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Segmentation of Flat curves for Generating Programs for a Robotic Manipulator Performing the Functions of Processing 3D Surfaces by Applying Coatings

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***Abstract* — This paper presents an algorithm for approximating planar curves with conjugated circular arcs and straight-line segments. Approximation of digital planar curves using line segments and analytic curve pieces is a problem of fundamental importance in image analysis and pattern recognition. A specific variation of this problem is approximation by conjugated arcs and line segments, which also arises in robotics, particularly in motion planning. We propose an algorithmic solution to this problem, designed for the automatic generation of programs for robotic manipulators in the AS language. The originality of the method lies in employing a special representation of planar curves that is invariant under translation, rotation, and scaling. Such a representation enables the development of an optimal algorithm that automatically determines the number and placement of conjugated arcs and line segments approximating a given curve with a prescribed accuracy. The simplicity, flexibility with respect to input data, and low computational requirements of the proposed method make it suitable not only for image recognition tasks but also for robotic trajectory planning.**

Keywords* — *approximation of planar curves, pattern recognition, motion planning, robotics.

I. Introduction

The problem of accurate geometric representation and segmentation of planar curves has long been fundamental in computer vision, image analysis, and robot trajectory planning. In modern industrial robotics, the demand for high-precision path generation has significantly increased due to the growing use of robotic manipulators in additive manufacturing, plasma spraying, and surface processing [1-3]. The ability to approximate complex trajectories by simple geometric primitives such as circular arcs and straight segments is crucial for generating executable robot programs while maintaining smooth motion and continuous curvature [4, 5].

In robotic machining and coating applications, discontinuities in curvature lead to undesirable dynamic effects, including abrupt changes in acceleration and tool wear. Therefore, many studies focus on continuous curvature path planning (G^2-continuity) and curvature-based representations of planar trajectories [6]. Modern approaches exploit differential geometry to describe trajectories invariant under translation, rotation, and scaling, ensuring robustness of motion planning algorithms [7].

Recent research has also emphasized shape descriptors based on curvature distributions, such as curvature-scale space and angular functions, which enable compact and invariant representations of planar curves [8, 9]. Such descriptors are widely applied not only in pattern recognition and digital shape analysis but also in the segmentation and smoothing of robot paths [10-12].

In the context of robotic program generation, segmentation of planar contours into conjugated circular arcs and straight-line segments provides a direct mapping to robot motion primitives such as MOVE and CIRCLE commands in industrial robot programming languages (e.g., Kawasaki AS, ABB RAPID, or KUKA KRL). This type of approximation ensures physically realizable trajectories and eliminates infinite acceleration points inherent in polygonal path definitions [13].

This paper presents an algorithm for approximating digital planar curves by conjugated circular arcs and line segments based on an invariant representation of the curve known as the angular characteristic function. The method automatically determines the optimal number and placement of segments to achieve a specified approximation accuracy. The proposed approach combines simplicity of implementation, low computational cost, and robustness to geometric transformations, making it suitable for both robotic trajectory planning and digital image analysis.

II. The Problem of Approximating a Planar Curve by a Sequence of Mutually Conjugated Circular Arcs and Straight-Line Segments

Some problems in motion planning and robotics require that the plane curve representing a spatial trajectory be approximated by a special type of line, which we will call polysegment lines, defined as follows:

Definition 1. We define a polysegment line as a pair (A, B), where $A = \{v_1, v_2, \dots, v_n\}$ is a sequence of n vertices $\forall j \in \{1, .., n\}\ v_j \in \mathbb{R}^2$ defining the polyline p, and $B = \{s_1, s_2, \dots, s_{n-1}\}$ is a sequence of $n-1$ segments. Each segment s_k where $k \in \{1, .., n-1\}$ is a plane curve with two ends at vertices v_k and v_{k-1} which is either a line segment or an arc of a circle, connecting vertices v_k and v_{k-1}. A polysegment line in which any two adjacent segments s_j and $s_{j+1}$are either conjugate circular arcs or a straight line segment, conjugate with an arc of a circle, we will call a conjugate polysegment line. An example of a polysegment curve is shown in Fig. 1.

