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Breakage rate measuring – a rock classification alternative?

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Abstract

A new method was proposed to characterize the strength properties of rocks. A crushing test was performed on sand-pairs with different parent rocks, using identical initial gradings. The data were analysed using the grading entropy theory, the grading curve variation was represented in the grading entropy diagram (with a coordinate uniquely related to the mean log diameter). The results with various rocks with the same conditions indicated the same entropy path, only the speed was different, indicating the possibility of a new testing method. As a by-product of the result, it is shown that the breakage path and the internal stability of soils seem to be linked giving the explanation why fractal distribution with fractal dimension n<3 is so frequent in nature.

Keywords: breakage, fractal, grading entropy, entropy principle

1. Introduction

The aim of the research is to study the breakage and degradation process in rocks. Based on this study, a new method to characterize the rock material properties in terms of degradation and/or fragmentation is proposed. In order to achieve this, a crushing test is combined with the grading entropy theory to describe the path and the rate of breakage in terms of grading curve (i.e. particle size distribution).

Similar paths and fractal dimensions occur in the nature [1] as in the laboratory tests. The breakage path is independent of the rock material and its rate is dependent on the material only. The linear part of path being completed by a theoretically computed starting point is proved to be a possible way for breakage rate characterization.

The paper starts by introducing the concepts of grading entropy [2 to 4], entropy coordinates and optimal (or fractal) grading curves. Subsequently, the (discontinuity of the) breakage path and a criterion for internal stability based on grading entropy concepts are presented.

These concepts enable presenting and analyzing some experimental breakage tests using an entropy diagram. It is found that the path may contain an initial discontinuity in the normalized diagram, a linear part and, a curved part where all distributions are fractal [1] with increasing fractal dimensions reaching a value of 3. The latter is the direct consequence of the entropy principle [3]. The samples are “young mixtures” where most grains are larger than the comminution limit ([5]).

2. Grading entropy

The grading curve is the distribution of the log diameter of the grains *d* by dry weight. In the grading curve measurement the sieve sizes, and as a result, the fraction limits are doubled. An abstract fraction system is defined as follows. The diameter range for fraction j (j =1, 2...j see Table 1, Lőrincz (1986)) are defined by using the integer powers of the number 2 (Imre et al., 2009).

$$2^{j} d_0 \geq d > 2^{j-1} d_0, \tag{1}$$

where d_0 is the smallest diameter which may be equal to the height of the SiO4 tetrahedron (2^{-22} mm). The $\log_2$ of the diameter limits are integers, called abstract diameters. The relative frequencies of the fractions x_i (i = 1, 2, 3...N) for each grading curve fulfil the following equation:

$$\sum_{i=1}^{N} x_i = 1, x_i \geq 0, N \geq 1 \tag{2}$$

where the integer variable N – the number of the fractions between the finest and coarsest non-zero fractions – is used. The relative frequencies x_i can be identified with the barycentre coordinates of the points of an N-1 dimensional, closed simplex (which is the N-1 dimensional analogy of the triangle or tetrahedron, the 2 and 3 dimensional instances) and, the space of the grading curves with N fractions can be identified with the N-1 dimensional, closed simplex. The vertices of the simplex represent the fractions, and the 2 dimensional edges are related to the two-mixtures etc. The sub-simplexes of a simplex are partly continuous, and partly gap-graded. The continuous sub-simplexes have a lattice structure, as illustrated in Fig 1.

The grading entropy S is a statistical entropy, modified for the unequal cells (fractions are doubled, Lőrincz (1986)). It can be separated into the sum of two parts. The grading entropy S:

$$S = S_0 + \Delta S \tag{3}$$

where S_0 is called the base entropy and ΔS the entropy increment. The coordinates:

$$S_0 = \sum x_i S_{0i}\ ,\ A = \frac{S_o - S_{omin}}{S_{omax} - S_{omin}}\ ,\ B = \frac{\Delta S}{\ln N},\quad \Delta S = -\frac{1}{\ln 2}\sum x_i ln x_i \tag{4}$$

where S_{0i} is the grading entropy of the i-th fraction, being identical to the fraction serial number (Table 1). The normalized or relative base entropy is A, where S_{0max} and S_{0min} are the entropies of the largest and the smallest fractions, resp. The entropy increment is ΔS normalized form: B.

Table 1. Definition of fractions

j	1	23	24
Limits	d_0 to 2 d_0	2^{22} d_0 to 2^{23} d_0	2^{23} d_0 to 2^{24} d_0
S_{0j} [-]	1	23	24

Any grading curve can be represented as a single coordinate pair in terms of the entropy coordinates. Four maps can be defined between the N-1 dimensional, open simplex (fixed N) and the two dimensional real Euclidean space of the entropy coordinates, the non-normalized [*S0,ΔS*], normalized Δ→[*A,B*], and partly normalized Δ→ [*A,ΔS*] or [*S0, B*].

