Attila Bencze, Virág Horváth, Miklós Berta: Causality detection in magnetic confined fusion plasma parameters

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Abstract: In this paper we introduce information theory-based transfer entropy method to analyse tokamak data, specifically in connection to Edge Localised Mode instability. Using synthetic data, we first show how the transfer entropy approach can be superior to linear methods, then we analyse actual experimental data measured by Li-beam emission spectroscopy and by magnetic Mirnov-coils using information theoretical methods in order to quantify the strength and the direction of causal relation between plasma measurement time recordings.

Keywords: causality; fusion plasmas; transfer entropy

1. Introduction

Modern causality detection methods are widely applied in different scientific fields such as ecology [1], economic and financial sciences [2], neuroscience [3]. Unfortunately, little has been done in application of these methods in time series recorded in magnetically confined fusion plasmas [4].

The present contribution aims at expanding the field of application for Li beam emission spectroscopy (LiBES) signals and Mirnov coil recordings collected on COMPASS tokamak. Edge Localized Modes (ELMs) are periodic magneto-hydrodynamic instabilities which are seen in all diagnostic signals. The ELMs are usually described by the so-called “peeling-ballooning” model, where the “peeling” part refers to an electric current instability (Mirnov signal) while the “ballooning” part is a pressure driven instability (LiBES signal). The causal relation between these two parts is a question investigated by the present contribution using transfer entropy as the measure of directed connection between plasma density and magnetic field strength. The results are compared with the more conventional correlation analysis and mutual information calculations.

2. Methods

2.1. Information theory basics

Information as a fundamental concept is at the core of every science, often not explicitly stated, e.g., in quantum mechanics there is a zeroth axiom which says that the information is preserved and cannot be destroyed. Let’s assume that we have a discrete random variable X and one of its realizations x_i. If the probability of observing x_i is $p(x_i)$, the Shannon information [5] content of observing x_i is:

$$\text{Shannon information} = log_2 \frac{1}{p(x_i)} = -\,log_2\, p(x_i) \text{ bits.}$$

From the formula it is clear, that the more unlikely the outcome is the more information it carries. If the random variable X has m different outcomes with probabilities:

$p(x_1), p(x_2), \ldots p(x_i), \ldots p(x_m)$, the average value of information content is called Shannon entropy:

$$H(X) = -\sum_{i=1}^{m} p(x_i) \, log_2 \, p(x_i).$$

In order to discover causal connection between different processes, described by random variables X and Y, we can try to estimate the common information content present in both processes. This can be done using the joint probability distribution and constructing the so-called mutual information [6]:

$$I(X, Y) = H(X) - H(X|Y),$$

where $H(X|Y)$ represents the uncertainty (entropy) remaining in variable X after knowing the value of Y (conditional entropy). This measure is symmetric with respect to X and Y. It would be very useful to somehow quantify the direction of possible information transfer, if any, between X and Y. The concept of transfer entropy has been developed exactly towards this end [7]:

$$T_{x \to y} = H(Y_t|Y_{t-1:t-L}) - H(Y_t|Y_{t-1:t-L}, X_{t-1:t-L}),$$

where the first term $H(Y_t|Y_{t-1:t-L})$ is the uncertainty remaining in Y at time t (present) after knowing the past values (L long history length, number of time steps) of Y, while $H(Y_t|Y_{t-1:t-L}, X_{t-1:t-L})$ is the uncertainty remaining about the present of Y knowing both the L-long past of Y and X. In summary one can say that the transfer entropy quantifies the amount of uncertainty reduction in the future values of Y knowing the past values of X given the past values of Y.

2.2. Edge Localized Modes in magnetically confined plasmas

High confinement mode (H-mode) in magnetically confined toroidal plasmas (tokamaks for short) has been discovered decades ago making commercial fusion energy production a realistic possibility [8]. One important characteristic of the H-mode plasma is the presence of periodic plasma burst called Edge Localized Modes (ELMs) [9], which are basically MHD instabilities causing the relaxation and rebuilt of the H-mode pressure profile. Qualitatively the pealing-ballooning model of the ELM cycle consists of two processes, see Figure 1, first the pressure gradient increases until the point where a self-driven plasma current, called bootstrap current, is induced. As the bootstrap current increases a current instability develops and the pressure profile crashes, and the cycle starts again.

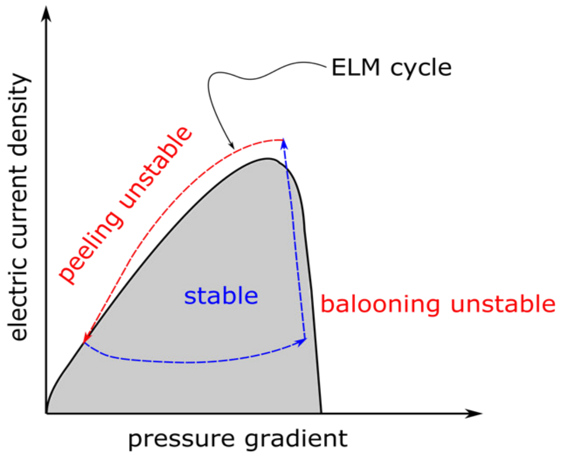

Figure 1 Stability diagram in the framework of peeling-ballooning instability model, showing the ELM cycle

From experimental/diagnostic point of view the effect of ELMs can be seen on almost all diagnostic signals. The most relevant measurement would be the simultaneous and fast local measurement of both plasma pressure and of the plasma current. Nowadays in tokamaks this is a difficult, if not impossible task, therefore some proxy measurements need to be performed in order to establish causal connection between different parts (quantities) of the model. In our case we use the Li-beam emission spectroscopy signal as the proxy of the local plasma density [10], and the Mirnov-coil signal [11] as the non-local approximation of the magnetic field strength.

