Robust and Optimal Control Design for Vehicle Suspension System

Muazu Adamu Modibbo[1], Aminu Babangida[2&3*] and Péter Tamás Szemes[2]

[1]Vehicles and Mechatronics Institute, Department of Electrical Engineering and Mechatronics, Faculty of Engineering, University of Debrecen, Ótemető u. 2-4, H-4028 Debrecen, Hungary

[2]Vehicles and Mechatronics Institute, Department of Vehicles Engineering, Faculty of Engineering, University of Debrecen, Ótemető u. 2-4, H-4028 Debrecen, Hungary

[3]Department of Electrical Engineering, Faculty of Engineering, Aliko Dangote University of Science and Technology, Wudil, Nigeria

xj1oyb@mailbox.unideb.hu; szemespeter@eng.unideb.hu

*Correspondence: aminu.babangida@eng.unideb.hu

Abstract: Developing a reliable control algorithm for a mechatronic suspension can be tricky because of the nonlinear nature of the system and the necessity to achieve a good trade-off between the requirements of road handling ability and the comfort of passengers. This paper assesses the performance of two control methods, the H-Infinity (H ∞) and linear quadratic regulator (LQR), applied to an active suspension system. This suspension system uses sensors and actuators in addition to the springs and dampers of the traditional suspension system. It combines software, electrical, and mechanical parts to improve the car handling and convenience of passengers on the road. A quarterly car automotive suspension model consisting of 2 degrees of freedom (2 DOF) is proposed as the case study. The suspension deviation, wheel displacement, and upward acceleration of the car frame are the performance parameters considered in this research. The aim is to strike a perfect balance while achieving minimum readings of car chassis acceleration and wheel deflection demonstrated by each controller when steady state error approaches zero from the suspension response. The body acceleration and wheel deflection affect the passenger comfort and road handling, respectively. Time-based simulation is carried out in MATLAB/Simulink environment to verify the effectiveness of the proposed control mechanisms. The results of the simulation demonstrate the effectiveness and robustness of the proposed control schemes.

Keywords: H-Infinity; LQR; MATLAB/Simulink; PID; Suspension System

1 Introduction

The function of the vehicle suspension system is to isolate the vehicle body from road disturbances to maximize passenger ride comfort and retain continuous road-wheel contact in one fell swoop. Since ride comfort considers the combined effects of suspension travels, wheel displacement, and upward acceleration of the vehicle body, the active suspension system (ASS) controller needs to minimize car chassis acceleration and wheel deflection to guarantee the ride comfort of passengers [1]. A qualitative suspension system must be designed to provide better isolation of passengers from the disturbances induced by road obstacles. Good ride comfort and road handling capability have been the conflicting criteria for achieving good-quality suspension [2]. The critical functionality of the vehicle suspension system is to support the vehicle body and provide comfortable driving to the passengers by rejecting the unpleasant vibratory motion induced by the irregular road input [3]. In [4], riding comfort becomes the focus of the automotive industry. The passengers feel the vibration effects due to the interaction of the vehicle and road surfaces. The ride comfort defined by [5] corresponds to the axis and angular acceleration of the vehicle's gravity centre; hence, minimizing the numerical body axis and angular acceleration equals a higher quality of ride comfort and adequate maneuver.

In the early stage of suspension system control, linear controllers like the Proportional-Integral-Derivative (PID) controller were applied. PID controllers have three parameters: proportional, integral, and derivative gains. The PID's simplicity and practicality make it widely used for industrial applications. It allows for a direct approach to control system design [6]. Up to date, the PID controllers frequently struggle with nonlinearities and system uncertainties, resulting in poor performance in car suspension systems [6]. Optimal control techniques were used to enhance suspension performance due to the limitations encountered in the case of PID and feedback controllers used in [7]. Due to these limitations, the PID has been enhanced, including the Fuzzy-PID for suspension system applications [8]. However, for some control applications, different variants of the PID were optimized as in [9, 10] based on fuzzy logic, such as the PID and PD, to improve system performance.

