Quantum Digitization of Electromagnetic Field States: A Computational Tool for Investigating Encoding-Dependent Quantum Information Measures

István Németh

Óbuda University, Kandó Kálmán Faculty of Electrical Engineering,
17 Tavaszmező Street, Budapest 1084
nemeth.istvan@kvk.uni-obuda.hu

Abstract: We present a comprehensive computational tool for investigating the quantum digitization problem: how to represent continuous electromagnetic field states as discrete qubit registers. The encoding of quantum field states into finite qubit systems is a fundamental challenge in quantum information theory, with significant implications for quantum computing, communication, and metrology. We introduce two Mathematica-based analysis functions, QubitInfoPanel[] and QubitInfoPanelGray[], which accept arbitrary density matrices describing quantized electromagnetic fields and compute a comprehensive suite of quantum information measures including entanglement, mutual information, and entropy. These tools enable systematic investigation of how encoding scheme choices (Binary vs Gray code) and measurement basis selections (Fock vs Phase) affect the observed quantum correlations. Our framework provides researchers with a validated platform to explore representation-dependent quantum properties in field quantization schemes.

Keywords: quantum digitization; qubit encoding; quantum entanglement measures; electromagnetic field quantization; Gray code representation

1 Introduction

The quantization of electromagnetic fields lies at the heart of quantum optics and quantum electrodynamics [11,12]. While the continuous-variable description of field states in the Fock space provides a complete mathematical framework, practical implementations of quantum information processing require discrete, finite-dimensional representations [1]. The process of encoding continuous field states into discrete qubit registers presents fundamental theoretical questions that have direct implications for quantum computing architectures, quantum communication protocols, and quantum measurement strategies.

This encoding process is far from trivial. The choice of how to map photon number states to computational basis states of qubits can fundamentally alter the quantum information content that is accessible through measurements. Similarly, the selection of which measurement basis to employ—whether the Fock basis that directly corresponds to photon numbers, or complementary bases such as the phase basis—can reveal different aspects of the quantum correlations present in the system.

Despite the fundamental importance of these questions, there has been a lack of comprehensive computational tools that enable systematic investigation of encoding-dependent quantum properties. Existing quantum information analysis packages typically assume a fixed computational basis and do not provide flexible frameworks for comparing different digitization schemes or exploring multiple measurement bases.

In this work, we present two complementary analysis tools, QubitInfoPanel[] and QubitInfoPanelGray[], implemented in Wolfram Mathematica. These functions accept as input an arbitrary density matrix describing a quantized electromagnetic field state and compute a comprehensive suite of quantum information measures, including:

- Bipartite entanglement quantified through negativity measures
- Tripartite genuine entanglement via three-tangle calculations
- Mutual information matrices revealing qubit-qubit correlations
- Von Neumann and linear entropies characterizing quantum uncertainty

The two functions differ in their encoding schemes: QubitInfoPanel[] implements standard binary encoding, while QubitInfoPanelGray[] employs Gray code encoding. This parallel implementation enables direct investigation of the fundamental question: does the choice of computational encoding affect measured quantum properties? Both tools support analysis in either Fock or phase measurement bases, allowing researchers to explore basis-dependent quantum correlations.

This paper provides a comprehensive description of the theoretical framework, computational implementation, and validation methodology for these tools. Our goal is to provide the research community with a robust, validated platform for investigating the rich landscape of representation-dependent quantum information in electromagnetic field systems.

2 The Quantum Digitization Problem

2.1 From Continuous to Discrete: The Fundamental Challenge

The quantum state of an electromagnetic field mode is naturally described in an infinite-dimensional Fock space, where the basis states $|n\rangle$ represent well-defined photon numbers n = 0, 1, 2, ... A general pure state can be written as a superposition:

$$|\psi\rangle = \Sigma\, c_n\, |n\rangle$$

where the probability amplitudes c_n are complex numbers satisfying $\Sigma|c_n|^2 = 1$. More generally, mixed states are described by density matrices ρ acting on this infinite-dimensional space.

For practical quantum information processing, we must work with finite-dimensional quantum systems. The most fundamental such system is the qubit, a two-level quantum system spanning a two-dimensional Hilbert space. To interface continuous field states with discrete quantum processors, we must therefore devise schemes to encode photon number information into registers of qubits.