3. Results

In this section we present the results of transfer entropy estimation in two different cases: first we use a simple nonlinear model to generate synthetic data to test the algorithm second, we apply this algorithm to real plasma signals.

3.1. Synthetic signal processing

As the most systems in nature, the fusion plasma is a complex nonlinear medium, therefore the usual linear approaches as cross-correlation and Granger causality analysis may lead to false conclusions. On the other hand, approaches based on information theory such as the transfer entropy can reveal nonlinear interactions even if the linear method fails. In order to demonstrate this point let's take two very simple nonlinearly connected random processes:

$$x_t = ax_{t-1} + b\epsilon_{x,t},$$

$$y_t = \mathrm{c}\sqrt{|x_{t-1}|} + \mathrm{d}\epsilon_{y,t},$$

where a, b, c, d are real, positive constants, and ϵ_x, ϵ_y are standard normally distributed random variables. From the formulae above it is clear that there is a nonlinear connection between X and Y and a directed causality $X \longrightarrow Y$.

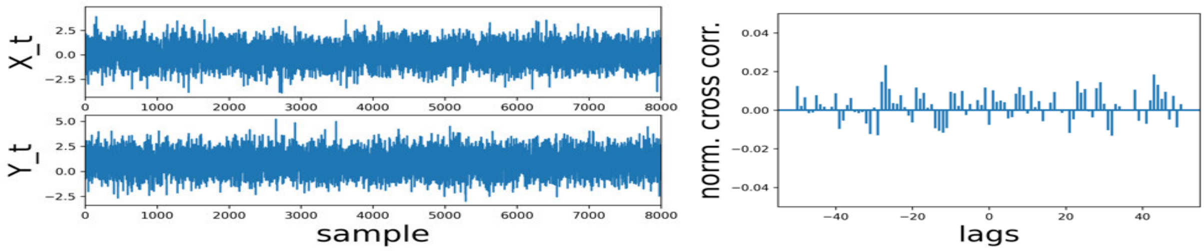

Figure 2
(left) 8000 samples long simulated synthetic signals a=0.2, b=c=d=1. (right) normalized cross-correlation function between X_t and Y_t

Figure 2 shows the nonlinearly coupled simulated signals (left), form the plot there is no visible connection between the signals, and this is confirmed by calculating the linear, normalized cross-correlation function (CCF) as a function of the time lag (right). The cross-correlation coefficients show very small correlation, below 2% at every time step.

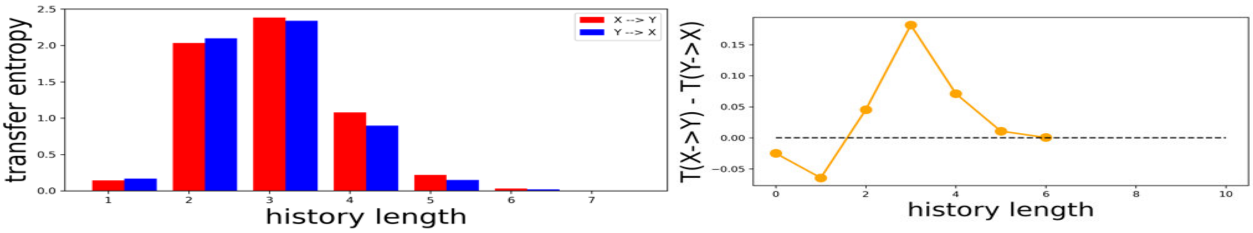

Figure 3

(left) transfer entropy estimation for simulated signals in both causality directions as a function of the history length L. (right) transfer entropy difference showing a peak at L=3 history length.

As it can be seen in Figure 3 using an optimal history length of $L = 3$ the transfer entropy calculation can reveal the correct nonlinear connection $X \longrightarrow Y$ well above the noise level, here we have estimated the noise level from the difference transfer entropy at 0 history length.

3.2. LiBES and Mirnov-coil signal processing

In our analysis we use data acquired at Compass tokamak at Institute of Plasma Physics in Prague by two different diagnostics [12]:

(1) Lithium Beam Emission Spectroscopy (LiBES) measures the light intensity originated from the interaction of Li atoms injected into the plasma edge with the plasma electrons [13]. This signal is proportional to the local plasma density. The spatial resolution of the LiBES diagnostic is 1 cm, while the sampling rate of the Avalanche Photo Diode array system is 2 MHz [14]. The downsampled signal (effective sampling rate of 153.8 kHz) can be seen on the upper panel of the Figure 4.