Moreover, another promising controller that has attracted a lot of interest recently in the field of vehicle suspension system control is model predictive control (MPC). The MPC has been applied to predict future states of the control systems and optimize control inputs. Therefore, MPC can solve an optimization problem at each control step [11]. The MPC has worked well for vehicle active suspension systems due to its versatility in managing restrictions and multi-variable control issues. In [12], the MPC and Kalman Filter hybrid has been proposed for vehicle suspension. The combined capabilities of both control strategies yielded a realistic performance for the suspension system. The need for a precise mathematical model could hinder robust performance in the case of MPC controllers. Therefore, the precision of the system model is a crucial factor that may limit the MPC performance [13]. However, the Linear Quadratic Gaussian (LQG) controller has been suggested as a promising control tool that can improve vehicle response using less actuator power [14]. Although the LQG control algorithm has advantages in theory, being prone to computational errors makes it not usually display the desired system's stability and robustness when it comes to practical implementations [15]. A flexible approach to controlling the nonlinearities and uncertainties involved in car suspension designs is provided by fuzzy sliding mode control [16].

The LQR is a promising optimal control strategy for suspension systems. The LQR controller reduces a cost function that balances state and control input energies. By considering the state space model of a control system, the LQR controller can provide more efficient performance compared to the PID control scheme, thus making it more capable of dealing with different interactions and system dynamics more effectively. In LQR implementation, we find the state feedback gains by solving the algebraic Riccati equation, which creates a control law that improves the specified cost function. However, the LQR control approach enhances ride comfort and road handling by decreasing vibrations and keeping the wheels in more significant contact with the road surface for vehicle suspension [17].

Several optimal and adaptive control policies have been adopted in literature to solve optimization problems [18] and multi-objective control methods [19]. In the early stage of the feedback control algorithm, optimization problems such as iterative feedback tuning algorithm were applied [20]. In recent years, the robustness and the need for optimal performance in the presence of uncertainty and external disturbances have led to the adoption of more robust controllers like $H\infty$. The purpose of $H\infty$ control method is to provide optimal and more robust performance even in the presence of external disturbances and model uncertainty [21]. Moreover, a hybridized $H2$ and $H\infty$ controller design that gets the benefits of both the $H2$ and $H\infty$ controllers. This combination has been applied to improve the system's robustness [22]. However, this combination cannot handle the bounded system's uncertainties. To handle this issue, control strategies such as the $H2$, $H\infty$, and Mu-synthesis (μ-synthesis) have been adopted [21]. The $H\infty$ control constrains the system's energy gain and provides robustness against unmodeled dynamics. In contracts, the $H2$ control scheme seeks to reduce the overall energy of the system response with the assumption that the disturbances are white noise. Despite system uncertainties, μ-synthesis provides a less conservative approach through D-K iteration, allowing for robust analysis and dependable performance [21]. Therefore, the above-mentioned controllers in literature and the associated limitations compared to robust and optimal control methods have motivated this research. This article proposes robust and optimal control strategies such as the $H\infty$ and LQR for vehicle active suspension systems. In addition, to handle the issue of the time delay and operational uncertainties in the vehicle suspension, we specifically use the μ-synthesis robust control method and the LQR optimal control strategy. The computer-aided simulations prove the effectiveness of the proposed control strategies carried out in the MATLAB/Simulink environment under different road conditions.

1.1 Unique Contributions

Although several new ideas and contributions in the design and evaluation of robust and optimal control strategies for active vehicle suspension systems have been made in the current state of the literature, this paper provides some contributions as follows:

- This paper systematically compares H-infinity (H∞) and LQR controllers for a quarter-car suspension system with 2 Degrees of Freedom (2 DOF) on the basis of different performance indicators achieving better tracking performance than most of the previous works, for example, the study in [23]. Many earlier works focused on a single control method; the comparison in this article clarifies their relative strengths.
- This article demonstrates that the H-infinity controller could serve as an alternative algorithm which provides superior robustness against disturbances, while the LQR controller offers optimal control performance for the proposed suspension system.
- Integrating robustness (H-infinity) and optimal performance (LQR) strategies provides insights into ride comfort and road handling trade-offs.
- In this article, detailed quantitative metrics (e.g., chassis acceleration and suspension deviation) to evaluate the controllers' effectiveness offer a unique and straightforward performance comparison.