Consider a qubit register of N qubits. This system spans a 2^N-dimensional Hilbert space with computational basis states that we can label by N-bit binary strings. The central question is: how should we map photon number states $|n\rangle$ to these computational basis states?

2.2 The Non-Triviality of Encoding Choice

One might initially assume that the choice of encoding is merely a matter of convention—a relabeling of states that cannot affect physical properties. However, this intuition is incorrect for several fundamental reasons:

1. Subsystem Structure: Different encodings define different subsystem decompositions of the total Hilbert space. When we identify individual qubits as subsystems, the entanglement between these subsystems depends critically on how photon numbers are distributed across the qubit register. For instance, photon numbers n and n+1 that differ by one photon might be encoded into qubit states that differ in one bit position (as in Gray code) or in multiple bit positions (as in binary). This affects which qubits become entangled.

2. Measurement Accessibility: When we measure individual qubits, the outcomes correspond to projections onto the computational basis of each qubit. Different encodings make different properties of the photon number distribution directly accessible through such measurements. The mutual information between qubits, which quantifies the correlations accessible through joint measurements, therefore depends on the encoding scheme.

3. Partial Trace Operations: Many quantum information measures, including entanglement negativity and mutual information, require computing reduced density matrices by tracing over subsystems. The partial trace operation depends explicitly on the subsystem structure, which is defined by the encoding choice. Different encodings therefore yield different reduced density matrices and different values for these information measures.

These considerations demonstrate that encoding choice is not a mere convention but a substantive physical choice that determines how quantum information is distributed across the qubit register and what correlations are accessible through measurements. A comprehensive tool for quantum field analysis must therefore support multiple encoding schemes and enable systematic comparison of their properties.

3 Encoding Schemes

3.1 Binary Encoding

The most straightforward encoding scheme is standard binary representation. In this scheme, a photon number state $|n\rangle$ is mapped to the computational basis state corresponding to the binary representation of the integer n.

For an N-qubit register, we can represent photon numbers from 0 to 2^N-1. The mapping is defined by expressing n in binary:

$n = \Sigma\, b_k\, 2^k$, *where* $b_k \in \{0,1\}$

The photon state $|n\rangle$ is then mapped to the qubit state $|b_{(n-1)}...b_1b_0\rangle$, where b_k represents the state of the k-th qubit.

Example: For a 3-qubit system (N=3):

- $|5\rangle \to |101\rangle$ (since 5 = 1×4 + 0×2 + 1×1)
- $|3\rangle \to |011\rangle$ (since 3 = 0×4 + 1×2 + 1×1)

Binary encoding has the advantage of being familiar and providing a direct correspondence between arithmetic operations on photon numbers and Boolean operations on qubit states. However, consecutive photon numbers can differ in multiple bit positions, potentially creating entanglement patterns that reflect the encoding more than the underlying physics.

3.2 Gray Code Encoding

Gray code provides an alternative binary encoding with a distinctive property: consecutive integers differ in exactly one bit position [10]. This single-bit-change property makes Gray code a natural candidate for investigating whether encoding choice affects quantum information measures.

The Gray code sequence can be generated recursively or through the formula:

$G(n) = n \text{ XOR } (n >> 1)$

where >> denotes right bit-shift and XOR is the exclusive-or operation.

Table 1
Example: For a 3-qubit system, the Gray code sequence is:

Photon n	Binary	Gray Code	Bits Changed
0	000	000	-
1	001	001	1
2	010	011	1
3	011	010	1
4	100	110	1
5	101	111	1
6	110	101	1
7	111	100	1

Note that in Gray code, moving from photon number n to n+1 always changes exactly one bit, whereas in binary encoding, the number of bit changes can vary (e.g., going from 3 to 4 in binary requires changing three bits: 011 → 100).

The single-bit-change property of Gray code makes it an important test case for investigating encoding dependence. By comparing quantum information measures calculated with binary versus Gray encoding, researchers can determine whether and how the encoding scheme affects observed quantum properties. Any differences observed would indicate that the subsystem structure defined by the encoding plays a genuine physical role in determining measured correlations.

