (29)-(30) holds true.

$$\langle f_1, f_2 \rangle \approx \vec{f_1} * \vec{f_2}(\tag{29}$$

$$\|f - g\| \approx |\vec{f} - \vec{g}| \tag{30}$$

Both Hilbert space and Euclidean space are metric spaces. A metric in space (a set of points D) may be defined by the distance function d, defined for all pairs of points $A, B \in D$, which satisfies four conditions:

1) $\forall A, B \in D\ \ d(A, B) = d(B, A)$
2) $\forall A, B \in D\ \ d(A, B) \geq 0$
3) $\forall A \in D\ \ d(A, A) = 0$
4) $\forall A, B, C \in D\ \ \rho(AB) \leq \rho(AC) + \rho(CB)$

As is known, in the Euclidean space $\mathbb{R}^N$, distance d between $\vec{A}, \vec{B} \in \mathbb{R}^N$ is defined as $d(\vec{A}, \vec{B}) = |\vec{A} - \vec{B}|$. Similarly, for a Hilbert space L^2 metric d^* is defined by equation (31), where $f_1, f_2 \in L^2$

$$d^*(f_1, f_2) = \|f_1 - f_2\| \tag{31}$$

It is proved that conditions 1) –4) are satisfied for the metric d^*. Suppose that for a given function of the angular characteristic θ, the function Ψ is determined by the ratio (21) and $\Psi \to \vec{\Psi}$. If we consider the vector $\vec{\Psi}$ as a featurevector of the Euclidean feature space, then based on the above considerations, it can be argued that such a method of feature extraction will be good suitable for curve classification tasks. But in practice we need to classify the step characteristic functions of broken lines, containing discontinuous jumps. It is highly desirable to smooth out such functions. To do this, we apply the operation described below to the vector $\vec{\Psi}$of the Fourier expansion coefficients of the function Ψ . Let denote by $g(x)$ Gaussian function, defined as,

$$g(x) = \frac{1}{\sqrt{2\cdot\pi\cdot\sigma}} \cdot e^{-\frac{x^2}{2\cdot\sigma^2}} \tag{32}$$

where $\int_{-\infty}^{\infty} g(x)\,dx = 1$,i.e. $g(x)$ is normalized. As g is an even function, equations (33)-(34), for arbitrary $\omega \in \mathbb{R}$ holds true.

$$\int_{-\infty}^{\infty} g(x) \cdot \cos(\omega \cdot x)\,dx = \int_{-\infty}^{\infty} g(x) \cdot e^{-i\omega x} dx \tag{33}$$

$$\int_{-\infty}^{\infty} g(x) \cdot cos(\omega \cdot x)\,dx = e^{-\frac{\sigma^2\cdot\omega^2}{2}} \tag{34}$$

For the values of the parameter σ, satisfying the non-equality$\sigma < \frac{2}{3}$, we can assume that$g(x) \approx x \; if \; x \notin [-1,1]$. Then the values g_k^* defined by (35) can be considered as the Fourier expansion coefficients of the function g^* , approximating the Gaussian function $g(x)$.

$$g_k^* = \begin{cases} 1 & if \; k = 0 \\ e^{-\frac{\sigma^2\cdot(k-1)^2}{2}} & if \; k > 1 \end{cases} \tag{35}$$

Let $\vec{\Psi} = (\varphi_1, \varphi_2, \dots, \varphi_N)$ be the vector of the Fourier coefficients of the function Ψ, and $\vec{g}^* = (g_1^*, g_2^*, \dots, g_N^*)$the vector of the Fourier coefficients of the functiong^*. We define the vector $\vec{\Psi}_s = (\varphi_{s1}, \varphi_{s2}, \dots, \varphi_{sN})$ by equalities (16).

$$\varphi_{sk} = \varphi_k \cdot g_k^* \tag{36}$$

According to the convolution theorem for Fourier series, we can consider the components of the vector $\vec{\Psi}_s$ as the Fourier expansion coefficients of the function $\Psi_s(x)$, which is a result of application of a filter with Gaussian kernel g^* to the function Ψ (37)

The choice of the number *n* and the values of the parameter *t* strongly affect the quality of recognition.

$$\Psi_s(x) = \int_{x-1}^{x+1} \Psi(t) \cdot g^*(t)\,dt \tag{37}$$

The most interesting result of this study is a practical confirmation of the applicability of the angular characteristic function for handwriting symbol recognition. Unfortunately, the limitations of this paper's volume did not allow including the discussion of a number of issues that arose during the research. In particular, this paper does not include a description of methods for eliminating artifacts that occur when converting an image into a planar graph, and methods for identifying symbols whose planar graphs contain loops. The authors are currently working on the angular characteristic function applying, particularly on segmentation of curves obtained by smoothing the step functions of the angular characteristic broken. Such segmentation will allow us to represent the curve as a sequence of geometric patterns - primitives (circular arcs, line segments, etc.). The recognition algorithm can be improved using segmentation methods, as well as applied to the recognition of a wider class of objects.

III. CONCLUSION

In our opinion, the most interesting result of the work is a practical confirmation of the applicability of the angular characteristic function for handwriting symbol recognition. Unfortunately, the limitations of the work volume did not allow us to include in it a discussion of a number of issues that arose during the work on the project. In particular, this work does not include a description of methods for eliminating artifacts that occur when converting an image into a planar graph, and methods for identifying symbols whose planar graphs contain loops. We are currently working on the father development of the idea of using the angular characteristic function, namely, we are trying to segment the curves obtained by smoothing the step functions of the angular characteristic broken. Such segmentation will allow us to represent the curve as a sequence of geometric patterns - primitives (circular arcs, line segments, etc.). We hope that our recognition algorithm can be significantly improved (in particular, using segmentation methods), as well as apply it to recognition of a wider class of objects.

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