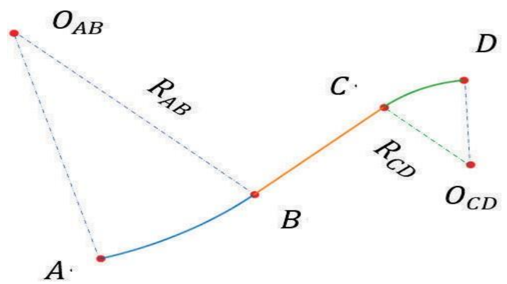

Fig. 1. Polysegment curve AD, composed of three segments AB, BC and CD, where AB and CD are circular arcs tangentially joined to the straight segment BC

As an illustrative problem arising in robotic automation of manufacturing, we address the automatic generation of an AS-language program for a robot manipulator that cuts contours from a flat metal sheet. In essence, an AS program is a linear sequence of commands with the mnemonics “MOVE” and “CIRCLE,” which direct the end effector to follow, respectively, straight segments and circular arcs. Consequently, the spatial path of the tool (the cutter in this context) and the corresponding trace left on the processed surface are described by polysegment curves.

Attempting to solve this task with an algorithm that approximates a smooth curve by a polyline leads to a fundamental difficulty. The operands of the MOVE and CIRCLE commands specify the tool's traversal speed; if the program prescribes motion along a broken line, thenregardless of how the speeds on the segments are chosen the manipulator cannot execute such a program exactly as written. From a physical standpoint the reason is straightforward: motion defined by a polyline entails infinite normal (radial) acceleration at the polyline's vertices. In practice, when such a program is loaded into the controller, the manipulator decelerates and effectively halts each time the tool passes a vertex of the notional polyline on the sheet surface (i.e., along the trajectory trace). For certain cutting processes (e.g., plasma cutting), the temporal accuracy of the trajectory is critical; if the tool speed deviates substantially from the setpoint, the operation may fail or the tool may even be damaged.

This problem is avoided by approximating the smooth curve with a conjugate polysegment rather than with a polyline. It is, of course, desirable to minimize the number of conjugation points along the polysegment that approximates a given smooth planar curve; at the same time, reducing the number of segments inevitably reduces the achievable approximation accuracy. Taking these two considerations into account, we formulate the optimal planar approximation problem as follows: given a maximum admissible error σ, find a polysegment with the minimum number of segments that approximates the specified curve with an error not exceeding σ.

III. Optimal approximation of a smooth curve by a polysegment line and the angular characteristic function

3.1 Motivation and Related Work

The problem of approximating a planar curve by a sequence of geometric primitives (mutually conjugated circular arcs and straight-line segments) is closely related to the choice of curve representation used in the approximation algorithm. Any such approximation can be regarded as a shape descriptor of the original curve. For a continuous planar curve with two endpoints, one can define such basic attributes as position, orientation, and scale. By applying translation, rotation, and uniform scaling transformations, an infinite number of curves differing only in position, orientation, and scale can be obtained, all sharing the same intrinsic shape.

From this general standpoint, it is desirable that the representation of the curve used in the approximation algorithm separates the description of the curve's shape from the parameters describing its position and orientation on the plane. The approximation algorithm should then operate directly on the shape descriptor.

In the algorithm presented in this paper for approximating a planar curve by a polysegment curve, we use as a shape descriptor a special function — the angular characteristic function of the curve. As will be shown below, any smooth planar curve can be uniquely associated with a continuous real-valued function of a single variable defined on the unit interval. A smooth curve can be reconstructed from its angular characteristic function and three additional parameters describing position, orientation, and scale (the latter corresponding simply to the curve length).

Originally, the angular characteristic function was used as a shape descriptor in image recognition tasks, specifically for developing an optical system for handwritten symbol recognition. This curve descriptor, invariant under translation, rotation, and scaling, proved to be very effective in digital image analysis. Later it was found that one of the mathematical properties of this construction allows one to design a highly efficient algorithm for planar curve approximation by polysegment lines. In fact, the angular characteristic function of a polysegment curve is a piecewise linear function. Consequently, the problem of optimal approximation of a smooth planar curve by a polysegment curve reduces to the well-known problem of polyline simplification, for which efficient algorithms such as the Ramer–Douglas–Peucker algorithm exist.

The question of selecting an appropriate descriptor capable of representing the shape of a planar curve naturally arises in image processing and recognition problems and has long attracted research attention. The idea of using the curvature distribution along the curve length as such a descriptor was proposed in earlier studies on curvature-based shape signatures [14]. More recently, several works have explored invariant representations of planar curves learned through machine learning methods [15], as well as curvature-continuous approximations suitable for robotic trajectory planning [16, 17].