4 Measurement Bases

Beyond the choice of encoding scheme, the selection of measurement basis provides another degree of freedom in quantum field analysis. Our tools support computation in two complementary bases: the Fock basis and the phase basis.

4.1 Fock Basis

The Fock basis $\{|n\rangle\}$ is the natural eigenbasis of the photon number operator $\hat{n} = a^\dagger a$, where $a^\dagger$ and a are the creation and annihilation operators [11,12]. In this basis, each basis state $|n\rangle$ represents a definite number of photons in the field mode.

For most cavity quantum electrodynamics systems, the Fock basis is the most natural choice because:

- The system Hamiltonian is typically diagonal or nearly diagonal in the Fock basis
- Photon number measurements directly project onto Fock states
- Standard quantum optical processes preserve or minimally mix Fock states

When working in the Fock basis, the density matrix elements $\rho_{nm} = \langle n|\rho|m\rangle$ directly represent the quantum coherences between different photon number states. The encoding process then maps these matrix elements to the corresponding qubit basis states according to the chosen encoding scheme.

4.2 Phase Basis

The phase basis provides a complementary representation of the electromagnetic field state. In this basis, states are labeled by definite phase values θ, forming a continuous basis conjugate to the photon number basis.

For practical computation with discrete systems, we work with a discretized phase basis. For an N-qubit system encoding up to 2^N photon states, we define 2^N phase basis states:

$|\theta_k\rangle = (1/\sqrt{(2^N)})\ \Sigma_n \exp(i\ n\ \theta_k)\ |n\rangle$

where $\theta_k = 2\pi k/2^N$ for $k = 0, 1, ..., 2^N-1$. This represents a discrete Fourier transform relationship between the Fock and phase bases.

The phase basis is particularly important because:

- It reveals phase coherences that may not be apparent in the Fock representation
- It provides information complementary to photon number measurements
- Different physical processes may be better characterized in phase space

To work in the phase basis, we transform the density matrix from Fock to phase representation before applying the qubit encoding. This transformation is accomplished through a discrete Fourier transform of the density matrix.

4.3 Basis-Dependent Quantum Information

An important conceptual point is that while the global quantum state $|\psi\rangle$ or ρ is basis-independent, the quantum correlations between subsystems can be basis-dependent. When we partition the system into qubit subsystems and measure properties like entanglement or mutual information, we are asking questions about correlations accessible through measurements on those specific subsystems. Since the subsystem structure depends on both the encoding scheme and the choice of basis, different basis selections can reveal different correlation patterns. Our tools enable systematic exploration of these basis-dependent quantum properties.

5 Calculated Quantities: Information Theory Measures

Our analysis tools compute several standard information-theoretic quantities that characterize the quantum state and its correlation structure. These measures provide complementary perspectives on the information content and uncertainty in the quantum field state.

5.1 Von Neumann Entropy

The von Neumann entropy is the quantum generalization of the Shannon entropy and provides a measure of the uncertainty or mixedness of a quantum state [1,2]. For a density matrix ρ with eigenvalues $\{\lambda_i\}$, it is defined as:

$$S(\rho) = -Tr(\rho \log_2 \rho) = -\Sigma_i \lambda_i \log_2 \lambda_i$$

The von Neumann entropy satisfies $S(\rho) \geq 0$, with equality if and only if ρ represents a pure state. For a maximally mixed state of dimension d, $S(\rho) = \log_2 d$. The entropy provides a measure of the number of qubits worth of uncertainty in the state.

Our tools compute both the total von Neumann entropy of the full qubit register and the entropies of individual qubits (obtained from their reduced density matrices). These quantities are useful for:

- Quantifying the total uncertainty in the quantum state
- Identifying which qubits carry the most information about the photon distribution
- Detecting transitions between different quantum phases or regimes

5.2 Linear Entropy

The linear entropy provides an alternative measure of state purity that is computationally simpler than the von Neumann entropy. It is defined as:

$S_l(\rho) = 1 - Tr(\rho^2)$

The linear entropy ranges from 0 for pure states to (d-1)/d for maximally mixed states of dimension d. It relates to the purity $P = Tr(\rho^2)$ through $S_l = 1 - P$.