However, the remaining part of this paper is organized as follows: Section 2 describes the mathematical modeling of the quarter-car active suspension system, including the system's state-space representation and key parameters. Section 3 presents the design of the H-infinity and LQR controllers, detailing the objectives, control strategies, and optimization processes. Section 4 provides the results and analysis of the simulation studies, comparing the H-infinity and LQR controllers' performance in chassis acceleration, suspension deflection, body travel, and control force. Section 5 concludes the paper by summarizing the key findings and suggesting directions for future research.

2 Mathematical Modeling

The traditional passive suspension system is characterized by the spring and shock absorber to provide mechanical support between the vehicle's tire assembly and chassis. The spring-damper characteristics are chosen to highlight one of the conflicting demands, such as ride comfort, suspension deflection, and the stability of the road [24]. However, in active suspension, the vehicle's chassis and tire assembly are connected via an actuator and feedback controller, enabling the achievement of the control objectives [24, 25]. Figure 1 shows a suspension model.

"The car's frame is indicated by mass m_b, and its wheel layout is indicated by mass m_w" [25]. The system's state equation, as expressed in [25], is as follows:

$$\dot{X} = Ax + Bu \tag{1}$$

$$x_b = z_1 \tag{2}$$

$$x_w = z_2 \tag{3}$$

$$r = z_r \tag{4}$$

$$k_1 = k_S \tag{5}$$

$$k_z = k_t \tag{6}$$

$$b_1 = b_s \tag{7}$$

$$\begin{bmatrix} \dot{z_2 - z_r} \\ \dot{z}_2 \\ \dot{z_1 - z_2} \\ \dot{z}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{k_2}{m_2} & \frac{b_1}{m_2} & \frac{k_1}{m_2} & \frac{b_1}{m_2} \\ \frac{b_1}{m_2} & \frac{k_1}{m_2} & -\frac{b_1 + b_2}{m_2} & \frac{k_2}{m_2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_2 - z_r \\ \dot{z}_2 \\ z_1 - z_2 \\ \dot{z}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{m_1}{m_2} \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} u \end{bmatrix} + \begin{bmatrix} -1 \\ \frac{b_2}{m_2} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \dot{z}_r \end{bmatrix} \tag{8}$$

Where k_s, b_s, and k_t are the spring, damper, and displacement of the tire, respectively. Moreover, the x_b, x_w, and r are the vehicle's movement, tire deflection, and road disruptions, respectively. The active suspension component corresponds to a feedback-controlled force, f_s exerted between the wheel's structure and the car body" [25]. The state parameters are the mass velocity of the tire construction, tire displacement, suspension deviation, and mass velocity of the automobile body. (z_r), u stands for the road disturbances and the control signal, to be exact. The elastic spring and damper have fixed variables, k_1, k_2, and b_1, b_2 [25].

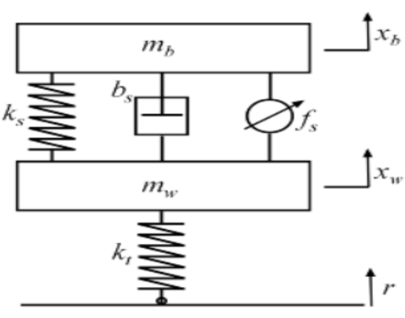

Figure 1

Quarter car suspension [25]

"Where the state parameters $x_1 = (z_2 - z_r)$ and $x_2 = (\dot{z_2})$, $x_3 = (z_1 - z_2)$ and $x_4 = (\dot{z_1})$, represent the rim deviation from the norm, tire construction mass velocity, deflection of suspension, and car frame mass velocity [25]. Table 1 presents the vehicle parameters as adopted from [24].