In robotics and motion planning, approximating planar trajectories with arcs and line segments remains highly relevant. Paths composed of alternating straight and circular elements exhibit desirable kinematic properties, such as bounded curvature and feasible steering control, as in the classical Dubins path problem [18]. Recent studies further develop this approach, introducing adaptive segmentation and curvature-continuous fitting of planar paths for robotic manipulators and autonomous vehicles [19].

Therefore, our choice of the angular characteristic function as the fundamental shape descriptor is justified by two factors:

1) the need for an invariant representation independent of position, orientation, and scaling;

2) the direct link between the curve's shape descriptor and its geometric composition from arcs and line segments, which simplifies algorithmic implementation.

The proposed method automatically determines the number and placement of conjugated arcs and straight segments to achieve the required approximation accuracy with minimal computational cost.

3.2 Definition and main properties of the angular characteristic function of a curve

Smooth curves on the plane with two endpoints are considered as smooth mappings of the form $\gamma: I \to \mathbb{R}^2$, where $I = [0, T]$ is an interval on the real axis. If we interpret the parameter $t \in I$ physically as time, then a plane curve given parametrically as $\gamma(t)$ can be viewed as the trajectory of a point moving on the plane, with the velocity vector defined as $\gamma^{\cdot}$. and the acceleration vector as $\gamma^{\cdot\cdot}$. In the special case when $\forall t \in I \ |\gamma^{\cdot}(t)| = 1$, the parameter t can be interpreted as the arc length s traversed by the moving point from the starting position. Such a parametrization is called the natural (arc-length) parametrization of the curve. Every smooth curve with two endpoints can therefore be represented in the form (1), where s denotes the natural parameter and L is the total length of the curve.

$$\forall s \in [0, L] \ \gamma(s) = \gamma(0) + \int_0^s \gamma^{\cdot}(s) ds \tag{1}$$

Since, under the natural parametrization of the curve, the condition is satisfied, the vector can be represented as (2).

$$\gamma^{\cdot}(s) = t(\alpha(s)), \tag{2}$$

where the mapping $t: S^1 \to \mathbb{R}^2$ is given by formula (3), which defines the vector components in an orthonormal basis, and the function $\alpha(s)$ determines the shape of the curve.

$$t(\alpha) = [\cos\alpha, \sin\alpha]^T \tag{3}$$

Differentiating both sides of equation (2) with respect to s yields formula (4) for the second derivative:

$$\frac{d^2}{ds^2}\gamma(s) = \frac{d}{ds} t(\alpha(s)) = \alpha^{\cdot}(s) n(\alpha(s)) \tag{4}$$

where $n: S^1 \to \mathbb{R}^2$ is defined by formulas (5):

$$n(\alpha) = \frac{d}{d\alpha} t(\alpha) = [-\sin\alpha, \cos\alpha]^T \tag{5}$$

Obviously, for any α the identities $|n(\alpha)| = 1$ and $(n(\alpha), t(\alpha)) = 0$, that is, $n(\alpha)$ is a unit vector orthogonal to $t(\alpha)$. Since $t(\alpha(s))$ is the unit tangent vector to γ at the point $\gamma(s)$, $n(\alpha(s))$ is the unit normal vector. As we see, the vectors $t(s)$ and $n(s)$ (the unit tangent and unit normal to the curve) satisfy the differential equation (6).

$$\frac{d}{ds} t(s) = k(s) n(s), \tag{6}$$

where, the real number $k(s)$ is called the signed curvature. It is straightforward to show that $|k(s)| = \frac{1}{R}$, where R is the radius of the circle tangent (the osculating circle) to γ at the point $\gamma(s)$. The sign of $k(s)$ depends on both the orientation of the plane and the orientation of the curve provided by the parametrization. From formulas (4) and (6) it follows that equation (7) holds, which determines the function $\alpha(s)$ up to an additive constant.

$$\forall s \in [0, L] \ \alpha^{\cdot}(s) = k(s) \tag{7}$$

It should be noted that formula (7) is directly related to another possible geometric interpretation of the concept of signed curvature. Recall that the angle $\varphi(s)$ between $t(s)$ and the positive direction of the x-axis is called the turning angle of the plane curve (see Fig. 2). The signed curvature $k(s)$ can also be interpreted as the rate of change of the turning angle with respect to the arc length, which corresponds to the formula $k(s) = \frac{d\varphi}{ds}(s)$.