The linear entropy has computational advantages because it only requires computing ρ^2, avoiding the eigenvalue decomposition needed for von Neumann entropy. It provides a useful first approximation to state mixedness and is particularly efficient for large systems.

5.3 Mutual Information

The mutual information quantifies the total correlations (both classical and quantum) between two subsystems [1,3]. For subsystems A and B described by reduced density matrices ρ_a and ρ_β, the mutual information is:

$I(A:B) = S(\rho_a) + S(\rho_\beta) - S(\rho_{a\beta})$

where $\rho_{a\beta}$ is the joint density matrix of the combined system. The mutual information is always non-negative and vanishes if and only if the subsystems are uncorrelated ($\rho_{a\beta} = \rho_a \otimes \rho_\beta$).

Our tools compute the complete mutual information matrix I(i:j) for all pairs of qubits i and j. This N×N matrix (for N qubits) provides a comprehensive map of the correlation structure:

- High mutual information indicates strong correlations between qubits
- The pattern of mutual information reveals which qubits are most strongly coupled
- Changes in mutual information patterns can signal quantum phase transitions

The mutual information matrix is particularly useful for understanding how the encoding scheme distributes correlations across the qubit register. Different encodings can produce dramatically different mutual information patterns for the same underlying field state.

6. Calculated Quantities: Entanglement Measures

Beyond information-theoretic measures, our tools compute several quantities that specifically characterize quantum entanglement—the quintessentially quantum correlation that distinguishes quantum information from classical information theory.

6.1 Bipartite Entanglement: Negativity

Entanglement between two subsystems (bipartite entanglement) is quantified through the negativity measure, which is based on the partial transpose criterion [4,5,6]. For a bipartite system AB with density matrix $\rho_{a\beta}$, we define the partial transpose with respect to subsystem B:

$\langle i,j|\rho^{TB}|k,l\rangle = \langle i,l|\rho|k,j\rangle$

The Peres-Horodecki criterion states that $\rho_{a\beta}$ is separable (non-entangled) if and only if ρ^{TB} has no negative eigenvalues. The negativity quantifies the violation of this criterion:

$N(\rho_{a\beta}) = (||\rho^{TB}||_1 - 1)/2$

where $\|\cdot\|_1$ denotes the trace norm (sum of absolute values of eigenvalues). The negativity vanishes for separable states and increases with entanglement strength.

Our tools compute negativity for all pairs of qubits, producing an N×N negativity matrix. The computational procedure involves:

- Computing the reduced density matrix ρ_{ij} for each qubit pair (i,j)
- Performing the partial transpose operation
- Computing eigenvalues of the partially transposed matrix
- Calculating the trace norm and extracting the negativity

The partial trace operation required for this calculation explicitly depends on the subsystem structure, and therefore on the encoding scheme. This makes the negativity matrix particularly sensitive to encoding choice.

6.2 Tripartite Entanglement: Three-Tangle

While bipartite entanglement characterizes correlations between pairs of qubits, genuine multipartite entanglement reveals richer correlation structures. For three-qubit systems, the three-tangle provides a measure of genuine tripartite entanglement that cannot be reduced to pairwise correlations [7,13].

The three-tangle τ(A:B:C) for three qubits A, B, and C is defined through the Coffman-Kundu-Wootters monogamy relation [7]:

$$C^2(A{:}BC) = C^2(A{:}B) + C^2(A{:}C) + \tau(A{:}B{:}C)$$

where C(A:B) denotes the concurrence between qubits A and B. The three-tangle quantifies the entanglement that is genuinely tripartite—it cannot be accounted for by any decomposition into bipartite entanglements.

For systems with N > 3 qubits, our tools compute the three-tangle for all triplets of qubits, providing a complete characterization of tripartite entanglement structure. This reveals:

- Which triplets of qubits exhibit genuine three-way entanglement
- Whether the system contains GHZ-type correlations
- How multipartite entanglement is distributed across the register

The three-tangle is particularly valuable for investigating encoding dependence because genuine multipartite entanglement is even more sensitive to subsystem structure than bipartite entanglement.

6.3 Computational Challenges and Numerical Methods

Computing entanglement measures for qubit registers encoded from electromagnetic field states presents several computational challenges:

1. Matrix Dimensionality: An N-qubit system has a $2^N \times 2^N$ density matrix. For typical systems (N = 6-10), this ranges from 64 to 1024 dimensions, requiring careful numerical handling.