Table 1
Vehicle Parameters [24]

Specification	Quantity (Unit)
Vehicle Mass (m_b)	285 (kg)
Mass of tire assembly (m_w)	60 (kg)
Rigidity of Suspension system (k_s)	25400 (N/m)
Shock absorber (b_s)	1300 (N.s/m)
Rigidity of tire (k_t)	200000 (N/m)

3 Controller Designs

This section presents the design of the H-infinity and LQR, outlining the objectives and controller structure.

3.1 H Infinity Control Design

The design of the H∞ control meets the operational criteria set by the nominal actuator [25]. The H-infinity controller design aims to minimize the norm of the transfer function from disturbance inputs to error outputs. The optimization problem is formulated to constrain the energy gain of the system's response to external disturbances while ensuring robust performance against model uncertainties. In this article, we designed the control structure with robust performance, taking into account the errors and uncertainty. Subsequently, μ-synthesis is developed to create a controller that accounts for its unpredictable nature while maintaining consistent outcomes for all actuator configurations [26]. The two inputs of P are the control signal, and road disturbances that serve as a reference are parameters u and w. The error outputs, z, are the system's two outcomes, and the variables being assessed, v, are estimated to guide its functioning [25]. The updated variable u is computed in K using v [4], [26]. Figure 2 illustrates the adopted H infinity configuration.

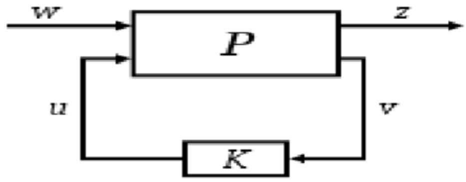

Figure 2
H infinity configuration [27]

The main goals of car suspension are passenger comfort and stability. These objectives are linked up with the car's chassis acceleration (a_b) and suspension deviation (s_d) [25]. The weighting functions specify the project's objectives and depict external disruptions, as illustrated in Figure 3 [25,26]. The actuator is powered by u, which is determined by the suspension's displacement and the car's change in velocity based on information obtained from y_1 and y_2. The road disturbance is d_1, and there are three external sources of disturbances: r. The term represents road disturbances of up to 7 cm. The sensor noise is d_2 and d_3. The objective of the control is to minimize the effect of the disturbance on the suspension's displacement (s_d), the chassis' acceleration (a_b), and the control signal (u). Considering the H∞ norm (peak gain), we will understand how much impact the disturbances could have. Therefore, developing a controller that lowers the H∞ norm from the road disturbance inputs to the error signals satisfies our control objectives [25].

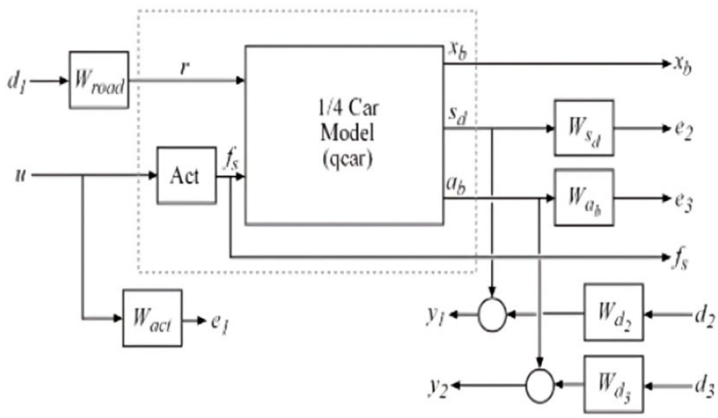

Figure 3
H Infinity formulation for disturbance rejection [25]

From Figure 3, the weighting functions $W_{s_d}, W_{a_b}, W_{road}, W_{act}$ were selected to balance the trade-off between robustness and performance. The weighting functions were tuned to achieve minimal chassis acceleration and suspension deflection while limiting the control force. The optimal controller gain K was computed using the

MATLAB *hinfsyn* function, which minimizes the H∞ norm. The robustness measure in H∞ control is defined as the worst-case energy gain of the transfer function T_{zw} between disturbance inputs w and controlled outputs z.

