Engineering Symposium at Bánki
(ESB 2025)

New smith predictive control applied in an electric water heater system

[1]José Vieira, [2]Alexandre Mota

[1]*Escola Superior de Tecnologia. Castelo Branco, Portugal, zevieira@est.ipcb.pt*
[2]*Departamento de Eletrónica e Telecomunicações e Informática. Aveiro, Portugal, alex@ua.pt*

Abstract

This paper presents the control results of a tankless electric water heater using a new Smith predictive control based on the physical internal model control structure.

The electric water heater was modelled with two variable blocks: a first order system and a time delay. The physical model of the electric water heater system was derived from energy-balance equations and validated using open-loop system data.

The new control algorithm explored is the Smith predictive control (SPC) based on the internal physical model control algorithm. It is more adequate to control this type of systems (first order system followed by a variable time delay). The new SPC achieves very good control results.

Keywords: tankless water heater, physical model, Smith predictive control, time delay system.

1 Introduction

Industrial control processes present many challenging problems, including non-linear or variable linear dynamic behaviour, variable time delays, which imply time-varying parameters. One of the alternatives to handle time-delay systems is to use prediction technique to compensate the negative influence of the time delay. Smith predictor control (SPC) is one of the simplest and most often used strategies to compensate time delay systems.

It has been shown that many industrial linear and nonlinear processes can be effectively modelled using black-box modelling techniques.

In previous work, these three modelling approaches were explored these three different modelling types: neuro-fuzzy [1], Hammerstein [2] and hybrid models that reflect the evolution of the knowledge about the first principles of the system. These kinds of models were used because the system had a non-linear actuator and time varying linear parameters [3].

At the beginning there was no knowledge of the physical model, so black-box and grey-box modelling approaches were used. Finally, the physical model was found, and a much simpler adaptive model was achieved (the physical model white box modelling).

This paper presents the Smith predictive controller based on the physical model of the system. The Smith predictive controller has a high mathematical complexity structure; it uses three internal physical models: one inverse model and two direct models and deals with the variable time delay of the system. The knowledge of the physical model allows changing the linear parameters correctly in time and gives an interpretable model that facilitates its integration into control schemes.

2 The electric water heater

The overall system has three main blocks: the electric water heater, a micro-controller board and a personal computer. The micro-controller board has two modules controlled by a flash-type micro-controller, PIC18F4550. First, the interface module composed by the microcontroller, electronics, sensors, and control the actuator. Second, the communication module composed by the serial interface used to transmit all important system variables to a personal computer. After this small description of the prototype system, the electric water heater characteristics and its first principles equations are presented.

2.1 Electric water heater description

The electric water heater is a multiple input single output (MISO) system. The controlled output water temperature will be called hot-water temperature (hwt(t)). This variable changes with cold-water temperature (cwt(t)), water flow (wf(t)), applied power (p(t)). The subtraction between hot and cold-water temperature is called delta water temperature (Δt(t)).

The electric water heater is physically composed of an electric resistance, a permutation chamber and several sensors used for control and security of the system as shown on Figure 1.

Operating range of hwt(t) is from 20 to 50ºC. Operating range of the cwt(t) is from 5 to 25ºC. Operating range of the wf(t) is from 0.5 to 2.5 litters per minute. Operating range of the p(t) is from 0 to 100% of the available power.

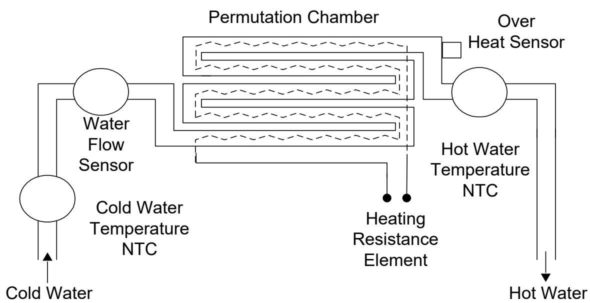

Figure 1. Schematic of the electric water heater: sensors and actuator.

The applied energy into the heating resistance is controlled using a window of 100 alternated voltage cycles (one second). Each iteration, the applied number of cycles is proportional to the delivered energy to the heating element.