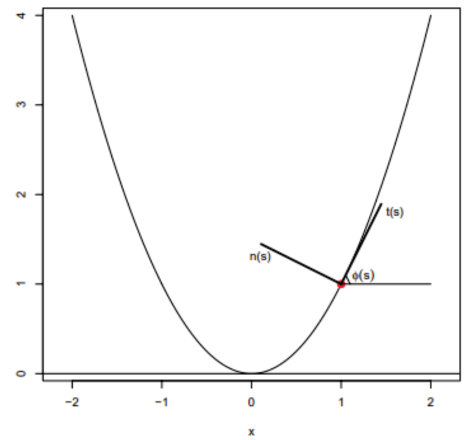

Fig. 2. The positions of $t(s)$, $n(s)$ and $\varphi(s)$ relative to an arbitrary red point $\gamma(s)$ on a quadratic curve

As we can see, it is, in principle, possible to define a shape descriptor of a smooth curve as the turning angle by choosing a fixed Cartesian coordinate system on the plane. However, we define the function $\alpha(s)$, which describes the shape of the curve, as the integral of the signed curvature in the form (8).

$$\alpha(s) = \int_0^s k(\tau)\, d\tau \tag{8}$$

Such a definition is not tied to the choice of a coordinate system on the plane and is invariant with respect to the group of plane transformations generated by translations and rotations. The function $\alpha(s)$ defined in this way also has a simple geometric interpretation (see Fig. 3).

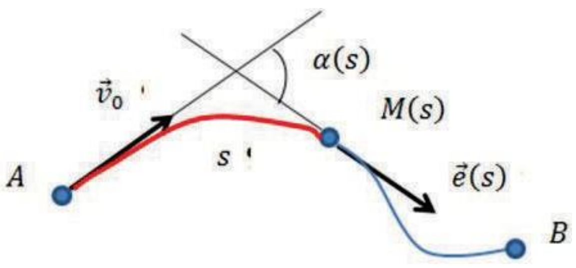

Fig. 3. $\alpha(x)$ represents the angle between the initial tangent vector and the tangent vector at a point along the curve

The function $\alpha(s)$ is defined on the interval $[0, L]$, where L is the length of the curve γ, and it is not invariant under the operation of scaling (uniform stretching of the plane). To obtain a shape descriptor of the curve that is invariant with respect to translations, rotations, and scaling, we define on the interval $I = [0,1]$ the function $\theta(x)$ by expression (9).

$$\theta(x) = \alpha(x \cdot L) \tag{9}$$

We will call $\theta(x)$ the function of the angular characteristic of the curve γ (or simply the angular characteristic of γ). The angular characteristic function of the curve can be defined analytically by formula (10).

$$\theta(t) = \int_0^{Lt} k(Ls)ds \tag{10}$$

Differentiating both sides of equation (10) gives formula (11), which expresses an obvious but important property of the angular characteristic function.

$$\forall t \in I \; \theta^{\cdot}(t) = Lk(Lt) \tag{11}$$

There exist only two types of curves with two endpoints that have constant curvature, namely, a circular arc and a straight-line segment. The angular characteristic functions of these curves have particularly simple forms. Let us consider these two cases separately.

First case, straight-line segment: obviously, the curvature of a straight line is zero at every point of the segment. From (11) and the initial condition $\theta(0) = 0$ we obtain the Cauchy problem for the differential equation $\forall s \in I \; \theta^{\cdot}(s) = 0$, whose solution is a constant function $\forall s \in I \; \theta(s) = 0$.

Second case, circular arc of radius R, and length l. The curvature of a circle is constant and equal in magnitude to $\frac{1}{R}$. In this case, equation (11) corresponds to the differential equation $\theta^{\cdot}(t) = lkt,\ t \in I$, where $|k| = \frac{1}{R}$. The solution of the Cauchy problem for this equation with the initial condition $\theta = 0$ has the form $\theta(t) = \frac{l}{R}t$. Here, $|\theta(1)| = \alpha$, where $\alpha = \frac{l}{R}$ is the central angle that subtends an arc of length l on the circle.

