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THE PERMEABILITY VARIABLES OF GRANULAR MATTER

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No "unique" relation between soil parameters and grading curves of granular matter can be elaborated. In case some statistics are used (e.g., d_{10}), then infinite many grading curves may be related to the same value of it. Moreover, several densities can be related to the same grading curve, and different permeability values are measured at different densities. A model discrimination study was started to be made to suggest a saturated water permeability k – grading curve – density relation using some new measurements. The following model variables were tested: (i) various grading curve statistics, (ii) combined variables of harmonic mean diameter and density like specific surface are or hydraulic radius, (iii) density parameters like void ratio or the newly applied relative density. Two newly measured data sets were used with identical soil composition and different density (in more dense and less dense states). Both data sets consisted of 3 series of optimal 2-fraction granular soils, the fractions were: 0.25 to 0.5 mm, 0.5 to 1 mm, 1 to 2 mm and 2 to 4 mm. Each series consisted of 5 mixtures with the same, fixed relative base entropy A values (i.e., mean log d values). In addition, some earlier data sets were used to test the extension of the relations. According to the results, the regression for the new data was acceptable if extra density parameters were applied with either the (i) or the (ii) type parameters. The addition of earlier data bases increased the regression coefficient R^2.

***Keywords**: entropy, grading curve, model discrimination, permeability, relative density, specific surface area*

INTRODUCTION

Permeability (k) is a measure of the ability of a porous material (typically, a rock or soil material) to transmit fluids (liquid or gas). Permeability is a function of the porosity, the particle diameter distribution and internal structure of the solid medium and the physical properties of the permeating fluid, and so, any measured value is specific to a particular combination of a porous medium and a permeating fluid.
It depends on the intrinsic permeability of the material and on the degree of saturation. Saturated water permeability describes water movement through saturated media (ms^{-1}).

Permeability variables

A particle size distribution curve is a compendium of much quantitative data, and it is difficult to define well and poorly graded soils in terms of a single basic data value from this curve. To overcome

this difficulty, a few derived quantities are defined in terms of the basic particle size distribution curve data. These are the d_{10}, d_{60}, the coefficient of uniformity C_U, given by d_{60}/d_{10}.
Early research established that the hydraulic conductivity may be related to other standard PSD descriptors such as d_{10}, d_{60}, C_U well as the void ratio e [1 to 4]. Recently the grading entropy theory is started to be used ([5, 6]) and the density [7], moreover the specific surface concepts ([8, 9, 10]).

Table 1. Definition and properties of fraction *j*.

j	**1**		**23**	**24**
Limits in d_0	1 to 2		2^{22} to 2^{23}	2^{23} to 2^{24}
S_{0j} [-]	1		23	24

The base entropy S_0 and the normalized form A read:

$$S_0 = \sum x_i S_{0i} \tag{1}$$

$$A = \frac{S_o - S_{omin}}{S_{omax} - S_{omin}} \tag{2}$$

where x_i is the relative frequency of fraction i, $S_{0i} = i$ is the i-th fraction entropy Table 1). The entropy increment ΔS and the normalized version B:

$$\Delta S = -\frac{1}{\ln 2} \sum_{xi \neq 0} x_i \ln x_i \tag{3}$$

$$B = \frac{\Delta S}{\ln N} \tag{4}$$

The harmonic mean diameter is:

$$\sum \frac{x_i}{d_i} \tag{5}$$

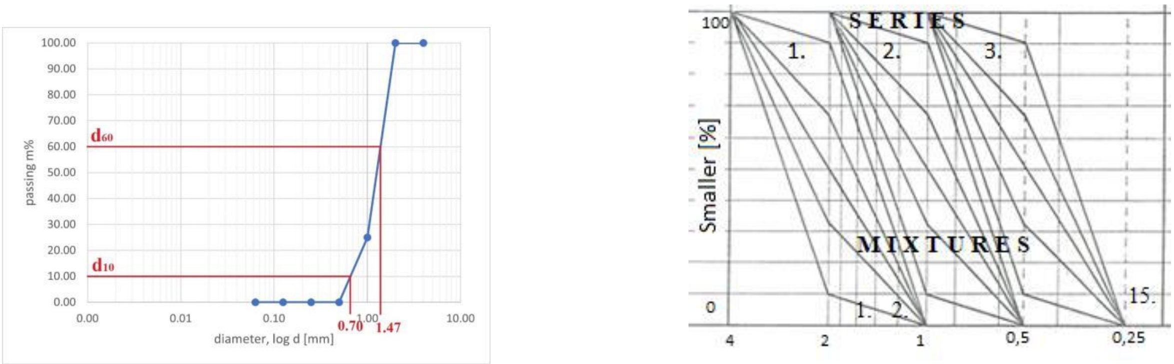
The harmonic mean diameter, d_h is a kind of equivalent grain diameter, being equal to the diameter of the sphere which has the same ratio of volume / surface area as the solid phase in soil. The specific surface area per mass / volume of a soil is the total surface area to the unit mass/ unit volume of grains. The specific surface area per mass of a single sphere and the soil is defined as:

$$S_m = 6/(1+e)/\rho_d/d_h \tag{6}$$

The particle surface area is a value in square metre, and the mass is grams, volume in cubic meters so the units are $m^2/g/$ and $1/m$. The specific surface area per volume is the ratio of the total surface area (m^2) to the volume of the soil (m^3) so it has a unit of m^{-1}. The specific surface area per volume of a single sphere and for the soil is defined as:

$$S_{sV} = 6/(1+e)/d_h \tag{7}$$

In an earlier study of [8], among specific surface per volume parameter (containing density information), harmonic mean d_h and d_{10}, it was found that the best regression explanation (higher R^2) was related to the specific surface variable.
In this work, the following variables were tested: (i) various grading curve parameters, (ii) specific surface area, (iii) void ratio and relative density [7]. Two new measured data sets were used with identical soil composition and different density.



(a) (b)

Figure 1. (a) Simple grading curve parameters. (b) The grading curve series 1, 2 and 3. These were numbered from 1 to 15.

MATERIALS AND METHODS

Earlier data bases

In Nagy data base [4] the artificial mixtures of natural soil mixtures were prepared from fluvial soil mixtures for permeability testing. The d_{10} or 10% diameter value of the measurement series were as follows:
series of measurements d_{10} = 0.004-0.006 mm,
measuring series d_{10} = 0.006-0.010 mm
measuring series d_{10} = 0.010-0.014 mm, and
measuring series d_{10} = 0.014-0.016 mm.
In Feng database (series V, [6]) d_{10} = 0.72 to 5.82 mm. In Pap database (series VI, [3]) d_{10} = 0.002 to 0.216. These are re-analyzed in this research [8].

New data

15 optimal 2-fraction soils, with fractal gradings were tested by means of constant head permeability tests. The composition of each series was: 25%, 33%, 50%, 67%, 75 % finer, A=0.75, A=0.67, A=0.5, A=0.33, A=0.25. Two different sample preparation techniques were used resulting in loose state (15 samples) and dense state (45 samples, 3 repeats) was tested (Figs. 1 to 3). The four fractions were: 0.25-0.5 mm, 0.5-1 mm, 1-2 mm and 2-4 mm. The tests with denser samples were repeated in 3 times.

Models

The models were as follows. The simplest relationship among the base entropy parameter S_0, the entropy increment ΔS and k [cm/s] in the equivalent forms are as follows:
In addition, the relationship among the base entropy parameter S_0, the entropy increment ΔS, k and e was determined in the following form.

$$k = \exp C_3 \Delta S^{C_1} S_0^{C_2} \tag{8}$$

$$\ln k = C_3 + C_1 \ln \Delta S + C_2 \ln S_0 \tag{9}$$

$$k = \exp C_4 \Delta S^{C_1} S_0^{C_2} e^{C_3} \quad (10)$$

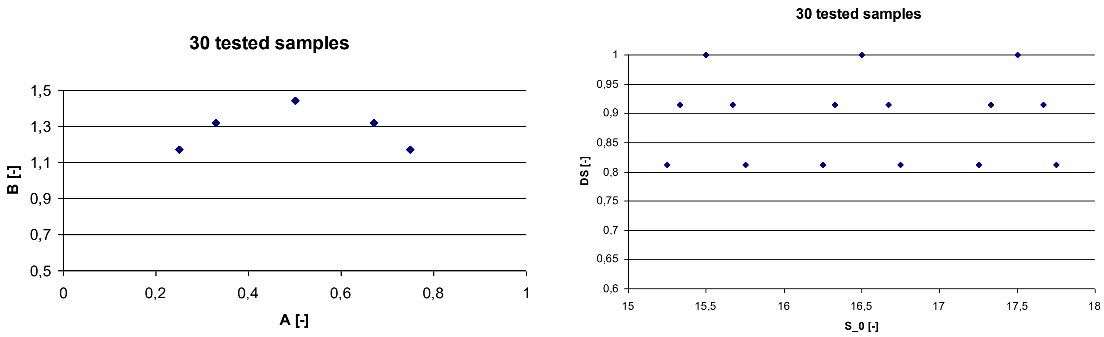
In the equation some other soil variables *p* were written instead of e, like specific surface area per unit volume or relative density:

$$k = \exp C_4 \Delta S^{C_1} S_0^{C_2} p^{C_3} \quad (11)$$

In the relationships the base entropy parameter S_0, the entropy increment ΔS were interchanged. In case of the most recent specific surface area approaches, the void ratio is generally included as extra variable. Using Ren & Santamarina (2018) [10], the following was suggested:

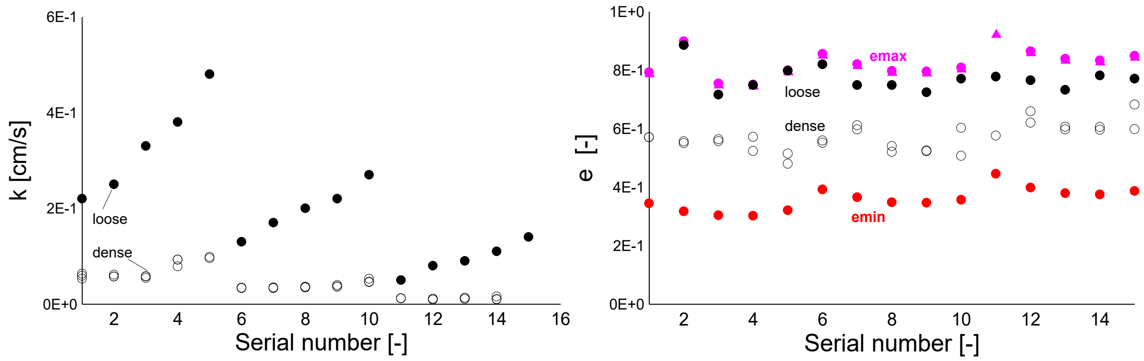
$$k \sim C\ S_S^{-2}\ e^D \quad (12)$$

where S_S is the specific surface area per unit volume or mass, the parameters may be obtained from a fitted power relation.



(a) (b)

Figure 2. The tested 30 mixtures in terms of entropy coordinates (a) Normalised entropy coordinates. (b) Non-normalised entropy coordinates. Due to the coincidences, any relations based on purely entropy coordinates are not suggested.



(a) (b)

Figure 3. (a) The permeability *k* in terms of serial number for tested grading curve series 1, 2 and 3. (b) The void ratios in terms of serial number for tested grading curve series 1, 2 and 3.

The parameters were determined by the Gauss Normal Equations [11]. The least-squares fitting process produces a value – r-squared (R^2) – which is 1 minus the ratio of the variance of the residuals to the variance of the dependent variable. It says what fraction of the variance of the data is explained by the fitted trend line.

RESULTS

New data

Results are shown in Figures 2 to 7 and Table 2. The tested 30 mixtures plot 5 points in terms of normalised entropy coordinates, and 15 points in terms of non-normalised entropy coordinates. The void ratio data were represented with maximum and minimum void ratios, assessed from earlier research, where e_{max} was minimal at A=2/3 in each series (Figure 3), according to measurement of Lőrincz [3]. The minimum void ratio e_{min} was estimated from that [5]. The k -relative density data - relation indicated large difference in k for the two data sets with smaller and larger density (Figure 4).

Model fitting

In this work it was found that the newly measured data separated considerably in terms of entropy coordinates – k and d_{10} – k for loose and dense samples (Figure 5). With extra density parameters, Eqs (10, 11) showed better the R^2 (void ratio e, specific surface area per volume S_{sv} or relative density R_d). The entropy parameters could have been substituted in Eqs (10,11) by d_{10} and Cu. The best the R^2 results were found with relative density. In case of Ren & Santamarina (2018) based relation Eq 12 - the R^2 was larger than in case of the specific surface variable alone. In case of the enriching of the newly measured data, when the Eqs (10, 11, 12) were fitted including the two additional data bases (with series III and V) and with different k range, the R^2 increased. Similar result was found for the case when specific surface area per unit volume parameter is used alone, the two additional data bases with different k range gave increased R^2.

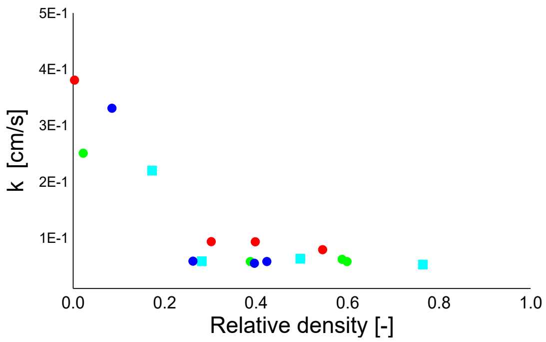


Figure 4. The permeability k in terms of relative density for tested grading curve series 1.