2.2 Electric water heater first principles

Applying the principle of energy conservation in the electric water heater system, Eq. 1 could be written. Eq. 1 was based on the previous work made in modelling the gas water heater system, first presented in [4].

$$\frac{dEs(t)}{dt} = Qe(t - td) - wf(t)hwt(t)Ce - wf(t)cwt(t)Ce \qquad (1)$$

Where dEs(t)/dt=MCe(dΔt(t)/dt) is the energy variation of the system in the instant t, Qe(t) is the calorific absorbed energy, wf(t)cwt(t)Ce is the input water energy that enters in the system, wf(t)hwt(t)Ce is the output water energy that leaves the system, and Ce is the specific heat of the water, M is the water mass inside of the permutation chamber and td is the variable system time delay. M is the mass of water inside of the permutation chamber (measured value of 0.09Kg) and

Ce is the specific heat of the water (tabulated value of 4186 J/(kgK)). The maximum calorific absorbed energy Qe(t) is proportional to the maximum electric applied power of 5.0 KW.

The absorbed energy Qe(t) is proportional to the applied electric power p(t). During each operation of the electric heater, it was considered that cwt(t) is constant; it may change from operation to operation, but in each operation it remains approximately constant.

Transforming Eq. 1 into the Laplace domain and considering a fixed water flow wf(t)=Wf and fixed time delay td, Eq. 2 can be written.

$$\frac{\Delta t(s)}{Qe(s)} = \frac{\frac{1}{WfCe}}{\frac{M}{Wf}s+1} e^{-s\,td} = \frac{\frac{1}{WfCe}\frac{Wf}{M}}{s+\frac{Wf}{M}} e^{-s\,td} \tag{2}$$

Changing to the discrete domain, with a sampling period (h) of 1 second and with discrete time delay, the final discrete transfer function is presented in Eq. 3.

$$\Delta t(k+1) = \left(e^{-\frac{Wf}{M}}\right)\Delta t(k) + \left(\frac{1}{WfCe}\left(1-e^{-\frac{Wf}{M}}\right)\right)(Qe(k)-\tau d(k)) \tag{3}$$

The real discrete time delay $\tau d(k)$ is given in Eq. 4, where $\tau d_1(k) = 3s$ is the fixed part of $\tau d(k)$ that comes from the transportation of energy and $\tau d_2(k)$ is the variable part of $\tau d(k)$ that comes from the time that the water flow wf(k) takes while circulating in the permutation chamber.

$$\tau d(k) = \begin{cases} 4 & to \quad wf(k) >= 1,75l/\min \\ 5 & to \quad 1,00l/\min < wf(k) < 1,75l/\min \\ 6 & to \quad wf(k) <= 1,00l/\min \end{cases} \tag{4}$$

Considering now that the water flow changes, in the discrete domain Wf=wf(k) and $\tau d_2(k)$, the final transfer function is given in Eq. 5.

$$\Delta t(k+1) = \left(e^{-\frac{wf(k-\tau d_2(k))}{M}}\right)\Delta t(k) + \left(\frac{1}{wf(k-\tau d_2(k))Ce}\left(1-e^{-\frac{wf(k-\tau d_2(k))}{M}}\right)\right)(Qe(k-\tau d(k))) \tag{5}$$

Observing the real data of the system, the absorbed energy Qe(t) is a linear static function f(.) proportional to the applied electric power p(t) as expressed in Eq. 6.

Finally, the discrete global transfer function is given by Eq. 6.

$$\Delta t(k+1) = \left(e^{-\frac{wf(k-\tau d_2(k))}{M}}\right)\Delta t(k) + \left(\frac{1}{wf(k-\tau d_2(k))Ce}\left(1-e^{-\frac{wf(k-\tau d_2(k))}{M}}\right)\right)\big(f(p(k-\tau d(k)))\big) \tag{6}$$

If A(k) and B(k) are defined as expressed in Eq. 7, the final discrete transfer function is given as presented in Eq. 7.

$$\Delta t(k+1) = A(k)\Delta t(k) + B(k)\big(f(p(k-\tau d(k)))\big) \tag{7}$$

2.3 Physical model

To validate the presented discrete physical model, it is necessary to have open loop data of the real system. The data are chosen to respect frequency and amplitude spectrum wide enough.

The validation data and the physical model error are illustrated in Figure 2. It shows the physical model error signal e(k), which is equal to e Δt(k)-Δt modelled(k). It can be seen from this signal, that the proposed model achieves good results with a mean square error (MSE) of 1.32ºC for the all-test set (1 to 1600). From the validation test, Figure 3 shows the two linear variable parameters expressed in Eq. 8 of the physical model used.