These two facts, together with the property of the angular characteristic function of a smooth plane curve described below, make it possible to understand the form of the angular characteristic function of a polysegment line. Let γ be an arbitrary smooth plane curve with two ends at points A and B. Denote by θ the function of the angular characteristic of this curve. Assume that the length of the γ be equal to L. Let us choose an arbitrary point on the curve M, and denote by l_1 the length of the arc AM of the curve γ (in this case, of course, the length of the arc MB will be equal to $L - l_1$). Let us denote by θ_1 the function of the angular characteristic of the curve AM and by θ_2 the function of the angular characteristic of the curve MB. Then the function θ can be expressed through the functions θ_1 and θ_2, and analytically the relationship between the functions phi, phi1 and θ_2 is expressed by formula (12).

$$\forall x \in [0,1] \;\; \theta(x) = \begin{cases} \theta_1\left(\frac{Lx}{l_1}\right), & if\ x \in [0, \frac{l_1}{L}) \\ \theta_1(1) + \theta_2\left(\frac{(x - \frac{l_1}{L})\,L}{L - l_1}\right), & if\ x \in (\frac{l_1}{L}, 1] \end{cases} \tag{12}$$

Since the characteristic functions of the individual segments of a polysegment curve are linear functions, the graph of the angular characteristic function is a polyline (see Fig. 4).

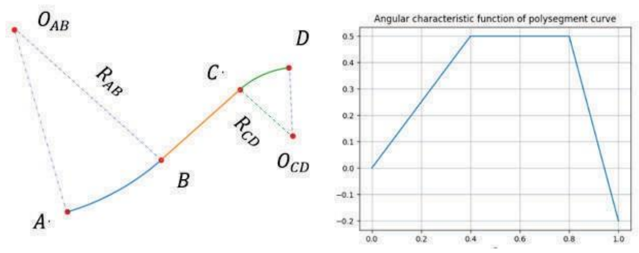

Fig. 4. The polysegment line AD (left) and the graph of its angular characteristic function (right). The graph of the angular characteristic function of the curve AD is a polyline whose three segments correspond to the three segments of AD

As will be shown below, the angular characteristic function of a curve, which describes its shape, together with the data on the curve's position, orientation, and length, contains complete information about the curve and can be used to define it. To demonstrate this, note that from representation (12), taking into account the results obtained above, formula (13) follows, which provides the reconstruction of the curve from its angular characteristic function.

$$\forall s \in [0, L] \; \gamma(s) = \gamma(0) + \int_0^s t\left(\theta\left(\frac{\tau}{L}\right)\right) d\tau \tag{13}$$

Naturally, any given angular characteristic function of a curve corresponds to an infinite set of plane curves, and therefore the obtained result can be interpreted in another way. For a given triple (v_0, e_0, l) where vector $v_0 = [v_{0x}, v_{0y}]^T$ defines a position of the point on the plane, $e_0 \in \mathbb{R}^2$ is a constant unit vector, and $l > 0$ is a scalar parameter, an arbitrary angular characteristic function φ uniquely determines a plane curve γ, given by equation (14).

$$\gamma(s) = v_0 + \int_0^l [R(\varphi(\tfrac{x}{l}))\, e_0]\, dx, \quad (14)$$

where $R(\alpha)$ is the orthogonal matrix of the two dimension rotation operator, defined by expression (15).

$$R(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \quad (15)$$

Let us denote by θ an arbitrary continuous function defined on the interval $[0,1]$ and satisfying the condition $\theta(0) = 0$. For this function θ and an arbitrary triple (v_0, e_0, l), the formula (15) uniquely determines a curve on the plane for which θ will be a function of the angular characteristic.