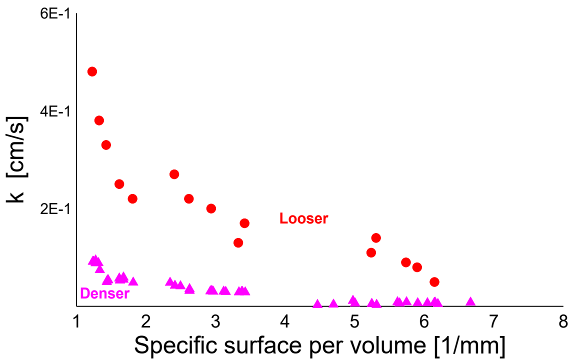


Figure 5. The measured permeability k in terms of specific surface area per unit volume, series 1 to 3, loose and dense samples.

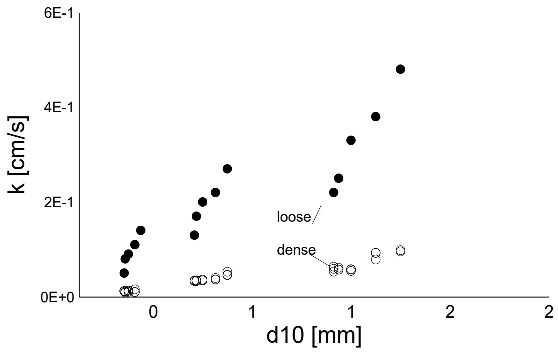


Figure 6. The measured permeability k in terms of terms of d_{10} for tested grading curve series 1, 2 and 3, loose and dense samples.

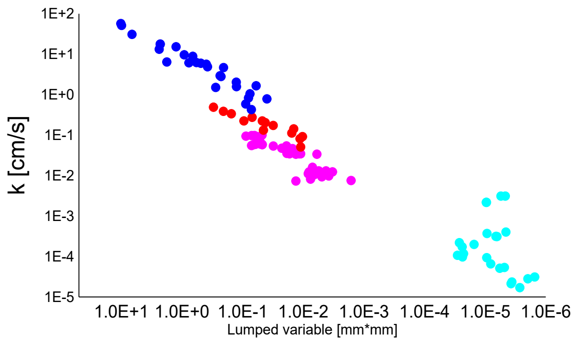


Figure 7. The *Ren & Santamarina* [10] based relation (R^2 = 0,8986). Red and pink: dense and loose data from newly tested series 1 to 3. Blue: database V. Light blue: database III.

Table 2. New 2-fraction mixtures data, R^2 values

extra density variable	**R^2**
Eq 8, no extra variable	0,4790
Eq 10, with void ratio, e	0,7701
Eq 11, with specific surface area per unit volume	0,7476
Eq 11, with relative density	0,8847
Eq 12, the *Ren & Santamarina* [10] based relation	0,7362

DISCUSSION

Multiple data bases

The Eq 8 type regression was used to visualize the results in the entropy diagram for large *k* ranges, using all previous data I to VI ([8], see Figure 8). The permeability zones of [6] are in accordance with the level lines presented by [8].
The relative density – entropy coordinate models can be extended and used for the entropy diagram representation purpose. The relative density variable may give a good normalisation since the e_{max} is dependent on the internal structure and on entropy coordinate *A* [12].

Fractal gradings and density

The optimal or finite fractal grading curves [12] are mean grading curves of all grading curves with fixed normalized base entropy parameter *A*, for a given fraction number *N* and given smallest fraction ([10,12]). Therefore, these are the best to be used to study the relation of permeability, relative density and grading curves.

