Products of the Second Pillar Pension

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***Abstract**: The second pillar pension guarantees an additional income for pensioners in their retirement. It is the main support for the first pillar or the state pension. The second pillar pension is inheritable during the savings period. The pension contract is a life insurance product on the basis of which annuity payouts are made to pensioners. This means that the agreed pension payments are guaranteed to pensioners until the end of their life. Recently the attention focuses mostly on pension funds which have different risk levels and strategies, but only a small attention is paid to the calculation of future pensions. Therefore, we offer several life insurance products which would be necessary to consider in the precise calculation of future pension annuity payouts.*

***Keywords**: second pillar pension; annuity; modelling; gender*

1 Introduction

Demographic trends - the low birth rate, the decrease in working-age population and especially the rapid increased life and health expectancy means that there will be less money for pensioners in the future. It is clear that the state pension alone is not enough if a man wants to maintain his/her current living standard. Recently in most countries the funding of pensions is based on three pillars: the 1st pillar - the compulsory, pay-as-you-go, old age state pension; the 2nd pillar - the supplementary, often funding-based, occupational pension and the 3rd pillar - individual savings.

The goal of the first pension pillar or state pension is to guarantee that all pensioners receive a pension on which they can subsist. The state pension insurance is financed through regular payments of the pension insurance premiums to the State Social Insurance Company which are redistributed in the form of the pension annuity for pensioners.

The amounts of the payouts made to pensioners from the first pension pillar are more or less the same and only slightly affected by earlier salaries and the amount of social tax paid. The social tax paid by the working population is the source of state pension insurance. The 2nd pillar pension guarantees additional income for pensioners in their retirement. It is the main support for the first pillar or the state pension. The second pension pillar is inheritable during the savings period. The private pension savings are administered by private Pension Fund management companies. A client of such a company contributes to a private savings account, while the savings the property of this client. If the case of death of the client, all money which was not used to repay the pension annuity becomes a subject-matter of the inheritance as a part of a decedent's estate or may be transferred to the person appointed by the client in the contract on retirement pension savings. All Pension Fund management companies must establish three kinds of pension funds based on various levels of the financial risk – conservative pension fund, balanced pension fund and a so-called growth pension fund. According to law, people have the right to start withdrawing the money they have saved in the second pension pillar when they attain retirement age. The amount of a pension payout depends on the period and frequency of payouts and the number of units accumulated. The third pillar is represented by the voluntary additional insurance, which is designed to keep a high standard of living after reaching the retirement age. The third pillar includes, for instance, an additional pension insurance, life insurance, investment to the mutual funds. The client fully controls the amount and the purpose of the contributions.

Recently the attention focuses mostly on pension funds which have different risk levels and strategies, but only a small attention is paid to the calculation of future pensions. Therefore, this paper offers three life insurance products which would be necessary to consider in the precise calculation of future pension annuity payouts. Moreover, this paper brings analysis of the impact of a ban on the use of gender in insurance, with special stress on the 2nd pillar pension annuity, according to the requirements of the European Court of Justice.

This paper is organized as follows. Section 2 presents the preliminaries. In Section 3 we introduce models of three insurance products which are suitable on an evaluation of pension annuities from the 2nd pillar pension. Section 4 discusses the impact of different input parameters on the size of monthly pension annuities. The last part contains the conclusions.

Products of the 2nd pillar pension were programmed by MS Office Excel 2010 VBA with using life tables obtained from web page of the Statistical Office of the Slovak Republic (Mortality tables, 2012).

2 Preliminaries

Pension contract is the life insurance product on the basis of which annuity payouts are made to pensioners. As a basic product of the second pillar pension we assume a single whole life pension insurance. This means that the agreed pension payments are guaranteed to pensioner until the end of his/her life. To modeling the products of the second pillar pension we can use multiple approaches. One of them is the deterministic.

At the beginning we recall the basic notations which are used on an evaluation of mentioned products. They are:

i - a technical interest rate;

q - an accumulated factor, $q = 1 + i$;

v - a discounting factor, $v = 1/q$;

m - a number of paid, resp. paid out annuities within one year;

y - a start saving age;

x - a retirement age;

n - a number of years of saving on personal pension account, $n = y - x$;

A - an yearly annuity during the period of saving;

l_x - a number of living at age x;

d_x - a number of deaths at age x;

ω - a maximum age to which a person can live to see (regarding used life tables is here $\omega = 100$;

p - a programed withdrawal from an accumulated value at the beginning of retirement time (in percent);

z - a quotient of the m-thly paid out pensions within t years (in percent);

t - a number of years of payment of a quotient of the certain pensions.