The images – the entropy diagrams – are compact, like the simplex (Figs 1, 2). The inverse image of the regular values is similar to an N-3 dimensional sphere, "centered" to the optimal point ([4]). The inverse image of the maximum normalized entropy increment lines *B* is the optimal line. The value of d_{min} is indifferent for the normalizd diagram, eg., all fractions map into A =1.

The optimal grading curve or simplex point with maximal *B* for a specified *A* is as follows. The entropy increment *B* is strictly concave function, with a unique conditional maximum point for each constant value of *A*. This single optimal point or unique optimal grading curve is defined as follows: The optimal - grading curve or point of the simplex maps at fixed *A* on the maximum *B*:

$$x_1 = \frac{1}{\sum_{j=1}^{N} a^{j-1}} = \frac{1-a}{1-a^N}, \quad x_j = x_1 \; a^{j-1} \tag{5}$$

where parameter *a* is the root of the following equation :

$$y = \sum_{j=1}^{N} a^{j-1} [j-1-A(N-1)] = 0. \tag{6}$$

The single positive root *a* varies continuously between 0 and ∞ as *A* varies between 0 and 1, a=1 at the symmetry point (A=0.5) and a>1 on the A>0.5 side of the diagram (Imre-Talata (2017)). The relation with fractal dimension:

$$F(d) = \frac{d^{(3-n)} - d_{\min}{}^{(3-n)}}{d_{\max}{}^{(3-n)} - d_{\min}{}^{(3-n)}}, \quad n = 3 - \frac{\log a}{\log 2} \tag{7}$$

where *d* is particle diameter, *n* is fractal dimension (Fig. 3 [4]).

Hence, the optimal grading curves have finite fractal distribution, the fractal dimension *n* varies between 3 and - ∞ on the A>0.5 side as *a* varies between 1 and ∞, *n* varies between 3 and - ∞ on the A<0.5 (left) side of the maximum normalized entropy increment line, as *a* varies between 1 and 0. The optimal grading curve is concave if A<0.5, linear if A = 0.5, convex if A > 0.5 (Fig 4). Some domains and points of the entropy diagrams were successfully related to internal or grain structure stability on the basis of vertical water flow tests by Lőrincz (2). On the basis of the suffusion test results, three basic types of soil structures were related to three domains of the normalized entropy diagram (Fig 4).

In Zone I (A < 2/3) no structure of the large grains is present, the coarse particles "float" in the matrix of the fines and become destabilized when the fines are removed by piping. In Zone II, the coarse particles start to form a stable skeleton and total erosion cannot occur. In Zone III, the structure of larger particles is inherently stable (i.e. most large particles are likely to be members of 'strong' force chains commonly observed on numerical DEM simulations).

Although the fractal dimension *n* may vary from minus to plus infinity as the relative base entropy A (normalized mean log scale diameter) varies between 0 and 1, in the function of N, only a few of them are stable. The fractal distribution is stable if $n< 2$ (independently of N and A).

Let us assume that the grading curve "continuously" varies due to breakage. If N varies, the non-normalized entropy path of the grading curve in terms of $[S_0, \Delta S]$ is continuous. However, the normalized entropy path of the grading curve in terms of $[A,B]$ is not continuous. Some formulae can be derived for the discontinuity. If some i zero fractions are added from smaller side:

$$B(N) - B(N+i) = \Delta S(N)\frac{1-\dfrac{\ln N}{\ln N + i}}{\ln N} \quad , \; A(N+i) - A(N) = \frac{i\left[1 - A(N)\right]}{N+i-1}) \qquad (8)$$

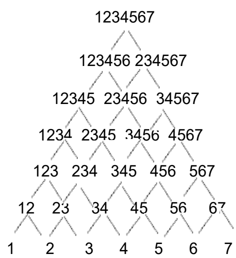

Figure 1. Lattice of continuous sub-simplexes (integers: fractions).

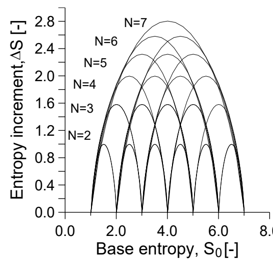

Figure 2. Simplex with N=7 optimal lines of continuous sub-simplexes in the non-normalised entropy diagram.

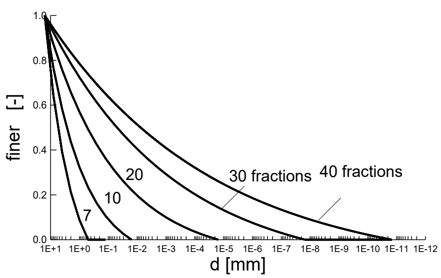

Figure 3. The optimal grading curves N varies, A=2/3.

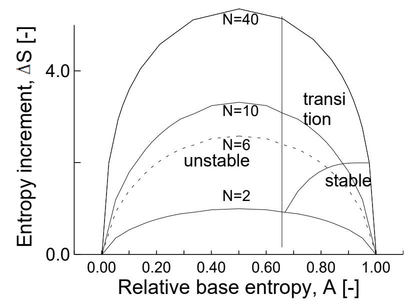

Figure 4. Internal stability criterion of Lőrincz (1986) in the partly normalized diagram.