The optimization goal is to minimize the H∞ -norm as formualted in [33] as follows:

$$||T_{zw}||_{\infty} \rightarrow min \tag{9}$$

Mathematically:

$$||T_{zw}||_{\infty} = \sup_{||w||_2 \neq 0} \frac{||z||_2}{||w||_2} \tag{10}$$

Here:

- w: Represents external disturbances (e.g., road bumps).
- z: Represents the system's error signals, such as chassis acceleration and suspension deflection.

This ensures that the impact of disturbances on system outputs is minimized, thereby enhancing robustness and ensuring the suspension system's stability and performance under uncertain conditions. While the H-infinity (H∞) controller provides robust performance by minimizing the worst-case energy gain of disturbances, it assumes unstructured uncertainties in the system. This assumption may lead to conservative designs that underperform in the presence of structured uncertainties. To address this, we employed the μ-synthesis approach.

3.2 Mu-synthesis Design

The technique of μ-synthesis builds upon H∞ synthesis to design robust controllers for uncertain systems, effectively addressing parameter and dynamic uncertainties [32]. The MATLAB musyn function was used to perform μ-synthesis, combining H∞ synthesis (K-step) with μ-analysis (D-step) through D-K iteration [31]. This iterative process optimizes the robust H∞ performance of the closed-loop system by explicitly accounting for structured uncertainties in the suspension model [32]. In this study, the μ-controller was developed to enhance the robustness of the H-infinity controller by adjusting weighting functions to prioritize uncertainty handling while maintaining performance objectives. The μ-controller's performance was then evaluated under the same conditions as the H-infinity and LQR controllers, showcasing superior robustness and stability across various road disturbances and system variations. Table 2 presents the robust performance of the controller according to the number of iterations.

3.3 LQR Control Design

The LQR controller aims to provide a control law that reduces the cost function or improves the performance index. The objectives of the control are to improve passenger comfort and road handling by minimizing suspension deviation, wheel displacement, and body acceleration. The optimization problem for the LQR controller is defined to minimize the quadratic cost function as represented by (11). Considering the performance index, the LQR controller provides promising outcomes [28]. The quadratic cost function for the LQR controller is expressed in [17] as follows:

$$J = \frac{1}{2}\int_0^t (x^t Q x + u^t R u)\, dt \tag{11}$$

Where x^t and u^t are the state vector which contains system variables and input vector which contains systems control input [17]. The matrices, R and Q, should be such that $R = RT \geq 0$ and $Q = QT \geq 0$, respectively [28]. Thus, this signifies positive definite as studied in [29]. The designer's preferences determine R and Q values. Figure 4 shows how the state variable feedback is set up. The analysis of this system needs proper design of the closed-loop control system to ensure its stability because stability analysis of the closed-loop control system is complicated since it is not based on accurate mathematical models of the process [30].

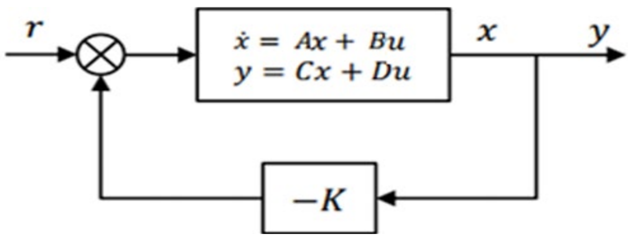

Figure 4
LQR block diagram [34]

“An appropriate linear full-state feedback control law used” as expressed in [28] as follows:

$$u(t) = -Kx(t) \tag{12}$$

The Gain matrix, K, used in this controller is:

$$K = R^{-1}B^T P \tag{13}$$

“The matrix P is estimated using the Algebraic Riccati Equation (ARE)”.

$$A^T P + AP + PBR^{-1}B^T P + Q = 0 \tag{14}$$

Matrix Q:

$$Q = \begin{bmatrix} 500 & 0 & 0 & 0 \\ 0 & 500 & 0 & 0 \\ 0 & 0 & 200 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix} \quad (15)$$

$$R = [0.1] \quad (16)$$

The Q matrix penalizes deviations in state variables, while the R matrix penalizes excessive control force. The state-feedback gain K was computed by solving the Algebraic Riccati Equation (ARE) using the MATLAB *lqr* function. The computed parameters reflect the performance indices, such as reduced chassis acceleration, suspension deflection, body travel, and control force magnitude. Table 3 and Table 4 present a detailed comparison of these indices for both controllers. The Simulink block diagram of an LQR controller is shown in Figure 5, where u is the control force, and w is the road disturbance input.