We remind so-called commutation functions which facilitate full actuarial modeling. The first one is

$$D_x = l_x \cdot v^x \tag{1}$$

and it means a number of living at age x discounted to the date of birth.

Commutation function N_x represents a sum of D_x up to age ω and is given as follows

$$N_x = \sum_{i=1}^{\omega-x} D_{x+i} \quad . \tag{2}$$

The following functions related to the number of deaths. A function C_x expresses the number of deaths at age x discounted to the date of birth, and is given by

$$C_x = d_x \cdot v^{x+1} \quad . \tag{3}$$

And finally, commutation function M_x represents a sum of C_x up to age ω and is given as follows

$$M_x = \sum_{i=1}^{\omega-x} C_{x+i} \ . \tag{4}$$

In our model we need also other financial and actuarial functions. These functions are usually evaluated for integer ages and terms, assuming that cash flows are payable annually. However, annuities are very often paid more frequently than annually, namely monthly, quarterly, but also semi-yearly. We can find in various sources well known formulas on evaluating of certain and expected annuities which are paid more frequently than annually. Here, we remind only those which we need for our modeling.

The accumulated value of temporary certain annuities of $1/m$ monetary unit $\ddot{s}_{n|}^{(m)}$ which are payable in advance during n years is as follows

$$\ddot{s}_{n|}^{(m)} = \frac{1}{m} \cdot q^{\frac{1}{m}} \cdot \frac{q^n - 1}{q^{\frac{1}{m}} - 1} \ . \tag{5}$$

The present value of temporary certain annuities of $1/m$ monetary unit $\ddot{a}_{t|}^{(m)}$ which are payable in advance during t years is as follows

$$\ddot{a}_{t|}^{(m)} = \frac{1}{m} \cdot q^{\frac{1}{m}} \cdot \frac{1 - q^{-t}}{q^{\frac{1}{m}} - 1} \ . \tag{6}$$

In our life insurance products we model as a single premium for whole pension insurance an accumulated amount S as follows

$$S = A \cdot \ddot{s}_{n|}^{(m)} . \tag{7}$$

A net single premium of the yearly whole life annuities in advance in the size of the one monetary unit for an x -aged insured is given by

$$\ddot{a}_x = \frac{N_x}{D_x}, \tag{8}$$

And a corresponding present value of m-thly paid annuities $\ddot{a}_x^{(m)}$ is the expected present value of the whole annuity of one per year, payable to the entry aged x with $1/m$ at the beginning of each m-thly period per year and it is given as follows

$$\ddot{a}_x^{(m)} \approx \ddot{a}_x - \frac{m-1}{2m}. \tag{9}$$

By A_x is denoted a net single premium for whole life insurance in the case of death for x-aged person with a policy sum of the one of monetary unit, and is given by

$$A_x = \frac{M_x}{D_x} \tag{10}$$

At this point we must reflect that the accumulated amount of savings actually represents a single gross premium and therefore we certainly will calculate the costs associated with the payment of pensions. We assume initial costs α which are usually calculated as a percentage of the accumulated amount, and usually are paid at the beginning of the insurance; administrative costs β and collection costs δ associated with the payment during of retirement which insurance companies calculated as a promile of the yearly pension annuity.

3 Products of the Second Pillar Pension

In this part we introduce three basic insurance products which could be used on an evaluation of pension annuities from the 2nd pillar pension. Observe, the third product represents a generalization of two previous ones, and practically on a programming of the all products it is sufficient to program just the third one. Moreover, we introduce formulas for evaluation of the corresponding provisions which insurers must create under the laws of individual countries.