2. Phase Basis Stability: The discrete Fourier transform required for phase basis representation can introduce numerical artifacts if not handled carefully. Our implementation uses stabilization techniques to maintain positivity and trace normalization of the density matrix.

3. Partial Transpose Eigenvalues: Computing negativity requires eigenvalue decomposition of the partially transposed matrix. Near-zero eigenvalues must be handled carefully to avoid spurious entanglement detection.

Our implementation addresses these challenges through adaptive numerical precision, careful matrix conditioning, and validation against known quantum states (discussed in Section 8).

7 Tool Implementation

7.1 Architecture Overview

Both QubitInfoPanel[] and QubitInfoPanelGray[] follow a common computational architecture with three main stages:

Stage 1: Input Processing and Basis Transformation

- Accept density matrix ρ in Fock basis
- If phase basis analysis requested, apply discrete Fourier transform
- Stabilize matrix to ensure proper normalization and positivity

Stage 2: Qubit Encoding and Subsystem Decomposition

- Determine number of qubits N needed to represent the system
- Apply encoding map (Binary or Gray) to reorder density matrix
- Establish subsystem structure for qubit partitioning

Stage 3: Quantum Information Calculation

- Compute reduced density matrices for all subsystems
- Calculate entropies, mutual information, and entanglement measures
- Format and return results

7.2 Key Functions and Modules

The implementation consists of several core functions:

InitialStateGeneration[]: Creates initial quantum states for testing purposes. Generates standard states including computational basis states, Bell states, GHZ states, and W states.

ReduceToSubset[]: Performs partial trace operations to compute reduced density matrices. Takes a density matrix and a specification of which subsystems to trace over, returning the reduced state of the remaining subsystems.

Negativity calculation modules: Compute partial transpose, diagonalize, and extract negativity measure. Includes specialized handling for numerical stability near zero eigenvalues.

Mutual Information modules: Calculate entropies of subsystems and joint systems, computing mutual information via S(A) + S(B) - S(AB).

Three-Tangle modules: Compute concurrences and apply the Coffman-Kundu-Wootters formula to extract genuine tripartite entanglement.

7.3 Input Specifications

Input Format:

- Density matrix ρ as a complex-valued matrix (dimensions must be a power of 2)
- Optional basis specification (Fock or Phase)
- Optional numerical precision parameters

7.4 Output Formats

Our tools provide output in two complementary formats designed for different use cases: machine-readable CSV files for automated analysis and human-readable PDF dashboards for visual inspection.

CSV Export for Automated Analysis:

All calculated quantum information measures are automatically exported to CSV (Comma-Separated Values) files. This enables:

- Integration into automated analysis pipelines
- Building large-scale measurement databases through loop execution
- Statistical analysis across parameter sweeps
- Import into external analysis tools (Python, R, MATLAB, etc.)

The CSV format includes all computed quantities:

- Total von Neumann entropy of the system
- Vector of individual qubit entropies
- Linear entropy and purity
- Complete N×N mutual information matrix
- Complete N×N negativity matrix for bipartite entanglement

- Three-tangle values for all qubit triplets
- Metadata including encoding type, basis, timestamp, and computation time

This machine-readable format is particularly valuable when conducting parameter studies that require hundreds or thousands of function evaluations. The functions can be called in loops to systematically explore the parameter space, with all results automatically archived in CSV format for subsequent analysis.

PDF Dashboard Visualization:

For human interpretation and quality control, the tools generate comprehensive single-page PDF dashboards that present all quantum information measures in a carefully organized visual layout. These dashboards are designed to provide information at a glance, enabling rapid assessment of:

- Overall quantum state structure through density matrix visualizations
- Qubit-by-qubit entropy and entanglement patterns
- Phase space representations and photon number distributions
- Correlation matrices showing qubit-qubit relationships
- Key statistical measures and quantum parameters

The dashboard layout includes multiple panels showing different aspects of the quantum state simultaneously. Density matrices are displayed in both standard matrix form and as graphical representations with arrows indicating phase relationships. Reduced density matrices for individual qubits are shown alongside their entropy values, providing immediate insight into which qubits carry the most information.