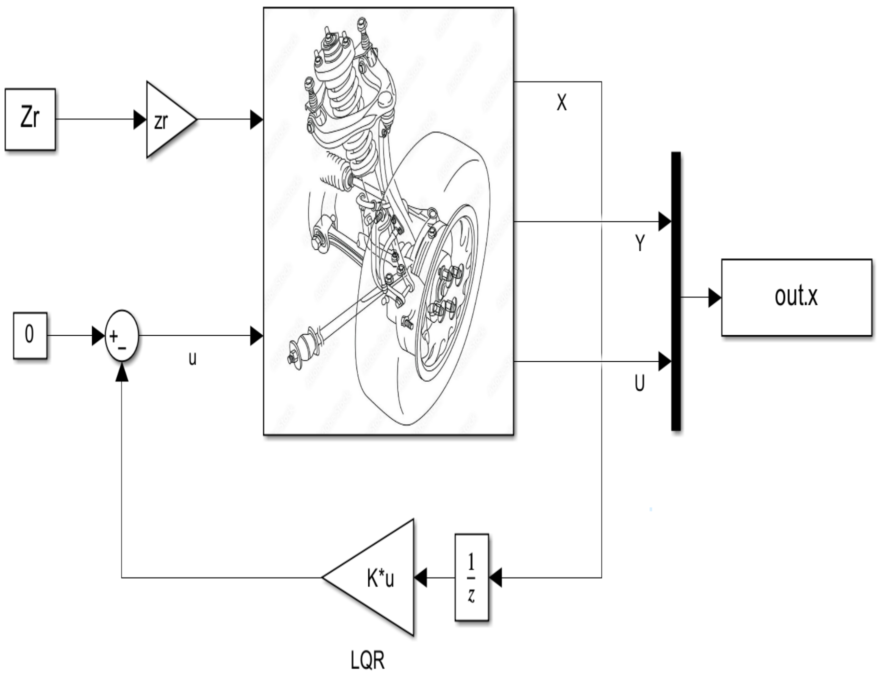

Figure 5

Linear Quadratic Regulator Simulink block diagram

However, to ensure the minimization of the cost functions, the following two conditions were fulfilled:

1) For the H-infinity controller, the weighting functions $W_{S_d}, W_{a_b}, W_{road}, W_{act}$ were carefully designed to prioritize disturbance rejection, control effort, and system output tracking. The MATLAB *hinfsyn* function was used to

numerically solve the H-infinity optimization problem, resulting in the controller K and minimized gamma-value (H-infinity norm). The performance was validated through simulations.

2) For the LQR controller, the Q and R matrices were selected to balance state performance and control effort. The MATLAB *lqr* function was used to compute the optimal state feedback gain K, which minimizes the quadratic cost function J. The controller performance was evaluated using chassis acceleration, suspension deflection, and control force.

4 Results and Analysis

In this section, the simulation results for both controllers are analyzed and compared, with emphasis on chassis acceleration, suspension deflection, control force and body travel.

3.4 Comparison of the Proposed Control Strategies

To ensure a fair comparison between the H-infinity and LQR controllers, we used the same mathematical model and state-space representation for the quarter-car active suspension system. Both controllers were designed using the same system dynamics (A, B, C, and D matrices), physical parameters, and external disturbances like a road bump. The performance indices, including chassis acceleration, suspension deflection, and control force, were evaluated under identical conditions. This ensures that any observed differences in performance are solely due to the control strategies and optimization methods of the H-infinity and LQR controllers.