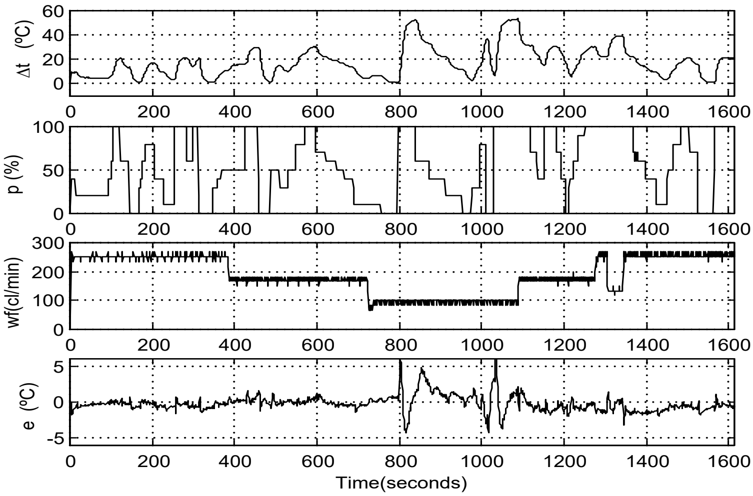

Figure 2. Open loop data used to validate the model.

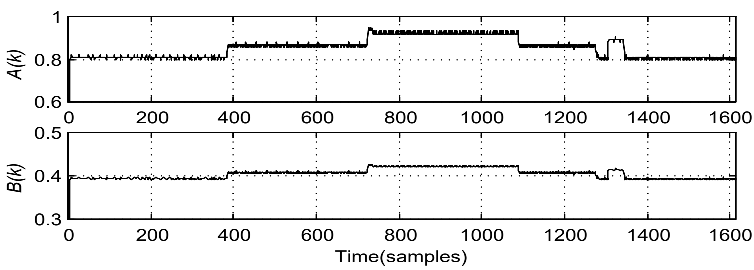

Figure 3. The two linear variable parameters A(k) and B(k) of the first order discrete model.

From the results, for low water flows the model presents a bigger error signal. This happens because of the low resolution of the water flow measurements.

3 New Smith predictive controller

The control loop tested is the Smith predictive control algorithm. This control strategy is particularly used to control systems with time delay. First, the control structure is described and its parameters and second the control results are shown.

3.1 New SPC structure

The new Smith predictive controller is based on the internal model controller architecture that uses the physical model presented in section 2, as illustrated in Figure 4. It uses two physical direct models: one with time delay for the prediction loop and another without the time delay for the internal model control structure.

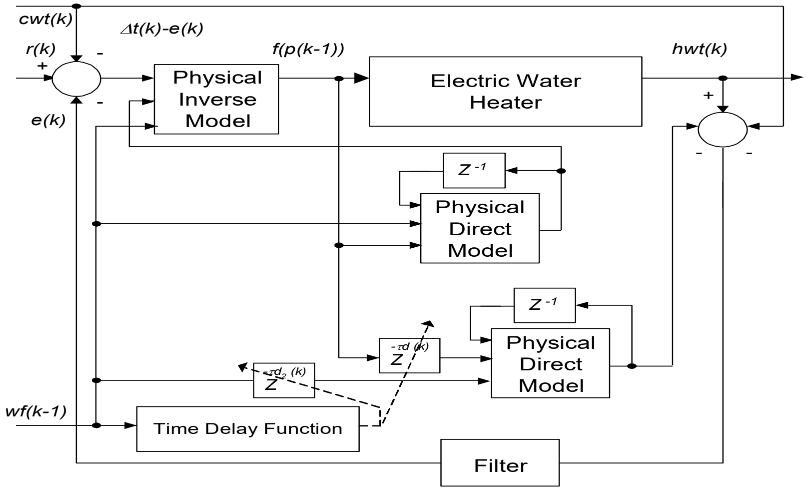

Figure 4. New SPC constituent blocks.

The new Smith predictive control structure has a special configuration, because the system has two inputs with two different time delays, so it uses two direct models, one model with time delay to compensate for its negative effect and another without time delay needed for the internal model control structure. The SPC separates the time delay of the plant from time delay of the model, so it is possible to predict the Δt(k), Td(k) steps earlier, avoiding the negative effect of the time delay.