3.1 Product 1

The first Product 1 assumes that the entire accumulated amount will be spent on pensions without residue. A net pension annuity $NP_{x1}^{(m)}$ paid out m-thly per year is given by

$$NP_{x1}^{(m)} = \frac{S}{m \cdot \ddot{a}_x^{(m)}} \tag{11}$$

and a corresponding gross pension annuity $GP_{x1}^{(m)}$ is as follows

$$GP_{x1}^{(m)} = \frac{S \cdot (1-\alpha)}{m \cdot \ddot{a}_x^{(m)} \cdot (1+\beta+\delta)}. \tag{12}$$

The following formulas express the level of provisions at the beginning

of r-th year of the payment of pensions. Formula (13) expresses a net provision ${}_rNV_{x1}$ at the beginning of r-th year and is given by

$${}_rNV_{x1} = m \cdot NP_{x1}^{(m)} \cdot \ddot{a}_{x+r}^{(m)}. \tag{13}$$

A gross provision ${}_rGV_{x1}$ at the beginning of r-th year is given by

$${}_rGV_{x1} = m \cdot GP_{x1}^{(m)} \cdot \ddot{a}_{x+r}^{(m)} \cdot (1+\beta+\delta). \tag{14}$$

Observe, in this case an insurance company must constitute provisions on costs which will need in the future.

3.2 Product 2

The second product is characterized in that it allows the client to choose at the beginning of retirement p percent of the saved amount as the first higher pension.

A net premium $NP_{x2}^{(m)}$ in this case we can evaluate as follows

$$NP_{x2}^{(m)} = \frac{S \cdot (1-p)}{m \cdot \ddot{a}_x^{(m)}}. \tag{15}$$

So, as in the previous product, in this insurer must calculate individual costs, which is expressed by the formula

$$GP_{x2}^{(m)} = \frac{S \cdot (1-p-\alpha)}{m \cdot \ddot{a}_x^{(m)} \cdot (1+\beta+\delta)} \tag{16}$$

which expresses m-thly per year paid out gross premium $GP_{x2}^{(m)}$.

Formulas (17), (18) express corresponding provisions at the beginning of r -th year of the payment of pensions. A net provision ${}_{r}NV_{x2}$ is as follows

$$ {}_{r}NV_{x2} = m \cdot NP_{x2}^{(m)} \cdot \ddot{a}_{x+r}^{(m)} \tag{17} $$

and a gross provision ${}_{r}GV_{x2}$ by

$$ {}_{r}GV_{x2} = m \cdot GP_{x2}^{(m)} \cdot \ddot{a}_{x+r}^{(m)} \cdot (1+\beta+\delta). \tag{18} $$

3.3 Product 3

The third product is a most general possibility of payment of pensions. In addition to the first higher income p percent of saving amount, this product also implies that in case of a pensioner death at the time of payment of the pension, to survivors or authorized persons will receive the guaranteed quotient z percent of his/her original pension during t years.

A net m -thly pension annuity $NP_{x3}^{(m)}$ we can evaluated by

$$ NP_{x3}^{(m)} = \frac{S \cdot (1-p)}{m \cdot \left(\ddot{a}_{x}^{(m)} + z \cdot \ddot{a}_{\overline{t}|}^{(m)} \cdot A_{x} \right)} \tag{19} $$

and a corresponding gross m -thly pension annuity $GP_{x3}^{(m)}$ as follows

$$ GP_{x3}^{(m)} = \frac{S \cdot (1-p-\alpha)}{m \cdot \left(\ddot{a}_{x}^{(m)} + z \cdot \ddot{a}_{\overline{t}|}^{(m)} \cdot A_{x} \right) \cdot (1+\beta+\delta)}. \tag{20} $$

The net and gross provisions ${}_{r}NV_{x3}$, ${}_{r}GV_{x3}$ at the beginning of r -th year of the payment of pensions are given as follows

$$ {}_{r}NV_{x3} = m \cdot NP_{x3}^{(m)} \cdot \left(\ddot{a}_{x+r}^{(m)} + z \cdot A_{x+r} \cdot a_{\overline{t}|}^{(m)} \right), \tag{21} $$

$$ {}_{r}GV_{x3} = m \cdot GP_{x3}^{(m)} \cdot \left(\ddot{a}_{x+r}^{(m)} + z \cdot A_{x+r} \cdot a_{\overline{t}|}^{(m)} \right) \cdot (1+\beta+\delta). \tag{22} $$

4 The Impact of the Individual Input Parameters on the Pension

As can be seen from preceding formulas, the amount of future pensions is influenced by a number of input parameters.