Figure 5. Reinforced crushing pot designed by Lőrincz [3]

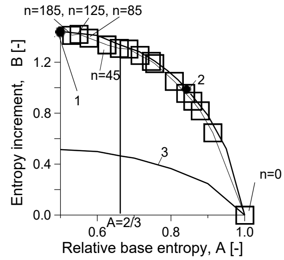

Figure 6. **Normalised e***ntropy path of a one-fraction soil. n: serial number of the crushing. 1: maximum B point. 2: maximum S point. 3: minimum B line [3]*

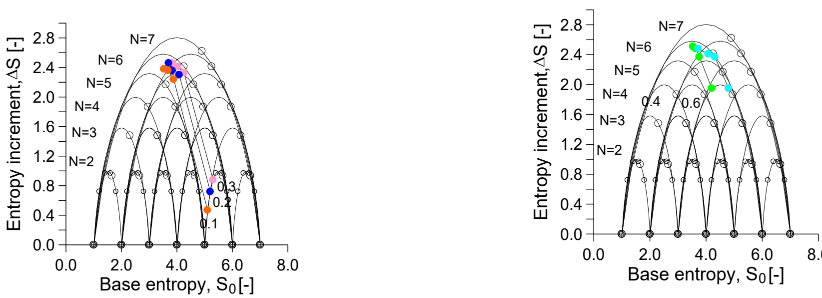

Figure 7. Non-normalized entropy path for initially N =2, Silica and carbonate soils, change in S_0 ~ 0,6 and ~ 1,5; change in ΔS ~ 0,7 and ~1,8, resp.

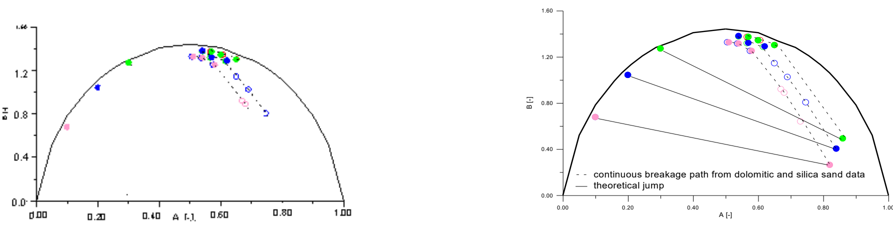

Figure 8. Normalized entropy path for initially N =2, Silica and carbonate soils. Measured data and computed discontinuity(Eq 8) in the normalised diagram (see Figs 4, 6).

3. Experiments

A specially reinforced crushing pot made at BME's Geotechnical Department was used). Each treatment involved the application of a compressive load of 25,000 N to the sample contained in the crushing pot, using a loading machine at the Department of Construction Materials and Engineering Geology, BME. After the compression of the sample, it was removed from the crushing pot for grading curve measurement and then was returned back into the pot for further successive crushing. The results with initially 1-fraction soil shown in Fig 6, 2-fraction of silica and carbonate sands from the same initial grading and testing conditions are shown in Figs 7, 8.

4. Discussion

The base entropy S_0 is a weighted mean of the fraction serial number (which depends linearly on the mean $\log_2$ diameter d). It is monotonically decreasing during breakage since particles become smaller.

The entropy increment ΔS is an entropy mean, a measure how much the soil behavior is really influenced by all of its N fractions. It is monotonically increasing during to breakage due to the entropy principle. The tests followed the same entropy paths, the rate of crushing was different.

The relative base entropy parameter A has a potential to be a grain structure measure of stability, possibly based on the simple physical fact that it expresses the ratio of the larger grains. If enough large grains are present in a mixture then these will form a skeleton (i.e. be part of a strong force chain). In case of an initially two-fraction soil, a discontinuity appears at the start of the normalized path at the appearance of finer fractions, which drifts the entropy path into the stable part of the diagram with great A values (Fig. 8). This explains how internally stable mixtures may form. After the jump, an opposite entropy path occurred (A decreased, B increased) until the maximum entropy increment line was reached, where every distribution is fractal.

5. Conclusion

The results of breakage tests with silica and carbonate sands, from the same initial grading and testing conditions followed the same entropy paths, only the rate of breakage was different. Starting from the same initial grading, the rate of the breakage paths seem to be unique for a given test. The base entropy S_o reflects the decrease in the mean diameter. The ΔS entropy increment increases likely due to the entropy principle. Both can be related to the rock type and quality.

A new laboratory rock qualification test is proposed on the basis of the result of this study. A few recommendations are however necessary. For example, the precise grading curve data are essential in computing the entropy path. Further research is suggested on the breakage rate definition, trying out different testing modes and some additional rock materials, on the comparison with standard rock tests.

6. References

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