Figures 1 and 2 show example dashboards for the W state, an entangled three-qubit state given by:

$|W\rangle = (1/\sqrt{3})(|001\rangle + |010\rangle + |100\rangle)$

This state exhibits symmetric pairwise entanglement but zero three-tangle, making it an ideal test case. Figure 1 shows the analysis with Binary encoding, while Figure 2 shows Gray code encoding of the identical quantum state. Comparing these dashboards reveals how encoding choice affects the distribution of quantum correlations across the qubit register while preserving global properties.

The combination of CSV and PDF outputs provides optimal workflow support: CSV files enable systematic large-scale studies and automated analysis, while PDF dashboards facilitate quality control, debugging, and physical interpretation of results.

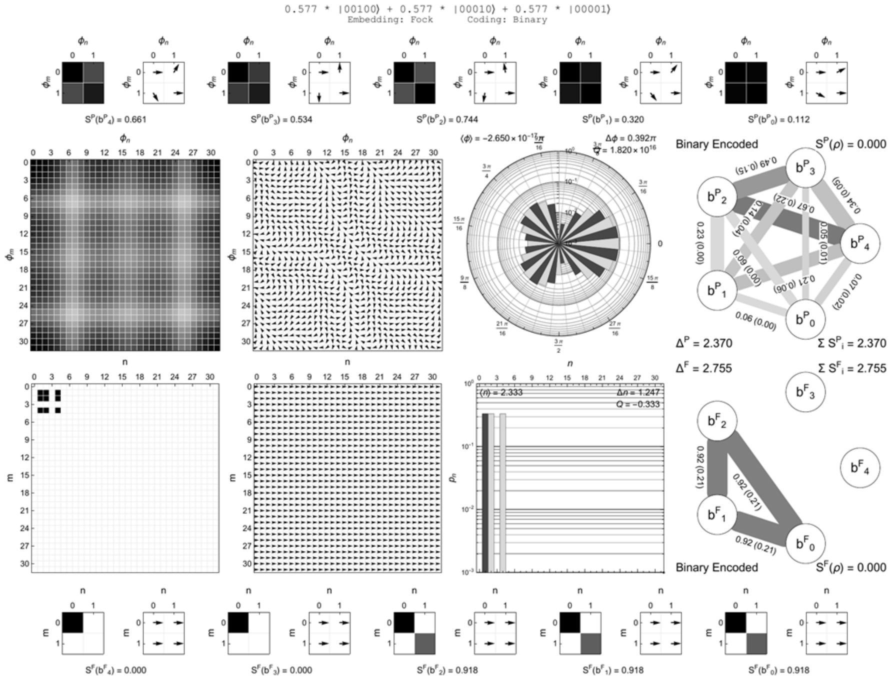

Figure 1

Dashboard for W state |W⟩ = (1/√3)(|001⟩ + |010⟩ + |100⟩) with Binary encoding in Fock basis. The visualization shows density matrix representations, reduced density matrices for each qubit with their entropies, photon number distribution, phase space representation, mutual information, and negativity patterns. Note the symmetric pairwise entanglement pattern characteristic of the W state.

