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Some notes on the use of a constitutive law for sands

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Abstract

A minimum dry density function is interpolated in the function of the grading curve on the grading entropy diagram on the basis of data measured by Lőrincz. The function can be extended onto the inverse image of the entropy diagram point. To illustrate the usefulness of the chart it is shown that it can be used for estimating the suction of sands from the grading curve on condition that the relative density and the degree of saturation is known.

Keywords: approximate interpolation, suction, unsaturated soils

1. A constitutive model for unsaturated soils

The constitutive model for the unsaturated soils with high degrees of saturation, the average pressure in the air-water mixture (Pande and Pietruszczak, 2008 [1]):

$$p = S_r p_w + (1 - S_r) p_a - \frac{\sqrt{1 - S_r}}{\rho_v} T \quad (1)$$

where Sr is the degree of saturation, pw/pa is the excess pore water/air pressure and T is surface tension. The constitutive model for the unsaturated soils with low degrees of saturation:

$$p_a - p_w = \frac{T}{(1 - S_r)\rho_v} \quad (2)$$

where ρv is average pore size. The aim of the paper is to assess average pore size – including density – information from the grading curve.

2. Derivation of mean pore parameter from the grading curve

The surface area per volume for a single sphere:

$$D/V = 6/d \quad (3)$$

Using this, the surface area of fraction *i*:

$$A_i = \frac{6V_i}{d_i} = \frac{6}{d_i} x_i V_s \tag{4}$$

where A_i and V_i are the surface area and the dry volume of fraction *i*, *di* is the mean diameter of fraction *i* and *Vs* is the total dry volume of all fractions or volume of solid phase, x_i is the relative frequency of the fraction *i*. The surface area of all fractions:

$$S = 6V_s \sum_{i=1}^{N} \frac{x_i}{d_i} \tag{5}$$

where S is the surface area of the solid phase (of all fractions), Vs is the volume of the solid phase (of all fractions), N is the fraction number. (Ordinates of the distribution of the surface area of a fraction are Ai/S). The d_h is the harmonic mean diameter, can be calculated as follows:

$$d_h = \frac{1}{\sum_{i=1}^{N} \frac{x_i}{d_i}} \tag{6}$$

The harmonic mean, dh is a kind of equivalent grain diameter. It can be seen from Eq 5 that the solid volume /solid surface ratio for the solid phase as a whole is d_h /6.

The mean pore diameter is defined ([2]) as the V_v volume of voids in solid phase over the S_{sA} surface area of pores in the solid phase:

$$\rho_v = \frac{V_v}{S_{sA}} = \frac{V - V_s}{6V_s \sum_{i=1}^{N} \frac{x_i}{d_i}} = \frac{1}{6} \frac{e}{\sum_{i=1}^{N} \frac{x_i}{d_i}} = \frac{e}{6} d_h \tag{7}$$

where S_{sA} is the surface area of pores in the solid phase (of all fractions), being equal to the S surface area of grains, V_v is the volume of voids in the solid phase, and *e* is the void ratio, $e = V_v/V_s$, V_s volume of the solid phase, V volume of soil.

To illustrate the difference with the specific surface area per soil volume S_{sm} (which is the ratio of total surface area (m^2) to the soil volume V (m^3), with units of m^{-1}), the following formula can be derived for the specific surface area per volume:

$$S_{sm} = \frac{6}{d_h(1+e)} \tag{8}$$

3. Minimum dry density (maximum dry void ratio) in the function of the grading curve

The grading entropy method ([3]) was applied together with advanced interpolation methods to establish empirical relationships between grading entropy coordinates and minimum dry density of sands. The databases of Lőrincz 1986 ([4]) consisting of 65 samples of artificial mixtures of natural sands with fractal or continuous grain size distributions were used to evaluate a linear empirical relationship. The results showed that there is a strong relationship between grading entropy coordinates and the minimum solid volume ration or maximum void ratio:

$$s_{\min} = C_1(S_0) + C_2(\Delta S) + C_3 \text{ or } s_{\min} = C_1(S_0) + C_2(\Delta S) + C_3 \qquad (9a,b)$$