IV. Algorithm for approximating a continuous planar curve by a polysegment line

As shown above, the graph of the angular characteristic function of a polysegment line is a polyline. The main idea of the proposed approximation algorithm is based on this fact and consists in constructing a piecewise-linear approximation $\theta^\wedge$ of the angular characteristic function θ of a given planar curve γ, and then reconstructing, from $\theta^\wedge$ a polysegment line $\gamma^\wedge$, and then reconstructing, from $\gamma^\wedge$ and γ coincide. For the computer implementation of the algorithm, it is convenient to use a discrete representation of continuous planar curves by polylines defined by a sequence of vertices $\{v_1, v_2, \ldots, v_N\}$, where.$\forall k \in \{1, \ldots, n\}$ $v_k \in \mathbb{R}^2$. For a curve given parametrically by the functions $x(t)$ and $y(t)$, the polyline representing the curve is obtained by discretizing these functions, with the coordinates of the polyline vertices defined as $v_k = [x_k, y_k]^T$. As will be shown below, the definition of the angular characteristic function naturally extends to the case of polylines. For given polyline $\{v_1, v_2, \ldots, v_n\}$ we denote $\alpha_k = \begin{cases} 0, & if\ k = 1 \\ \angle(e`_k, e`_{k-1}), & otherwise \end{cases}$. Let $\theta_k = \sum_{i=1}^{k} \alpha_k$ and we define the sequence $\{s_k\}$ by the formulas $s_k = \begin{cases} 0, & if\ k = 1 \\ \frac{\sum_{j=1}^{k} |e`_j|}{L}, & otherwise \end{cases}$, where $L = \sum_{j=1}^{n-1} |e`_j|$ is a total length of the polyline. Then the angular characteristic function $\theta(x)$is a piecewise constant function defined as $\theta(x) = \theta_k$ if $x \in [s_k, s_{k+1})$ (see Fig. 5).

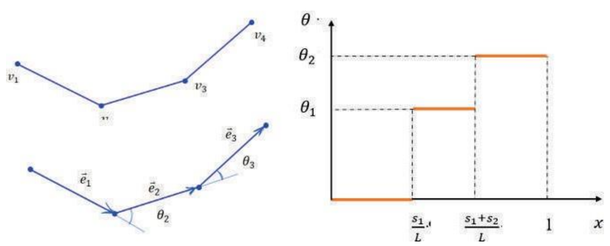

Fig. 5. The angular characteristic function of a polyline is a piecewise constant function

As will become evident below, working with a discretely sampled angular characteristic function is much more convenient than with a piecewise constant one. Therefore, in the algorithm implementation, we discretized the computed angular characteristic functions, and throughout the following discussion we will assume that the angular characteristic function is given as a list of samples. An example of the graph of an angular characteristic function computed in this way for a synthetic example is shown in Fig. 6.

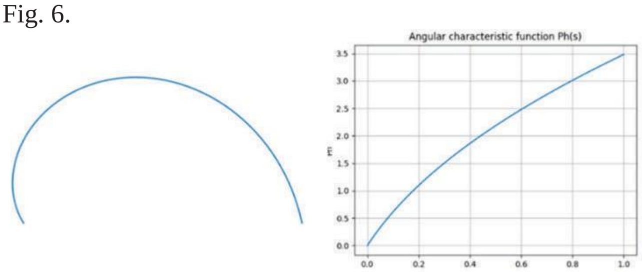

Fig. 6. A plane curve defined by the parametric equations $x(t) = -(1 + 2t)\cos t$ and $y(t) = (1 + 2t)\sin t$ (left) and the graph of its angular characteristic function (right)

The problem of optimal piecewise-linear approximation of a function given by discrete samples reduces to the problem of optimal polyline decimation. For approximating the angular characteristic function, we used the Ramer–Douglas–Peucker algorithm (also known as the Douglas–Peucker or iterative end-point fit algorithm). This algorithm simplifies a curve composed of line segments to a similar curve with fewer points.

An important property of the Ramer–Douglas–Peucker algorithm is its ability to automatically select the optimal subset of vertices from the original polyline to be retained during decimation, thereby ensuring the user-defined approximation accuracy.

Fig. 7 shows an example of applying the Ramer–Douglas–Peucker algorithm with different values of the maximum allowable approximation error parameter.

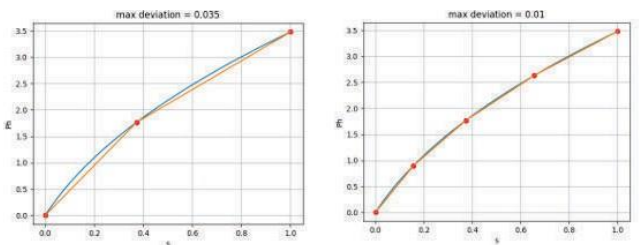

Fig. 7 Application of the Ramer–Douglas–Peucker algorithm for constructing a piecewise-linear approximation of the angular characteristic function