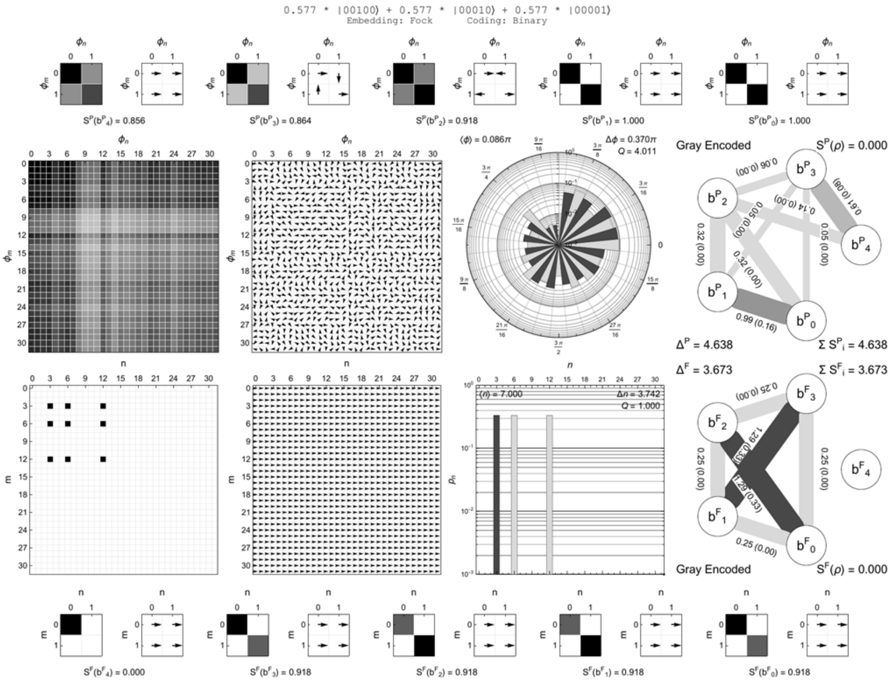

Figure 2

Dashboard for the same W state with Gray code encoding in Fock basis. Comparing with Figure 1 reveals how encoding choice redistributes quantum correlations across the qubit register. While global properties (total entropy, state purity) remain invariant, the subsystem entanglement patterns differ significantly, demonstrating the encoding-dependent nature of qubit-qubit correlations.

8 Validation Examples

To ensure correctness of our computational tools, we perform extensive validation using quantum states with known entanglement and information-theoretic properties. This section describes our validation methodology and results.

8.1 Standard Test States

We validate against several classes of standard quantum states:

1. Product States:

- Computational basis states $|000\rangle$, $|001\rangle$, $|010\rangle$, etc.

- Expected: Zero entanglement, zero mutual information
- Result: All entanglement measures correctly return zero

2. Bell States:

- Maximally entangled two-qubit states: $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$
- Expected: Negativity = 0.5, Mutual Information = 2 bits
- Result: Computed values match theoretical predictions within numerical precision

3. GHZ States:

- Three-qubit maximal entanglement: $|GHZ\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$ [8]
- Expected: Zero pairwise entanglement, maximal three-tangle
- Result: Three-tangle = 1.0, pairwise negativities vanish correctly

4. W States:

- Three-qubit state: $|W\rangle = (|100\rangle + |010\rangle + |001\rangle)/\sqrt{3}$ [8]
- Expected: Non-zero pairwise entanglement, zero three-tangle
- Result: Pairwise negativities correctly computed, three-tangle vanishes

8.2 Validation Metrics

For each test state, we verify:

- **Trace preservation:** $\mathrm{Tr}(\rho) = 1$ for all reduced density matrices
- **Positivity:** All eigenvalues of ρ are non-negative
- **Hermiticity:** $\rho = \rho^\dagger$ for all density matrices
- **Entanglement bounds:** $0 \leq N(\rho) \leq (d-1)/2$ for d-dimensional systems
- **Information bounds:** $0 \leq I(A:B) \leq 2 \min(S(A), S(B))$

All validation tests pass with numerical precision better than 10^{-10} for double-precision arithmetic.

8.3 Encoding Consistency Checks

An important validation concerns the relationship between Binary and Gray encodings. While these encodings can produce different entanglement values (demonstrating encoding dependence), certain global properties should remain invariant:

- **Total system entropy:** $S(\rho\_total)$ should be identical for both encodings
- **Global purity:** $Tr(\rho^2)$ should be encoding-independent
- **Sum rules:** Certain sum rules for entanglement must hold regardless of encoding

We verify these consistency conditions for all test states, confirming that encoding differences affect subsystem properties but preserve global invariants as expected.

9 Discussion and Applications

9.1 Investigating Encoding Dependence

The primary application of these tools is to systematically investigate whether and how encoding choices affect quantum information measures. Key questions that can be addressed include:

- Do different encoding schemes (Binary vs Gray) produce different entanglement patterns for the same field state?
- Which qubits show the strongest encoding dependence in their correlations?
- Are there parameter regimes where encoding choice becomes particularly important?
- Can we identify universal features that are encoding-independent?