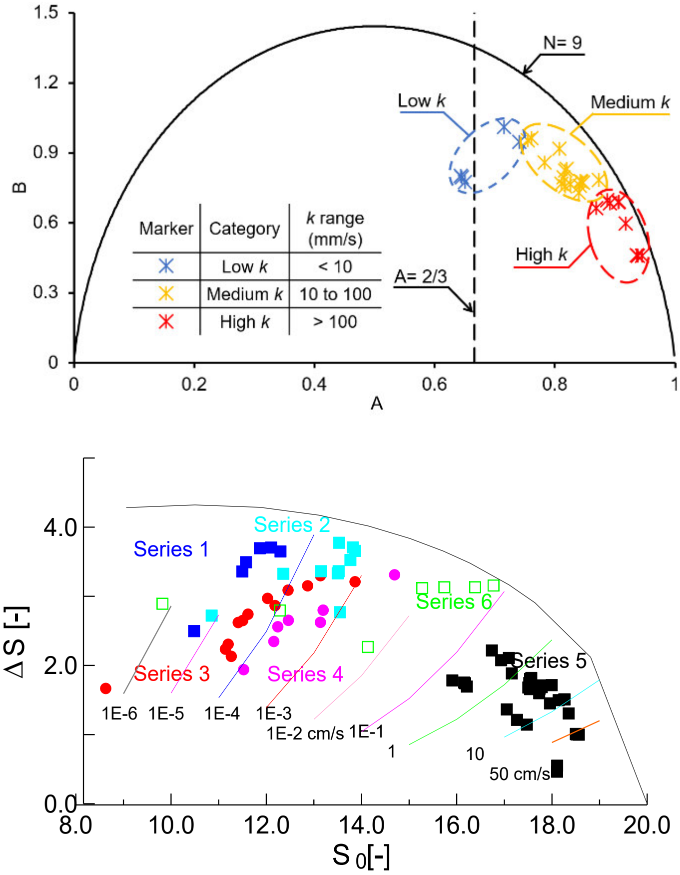


Figure 8. (a) Identified permeability zones shown on the normalized entropy diagram (adapted from Feng et al, 2019a) (b) The level lines determined for Eq. 8, using data bases of 1 to 6

CONCLUSION

The permeability relations can be based either on grading curve statistics like the non-normalised entropy coordinates, or on a specific surface area type variable. In this work it was found that the in each case some additional parameter for the density is needed. The relative density - a newly tested parameter - provided the largest R2 values. The relative density gave a good normalisation in terms of density since - for fractal (or mean) gradings -, the e_{max} is dependent on the entropy coordinate A. In further research, the model discrimination study is suggested to be continued, including sensitivity analyses of the various models. Some additional measurements are suggested on the variation of the saturated k in terms of relative density. The relative density – entropy coordinate models are suggested to be extended and used for the entropy diagram representation.

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REFERENCES

[1] Hazen, A.: Some physical properties of sand and gravels. 1902; Massachusetts State Board of Health, Wright & Potter Printing, Boston, MA, USA, Annual Report 24. 1-12.

[2] Chapuis, R. P.: Predicting the saturated hydraulic conductivity of sand and gravel using effective diameter and void ratio. Can. Geotech. J., 2004; 41(5):787–795., https://doi.org/10.1139/t04-022.

[3] Pap, M., A. Mahler: Comparison of Different Empirical Correlations to Estimate Permeability Coefficient of Quaternary Danube Soils. Period. Polytech. Civil Eng., 2019; 63(1): 25-29., https://doi.org/10.3311/PPci.13108

[4] Nagy, L.: Permeability of well graded soils. Per. Pol. Civil Eng., 2011; 55(2): 199-204., https://doi.org/10.3311/pp.ci.2011-2.12

[5] Lőrincz, J.: Grading entropy of soils. 1986; Doctoral Thesis, Technical Sciences, TU of Budapest (in Hungarian).

[6] Feng, S., P. J. Vardanega, E. Ibraim, I. Widyatmoko, and C. Ojum: Permeability assessment of some granular mixtures. Géotechnique, 2019; 69(7): 646–654., https://doi.org/10.1680/jgeot.17.T.039

[7] Imre, E., Fityus, S., Keszeyné, E., Schanz T.: A comment on the ratio of the maximum and minimum dry density for sands. Geotechnical Engineering 2011; 42(4): 77-82.

[8] Imre, E., Á. Bálint, L. Nagy, J. Lőrincz, and Z. Illés: Examination of saturated hydraulic conductivity using grading curve functions. 2021; In: Proceedings of 6th International Conference on Geotechnical and Geophysical Site Characterisation: Toward synergy at site characterisation Paper: ISC2020-373, 12 p.

[9] Ren, X. W. and J. C. Santamarina: The hydraulic conductivity of sediments: A pore size perspective. Engineering Geology, 2018; 233: 48–54., https://doi.org/10.1016/j.enggeo.2017.11.022

[10] Imre, E., Lőrincz J, Barreto D, Singh V. P., et al.: Statistical parameters and grading curves. 20ICSMGE submitted.

[11] Press, W. H., S. A. Teukolsky, W. T. Wetterling, B. P. Flannery: Numerical recipes: the art of scientific computing. 2007; Cambridge, UK; New York: Cambridge University Press. ISBN 978-0-511-33555-6.

[12] Imre, E., Lőrincz J, Talata I, Baille W., Rahemi N., Goudarzy M., Barreto D, Singh V. P.: Grading curves and internal stability. Dunakavics. 2019; VIII(III): 37-50.