To construct a polysegment line approximating a given continuous planar curve, we need an algorithm that reconstructs a polysegment line from its piecewise-linear angular characteristic function. Obviously, the graph of a piecewise-linear function $\theta(t)$, defined on the interval $I = [0,1]$ is a polyline, and such a function is completely determined by specifying the vertices of this polyline. Each vertex v of the polyline is described by a coordinate pair (t, φ), where $t \in I$ and $\varphi = \theta(t)$. We interpret this pair as the coordinates of a vector in the Cartesian basis, i.e. $v = [t, \varphi]^T \in \mathbb{R}^2$. We will assume that the sequence of vertices $\{v_1, v_2, \ldots, v_N\}$ defining the piecewise-linear angular characteristic function $\theta(t)$ is ordered in such a way that for any two consecutive vertices $v_k = [t_k, \varphi_k]^T$ and $v_{k+1} = [t_{k+1}, \varphi_{k+1}]^T$ the condition $t_k < t_{k+1}$ is satisfied. At the same time, the sequence of vertices $\{v_1, v_2, \dots, v_N\}$ starts with the zero vector $v_1 = [0,0]^T$. Let us denote by $d_k = v_{k+1} - v_k = [\Delta t_k, \Delta\varphi_k]^T$, where $\Delta t_k = t_{k+1} - t_k$ and $\Delta\varphi_k = \varphi_{k+1} - \varphi_k$ is a vector the one corresponding to the k-y th segment of the polyline. $\{v_1, v_2, \dots, v_N\}$. We associate with the segment d_k of the piecewise-linear angular characteristic function a vector $e_k = \Delta t_k R_{\varphi_k}(q_k)$, where R_α is the linear rotation operator through the angle α, and the vector q_k is defined by expression (16).

$$q_k = \begin{cases} \frac{1}{\Delta\varphi_k} \cdot [sin(\Delta\varphi_k), 1 - cos(\Delta\varphi_k)]^T, if\ \Delta\varphi_k \text{ G } 0 \\ [1,0]^T. \quad otherwise \end{cases} \tag{16}$$

It is not difficult to show that the polyline defined by the sequence of vertices $\{p_1, p_2, \dots, p_N\}$, where $p_1 = [0,0]^T$ and the positions of the remaining vertices are given by the recurrence relation $p_{k+1} = p_k + e_k$ orresponds to a unit-length polysegment line for which $\theta(t)$ is the angular characteristic function. The vertices $\{p_1, p_2, \dots, p_N\}$ lie on this polysegment line and correspond to the junction points of its segments. Note that the edges e_k, of the polyline, for which $\Delta\varphi_k$ G 0 correspond to the segments of the polysegment curve that are circular arcs of radius $R_k = \frac{\Delta t_k}{\Delta\varphi_k}$, while the segments corresponding to straight-line portions coincide with the respective edges e_k. The problem of selecting the translation vector, rotation angle, and scaling factor that transform the constructed polysegment line into a planar curve whose endpoints coincide with those of the original curve is completely straightforward. With a reasonable choice of the tolerance parameter in the Ramer–Douglas–Peucker algorithm, the proposed method makes it possible to achieve high-accuracy approximation of a planar curve by a polysegment line, as illustrated by the example shown in Fig. 8.

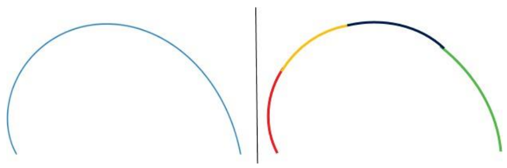

Fig. 8. Result of applying the polysegment approximation algorithm

The reconstructed polysegment closely tracks the reference curve while using few junctions; the maximum deviation remains within the prescribed tolerance, confirming accurate and curvature-consistent approximation suitable for direct MOVE/CIRCLE program generation.

V. Conclusion

This paper proposed an efficient algorithm for approximating planar curves by conjugated circular arcs and straight-line segments, based on an invariant representation of the curve known as the angular characteristic function. The approach enables automatic segmentation of digital curves into a minimal number of geometrically consistent elements while preserving smooth curvature continuity. The use of the Ramer–Douglas–Peucker algorithm provides an adaptive control of accuracy and computational cost, ensuring compact and precise curve reconstruction. The resulting polysegment curves are directly applicable to robotic programming languages such as AS, enabling automatic generation of motion trajectories for surface processing and coating tasks. The simplicity, invariance properties, and low computational requirements of the proposed method make it well suited for both image analysis and robotic trajectory planning. Future work will extend the method to 3D curves and embed dynamic constraints for real-time robotic manufacturing.

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