3.5 H Infinity Results

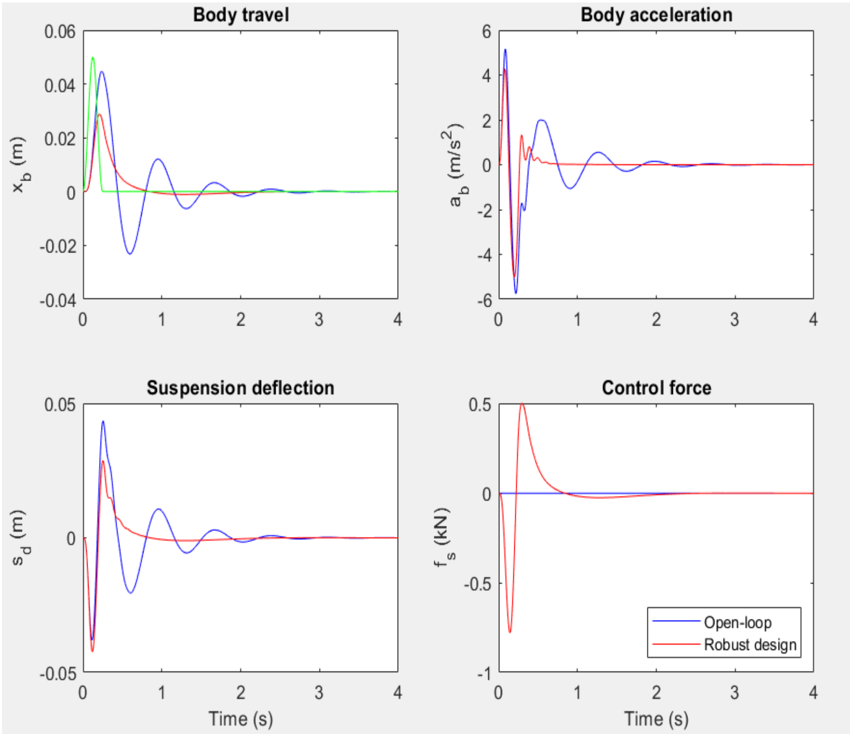

Figure 6
x_b, a_b, s_d and f_s when using μ controller

Table 2
Robust performance of Mu (μ) Controller

	Robust Performance			Fit Order
Iterations	K Step	Peak MU	D Fit	D
1	1.385	1.339	1.353	4
2	1.327	1.272	1.285	4
3	1.257	1.203	1.216	4
4	1.188	1.141	1.154	4
5	1.128	1.083	1.095	4
6	1.078	1.043	1.055	8
7	1.044	1.019	1.029	8
8	1.023	1.004	1.016	10
9	1.013	0.9984	1.01	10
10	1.009	0.9962	1.008	10
Best Achieved Robust Performance: 0.996				

Table 3
Comparison of H-infinity and LQR

Controller	Bump Response			
	Car body travel (m)	Acceleration of Chassis (m/s^2)	Suspension Displacement (m)	Control Force (N)
LQR controller	0.078	4.530	0.026	10.00
Mu controller	0.030	4.480	0.024	500.00