The time delay is a known function that depends on the water flow wf(k). The incorrect prediction of the time delay may lead to aggressive control if the time delay is underestimated or conservative control if the time delay is overestimated [6]. The low pass filter used in the error feedback loop is a digital first order filter used to filter the feedback error and indirectly to filter the control signal f(p(k)). The time delay function is a function of the water flow, which is explained in section 2 and expressed in Eq. 4.

3.2 New SPC results

The SPC results are shown in Figure 5. Based on previews work it is expected that the results should be better for reference changes and water-flow changes. The behaviour of the closed loop control system is similar in every working point.

For low wf(t) the measurement resolution is low that makes the control signal a bit aggressive, but it does not affect the output hot-water temperature. In this situation there is another problem with the multiplicity of the time delay and its resolution. With a sampling period of 1 second it is not possible to use fractional time delays that really happen. This makes the control results a bit aggressive. The final MSE control criterion achieved with the SPC is presented in table 1

Table 1. Mean Square Errors of the Control Results.

Algorithm NSPC	MSE 3.56 Test Set

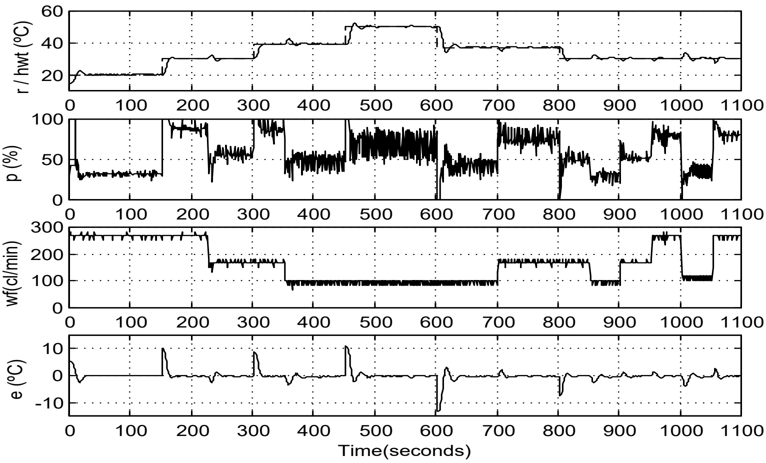

Figure 5. New SPC results.

4 Conclusions and Future Work

This work presents and validates the physical model of the electric water heater. This model was based on the physical model of a gas water heater from previews work because of the similarities of both systems. A good control structure for first order systems with time delay that changes is the Smith predictive controller. The new SPC reacts in a predictive way changing the control signal and it presents better performance for cold-water temperature changes. This controller is mathematically easy to implement in 8-bit microcontroller with reduced resources. This type of modelling has reduced mathematical complexity and interpretable.

For future work, some improvements should be made as an increase in the resolution of the used water flow and the redefinition of the time delay function.

5 References

[1] J. Vieira, A. Mota. Smith Predictor Based Neural-Fuzzy Controller Applied in a Water Gas Heater that Presents a Large Time-Delay and Load Disturbances. Proceedings IEEE International Conference on Control Applications, Istanbul, Turkey, 23 a 25 June 2003, 1, 362-367.

[2] J. Vieira, A. Mota. Parameter Estimation of Non-Linear Systems with Hammerstein Models Using Neuro-Fuzzy and Polynomial Approximation Approaches. Proceedings of IEEE-FUZZ International Conference on Fuzzy Systems, Budapest, 25 a 29 July 2004, 2, 849-854.

[3] J. Vieira, F. Dias and A. Mota. Hybrid Neuro-Fuzzy Network-Priori Knowledge Model in Temperature Control of a Gas Water Heater System. Proceedings of 5th International Conference on Hybrid, Intelligent Systems, Rio de Janeiro, 2005.

[4] J. Vieira, A. Mota. Water Gas Heater Non-Linear Physical Model: Optimization with Genetic Algorithms. Proceedings 23rd Int. Conf. on Modelling, Identification and Control, 2004.

[5] D. C. Psichogios and L. H. Ungar. A hybrid neural network-first principles approach to process modelling. AIChE Journal, 1992; 38(10); 1499-1511.

[6] Y. Tan, M. Nazmul Karim. Smith Predictor Based Neural Controller with Time Delay Estimation. Proceedings of 15th Triennial World Congress, IFAC, 2002.