(2) Poloidal array of 24 discrete Mirnov coils (called poloidal ring B) measuring the local B_θ (poloidal magnetic field) on 24 positions around plasma on toroidal angle 215.5° (for reference, the LiBES system is at a toroidal location of 180°), at 2 MHz sampling frequency. This Mirnov coil ring is the closest of the 3 rings to the LiBES system, therefore we used data from ring B, coil #1. The downsampled and pre-processed signal (effective sampling rate of 153.8 kHz) can be seen on the lower panel of the Figure 4.

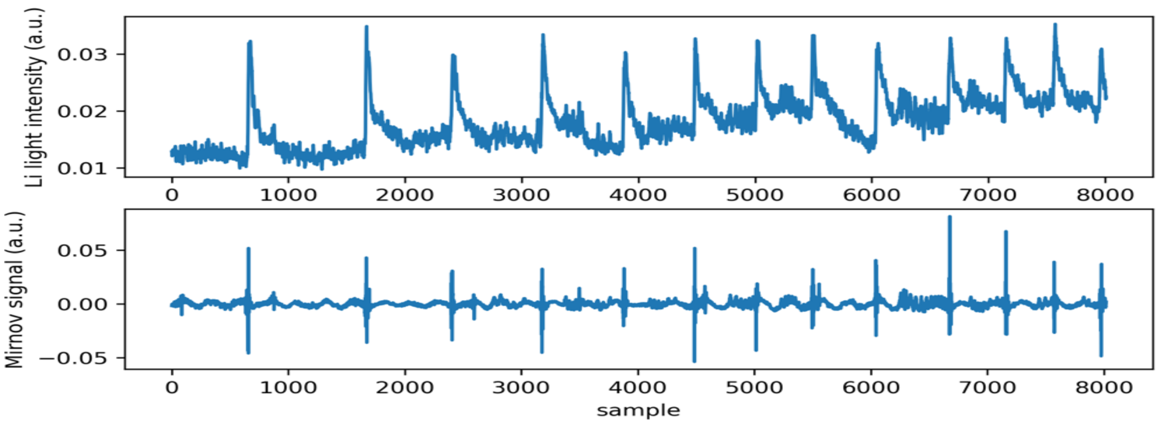

Figure 4

52 ms long portion of LiBES data from an ELMy H-mode (upper panel). 52 ms long Mirnov coil signal measuring the poloidal magnetic field at the outer midplane of the tokamak

The pre-processing of the signals consisted of two steps: a linear detrend and an amplitude rescaling to get the two signals in the same amplitude range. The critical point in the transfer entropy calculation is the estimation of the probability distributions for the individual signal as well as for the joint process (joint PDF). We have approximated these PDFs by the amplitude histograms using 20 bins for each variable. We have to note that the choice of bin number is critical and should be carefully analysed case by case.

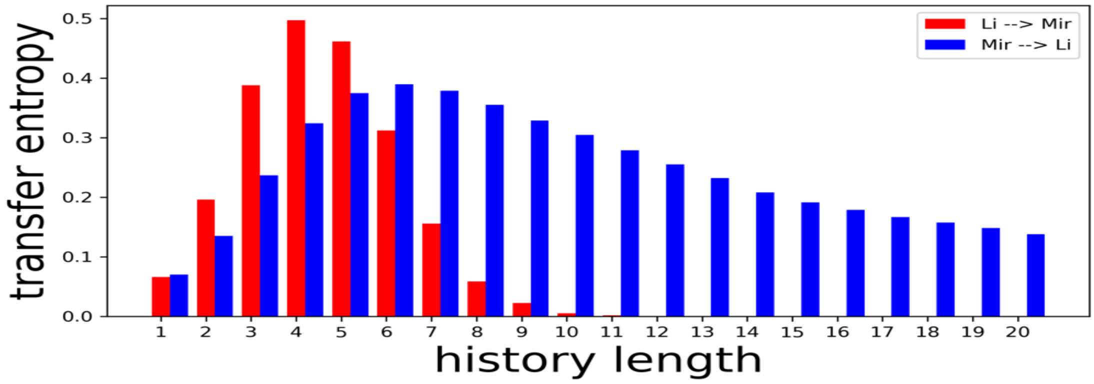

Figure 5
Transfer entropy estimation for real experimental signals in both causality directions as a function of the history length L.

Figure 5 shows that for history length longer then $L = 5$ the amount of information transferred from magnetic signals to density signals increases firmly establishing a Mirnov → LiBES causal relation.

Conclusions

In the present contribution we have shown for nonlinearly interacting random processes, when the linear correlation cannot reveal any connection, that the transfer entropy can detect directed information flow. Analysing the information transfer between LiBES data and Mirnov coil data, we have found that the information is flowing from the magnetic signal towards the density signal at the plasma edge. This seemingly contradicts the ballooning-peeling paradigm presented in section 2. The explanation for this surprising conclusion can be that the LiBES signal captures the density pulse when the ELM filament propagates radially into the scrape-off-layer after the peeling phase which is located more towards the plasma core.

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