3.6 LQR Results

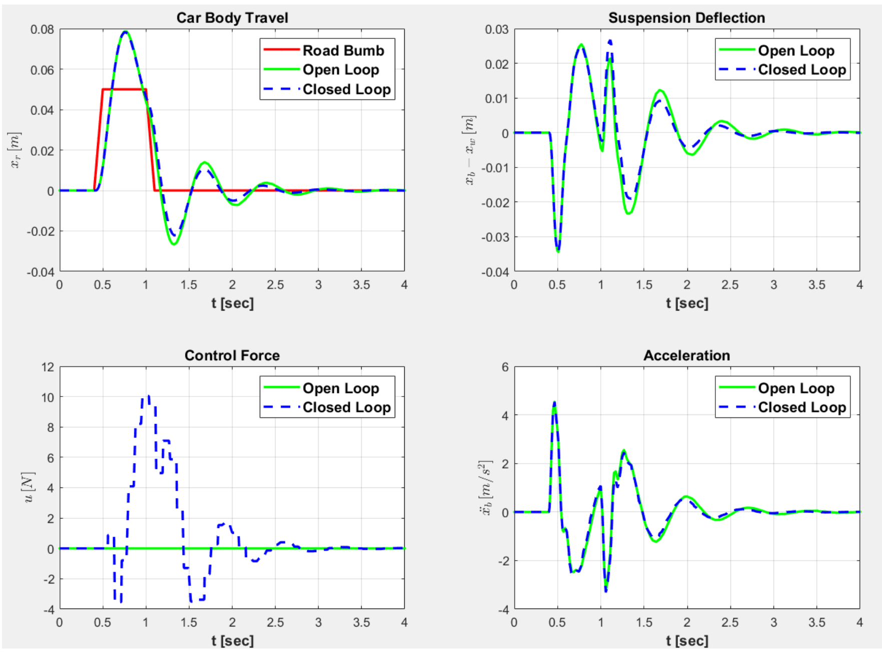

Figure 7
Car body travel, Acceleration, Displacement of suspension and Actuator force when using LQR controller

4.3 Results Analysis

In this paper, controllers have been designed to improve the performance of the vehicle suspension in terms of the car's handling and the comfort of the passenger. The goal is to produce measurements for suspension displacement, wheel deflection, and vehicle chassis acceleration at small magnitudes. The road disturbance demonstrated a 5 cm elevation road bump. Hence, a reasonable compromise has been reached with regard to the suspension deflection and the car's chassis acceleration force, which the balanced controller provides. Figures 6 and 7 shows the response from applying the μ-controller and the LQR. The open-loop response represents the passive suspension, and the closed-loop represents the active suspension. This study compares the performances of the control strategies using the error values presented in Tables 3 and 4. Table 3 presents the comparison for the maximum values for car frame acceleration, vehicle travel, suspension displacement, and control force between the H∞ and LQR controllers. It compares the peak of the error values (x_b, a_b, s_d, f_s) after encountering the road bumps. This shows the superiority of the Mu controller as it has lesser body travel, acceleration and suspension displacement when in contact with a road bump, which means a more comfortable and stable ride even though it uses a higher feedback-controlled force. Table 4 presents the percentage decrease in acceleration, body travel, and suspension deflection values for the passive suspension response against the active suspension with LQR and H ∞ response. The body travel, acceleration and deflection values were measured for the passive suspension system (without H∞ and LQR), and then the measurements were repeated after using H∞ and LQR. This shows by how much the controllers reduced the error values (x_b, a_b, s_d) compared to the passive suspension. Table 4 shows that the Mu (H∞) controller achieves higher performance than the LQR regarding ride comfort and stability, as it has a higher percentage decrease in error values.

Table 4

Percentage decrease comparison between H∞ and LQR

Performance Metrics	Percentage Decrease	
	Passive to H-infinity (%)	Passive to LQR (%)
Max acceleration (m/s^2)	13.0	0
Max Body travel (m)	34.8	-5.1
Max Deflection (m)	36.8	0

Conclusion

This article has presented the optimal and robust control design for the active vehicle suspension system. The main goal of the control is to improve the balance between road handling ability and ride comfort. After obtaining the mathematical equation for a quarter car framework, H∞ and LQR controllers were used to evaluate the system's performance. The active suspension's suspension travel was decreased by using the H∞. The LQR controller decreased the passenger's acceleration range.

In conclusion, the simulation results prove that the Mu controller is the best and superior controller in road handling and passenger comfort compared to the LQR controller concerning body travel, acceleration, and suspension deflection. Therefore, it can be recommended that advanced sensors, including accelerometers and gyroscopes, be employed in future research to provide real-time data to the controller for improved machine learning and artificial intelligence decisions. In addition, the developed controllers should be tested on the actual vehicle for the practical, real-world implications of the suggested algorithms on the vehicle suspension system.

Acknowledgement

This work was supported by the TKP2020-NKA-04 project implemented with support from Hungary's National Research, Development, and Innovation Fund, financed under the 2020-4.1.1-TKP2020 funding scheme. The authors wish to thank the Hungarian Research Fund (OTKA K143595).

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