By running both QubitInfoPanel[] and QubitInfoPanelGray[] on the same input state and comparing results, researchers can quantify encoding effects and determine their relevance for specific applications.

9.2 Basis-Dependent Quantum Correlations

The ability to compute in both Fock and phase bases enables investigation of complementary aspects of quantum correlations:

- Which correlations are enhanced in phase basis versus Fock basis?
- Are there states where phase basis reveals entanglement invisible in Fock basis?
- How do basis and encoding choices interact to determine measured properties?

These questions are relevant for designing optimal measurement strategies and understanding the full structure of quantum correlations in field systems [9,16].

9.3 Application to Various Quantum Systems

While our motivation comes from cavity quantum electrodynamics, these tools have broader applicability:

- **Quantum optics:** Analyzing squeezed states, coherent state superpositions, and cat states
- **Circuit QED:** Studying qubit-resonator systems and photon state engineering
- **Quantum information:** Investigating optimal encoding for quantum communication
- **Quantum metrology:** Understanding how digitization affects measurement precision

Any system involving quantized bosonic modes that needs to be interfaced with discrete qubit processors can benefit from these analysis tools.

9.4 Representation Engineering

An emerging concept enabled by these tools is representation engineering: the deliberate choice of encoding and measurement basis to optimize specific quantum information properties. For instance:

- Maximizing accessible entanglement for specific field states
- Minimizing decoherence effects through appropriate encoding
- Enhancing specific correlation patterns relevant for applications
- Balancing multiple objectives through encoding optimization

Our tools provide the computational infrastructure needed to explore this space systematically and design optimal encoding strategies for specific goals.

10 Conclusions

We have presented QubitInfoPanel[] and QubitInfoPanelGray[], comprehensive computational tools for investigating the quantum digitization of electromagnetic field states. These Mathematica-based functions provide researchers with the capability to:

- Encode arbitrary field density matrices into qubit registers using Binary or Gray code schemes
- Compute comprehensive quantum information measures including entanglement, mutual information, and entropy
- Analyze states in both Fock and phase measurement bases
- Systematically investigate encoding-dependent and basis-dependent quantum properties

The quantum digitization problem—how to represent continuous field states as discrete qubit registers—is fundamental to the interface between quantum field theory and quantum information processing. Our work demonstrates that this is not merely a matter of convention: the choice of encoding scheme substantively affects the quantum correlations that are accessible through measurements on the qubit subsystems.

The parallel implementation of Binary and Gray encodings enables direct investigation of encoding effects. By comparing results from QubitInfoPanel[] and QubitInfoPanelGray[] on identical input states, researchers can quantify how encoding choice influences entanglement patterns, mutual information structures, and other quantum information measures.

Similarly, the support for both Fock and phase bases allows exploration of basis-dependent quantum correlations, revealing complementary aspects of the quantum state that may not be apparent in a single representation.

Extensive validation against known quantum states (Bell states, GHZ states, W states, and product states) confirms the correctness of our implementation. All computed measures satisfy appropriate bounds and reproduce theoretical predictions to high numerical precision.

These tools open new avenues for research at the intersection of quantum field theory and quantum information science. Potential applications include:

- Systematic exploration of representation-dependent quantum properties
- Design of optimal encoding schemes for specific quantum information tasks
- Investigation of quantum phase transitions and critical phenomena in encoded systems
- Development of novel quantum measurement strategies

Looking forward, several extensions are possible. Additional encoding schemes beyond Binary and Gray (such as qutrit encodings or custom optimized encodings) could be incorporated. More sophisticated multipartite entanglement measures beyond three-tangle could be added for larger systems. Integration with quantum simulation packages could enable direct application to specific physical systems.

The fundamental insight enabled by these tools is that the quantum digitization problem is not merely a technical detail but a substantive physical choice that shapes the observable properties of quantum systems. As quantum technologies increasingly require interfaces between continuous and discrete quantum systems, understanding and optimizing these encoding choices will become crucial.

We have provided the research community with validated, flexible tools to explore this landscape systematically. By enabling direct comparison of different encoding schemes and measurement bases, QubitInfoPanel[] and QubitInfoPanelGray[] facilitate investigation of representation-dependent quantum information—a frontier that promises both fundamental insights and practical applications in quantum technology.

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