4. Discussion, conclusion

To illustrate the usefulness of Eq 9 and chart it is shown that it can be used for estimating the suction of sands from the grading curve, the relative density and the degree of saturation. Combining Eqs 2 and 7:

$$p_a - p_w = \frac{6T}{(1-S_r)ed_h} \qquad (10)$$

The experimental works by Kabai (1968, 1972, 1974) ([6 to 8])in relation to the dry density of sands from the Danube river concluded that the ratio of the minimum to the maximum dry density was basically constant, although it decreased as the soil became slightly plastic.

Equation (9) or its graphical representations in Figures 1 and 2 can be used for estimating the maximum void ratio. It follows that from this, the minimum void ratio can be computed with the Kabai's constant, and from the relative density, the actual void ratio can be determined.

In conclusion, Eqs 9, 10 can be used for estimating the suction of sands from the grading curve, the relative density, the degree of saturation.

Table 1 Coefficients identified by LS method

Equation, Variable	C_1	C_2	C_3	r^2
Eq 9a, s	0,02	0,03	0,23	0.96
Eq 9b, e	-0,06	0,10	1,89	0.96

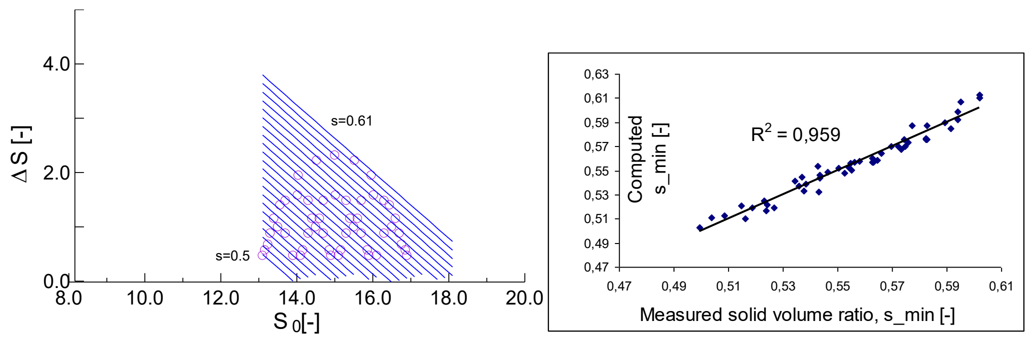

Figure 1. Lőrincz data (a) lines of equal minimum dry density (smin) according to Eq (9a) shown with gradings of experiments using on the non-normalised entropy diagram, (b) the representation of the measured and fitted data for Eq (9a) ([5]).

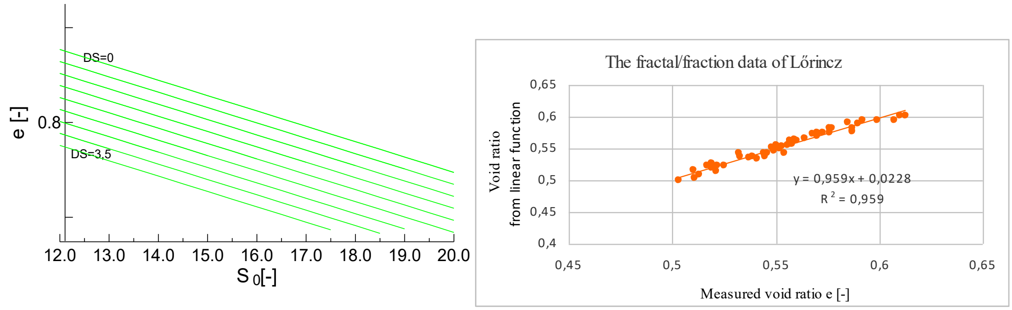

Figure 2. Lőrincz data (a) lines of equal ΔS in the maximum void ratio-base entropy coordinate system, according to Eq (9b) , (b) the representation of the measured and fitted data for Eq (9b).

5. References

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