Let consider for illustration 25-year-old person (unisex) who saving for retirement 30 euros monthly up to 65 his/her age. When he/she will be 65-year-old, he/she can buy monthly whole life pension annuity in the form of the single gross premium. Moreover, let consider a possibility of the choice of the first higher pension 20 percent of the saving amount and the certain quotient 50 percent of the monthly pension annuity during next 5 years. Individual premiums and provisions (for the first ten years) are at the Table 1.

Individual costs are calculated as follows: $\alpha = 5\%$, $\beta = 3$ ‰, $\delta = 1$ ‰.

Corresponding monthly premiums consequently for our three products are $GP_{x1}^{(12)} = 148.75$ euros, $GP_{x2}^{(12)} = 117.43$ euros, $GP_{x3}^{(12)} = 104.74$ euros, see also Table 1 and Figure 1.

Table 1

Net and gross monthly premiums and provisions according to individual products

Accumulated	Net Monthly Pension (euro)			Gross Monthly Premium (euro)		
Sum (euro)	Product 1	Product 2	Product 3	Product 1	Product 2	Product 3
24 592,27	157,2	125,76	112,17	148,75	117,43	104,74
Year	Net provisions			Gross provisions		
of the Pension	Product 1	Product 2	Product 3	Product 1	Product 2	Product 3
0	24 592,27	19 673,81	19 673,81	23 362,65	18 444,20	18 444,20
1	23 734,94	18 987,95	19 097,19	22 548,19	17 801,20	17 903,62
2	22 879,49	18 303,59	18 521,84	21 735,52	17 159,62	17 364,22
3	22 022,02	17 617,61	17 945,12	20 920,92	16 516,51	16 823,55
4	21 135,96	16 908,77	17 349,17	20 079,16	15 851,97	16 264,85
5	20 238,34	16 190,68	16 745,46	19 226,43	15 178,76	15 698,87
6	19 338,17	15 470,53	16 140,02	18 371,26	14 503,63	15 131,27
7	18 462,51	14 770,01	15 551,07	17 539,38	13 846,88	14 579,13
8	17 589,05	14 071,24	14 963,59	16 709,59	13 191,78	14 028,37
9	16 722,38	13 377,90	14 380,69	15 886,26	12 541,78	13 481,90
10	15 872,17	12 697,73	13 808,86	15 078,56	11 904,12	12 945,80

Figure 1 illustrates the development of the technical provisions for our mentioned pensioner in individual years of retirement according to offered insurance products for the 2nd pillar pension. The development of provisions is obvious. Observe, approximately from 83 years the provisions for the third product are higher than for others. This is due to the fact that the probability of death from that age is high and the companies have to create higher provisions for the payment of certain pensions. For the same reason, the provisions for the second product are higher compared to the first one.

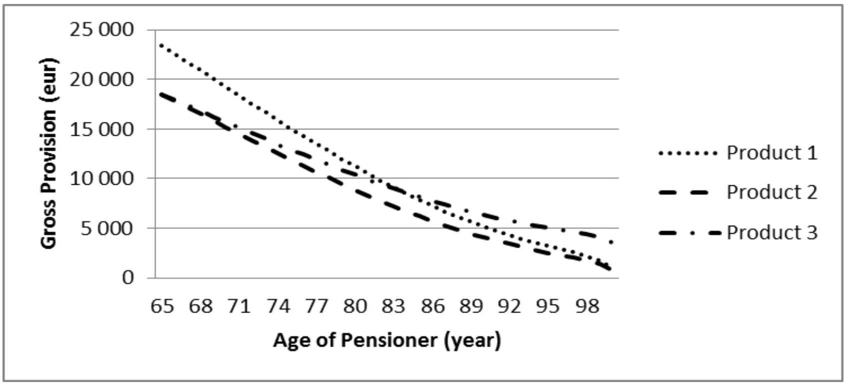

Figure 1

Gross provisions at r -th year of the receiving of the pension

These gross premiums are lower by 6.21 % in average against to pensions if we do not assume costs of an insurance company.

On the Figure 2 you can see the development of monthly pension annuities for our pensioner according to retirement age.

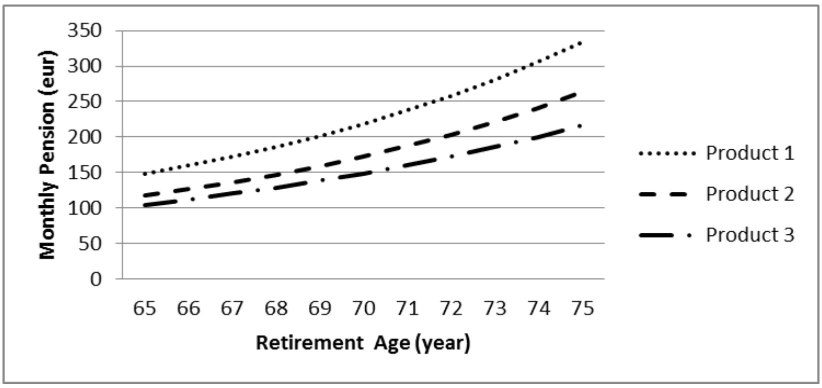

Figure 2

Monthly pension annuities according to retirement age

With the same conditions and with different costs in the size $\alpha = 1\,\%$, $\beta = 2$ ‰, $\delta = 1$ ‰ gross monthly premiums are lower by 1.46 % in average against to the pensions if we do not assume costs.

Figure 3 shows how much monthly pension annuities would increase if the costs $\alpha = 5\%$, $\beta = 3$ ‰, $\delta = 1$ ‰ decrease to the level $\alpha = 1\%$, $\beta = 2$ ‰, $\delta = 1$ ‰. It is clear that increase of the monthly pensions annuities for the two first products are the same.

Figure 3
Increase of the monthly pension by reducing costs

On the Figure 4 are illustrated monthly pension annuities with respect to unisex life tables and life tables for both genders, too.

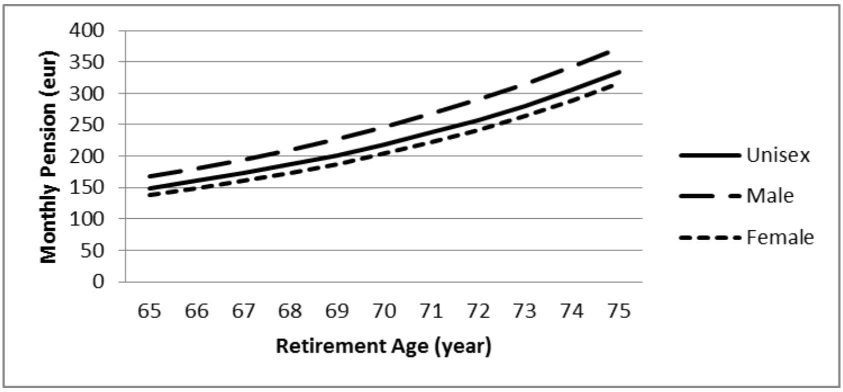

Figure 4
Monthly pension annuities according to retirement age

According to requirements of ECJ all insurance companies must from 21st December 2012 to use on evaluation of premium in all life insurance products unisex life tables. In the calculation by unisex life tables monthly pension annuities for men decreasing by 11% in average and for women increasing by 7% in average.

Conclusions

In this paper we offered products which could be used by insurance companies for future pensioners. However, there exist another possibilities on an extension of products from the 2nd pillar pension. In our next investigation in this area we want to study products which are based on so-called joint annuities. In this case we can use deterministic approach, too. However, in this case we assume independence of the joint lives. The copula models became an increasingly popular tool for modeling dependencies between joint lives. In actuarial studies, it is often of interest to examine the joint mortality pattern of groups of more than a single individual. This group could be, for example, a husband and wife, a family with children, or twins. Intuitively, pairs of individuals exhibit dependence in mortality because they share common risk factors. of joint survival analysis and particular problems of two associated lives. It however must be understood that even substantial development of second and third pillar pensions will probably not be sufficient to compensate both longer life-expectancy and a rising proportion of people over 65 years. With good health expectancy it is not only possible but also essential to plan for flexible extension of working life. Pension funds must encourage and facilitate this extension which will also benefit the insurance sector workforce.

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