Avoiding Lyapunov Functions in MRAC Control: a Comparative Simulation Study for Controlling an EμA

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***Abstract**: In this contribution a "traditional" and a "novel" approach to the Model Reference Adaptive Control (MRAC) are comparatively studied via simulation using the model of an Electrostatic Microactuators (EμA). The new method avoids the use of the Lyapuniov function based technique, so it is relatively simple as far as its mathematical structure is concerned. It works by the use of convergent Cauchy sequences generated by contractive maps. According to the simulations the new approach seems to be more precise and efficien than a traditional imlementation also investigated.*

***Keywords**: Adaptive Control; Model Reference Adaptive Control; Robust Fixed Point Transformations; Iteration; Cauchy Sequences*

1 Introduction

The "Model Reference Adaptive Control (MRAC)" is a popular approach from the early nineties to our days (e.g. [1], [2], [3], [4]). The essence of the idea of the MRAC is the transformation of the actual system under control into a well behaving reference system (reference model) for which simple controllers can be designed. In the practice the reference model used to be stable linear system of constant coefficients, but in principle it can be any type of prescribed "nominal" reference system. In [2] e.g. C. Nguyen presented the implementation of a joint-space adaptive control scheme that was used for the control of a non-compliant motion of a Stewart platform-based manipulator that was used in the Hardware Real-Time Emulator developed at Goddard Space Flight Center to emulate space operations. In [3] Somló, Lantos, and Cát suggested and investigated a local, robust MRAC axis control for robots via simulations. The method is also attracting in the control of teleoperation systems [4].

The above mentioned examples of MRAC controllers as well as their appearance in the mainstream of control literature applies Lyapunov's "direct method" that originally was elaborated for the investigation of the stability of dynamical systems in his PhD dissertation in 1892 [5], [6]. This method is quite ingenious because on the basis of relatively simple estimations the stability (either global or local, "common", exponential or asymptotic) can be determined by its use without obtaining and studying the solutions of the equations of motion. (It is well known that most of the practically occurring problems do not have analytical solutions in closed form, while the numerical solutions are normally valid only for the limited time-span of investigations and without deeper mathematical background their results cannot be extrapolated.) However, in spite of the essential conceptional simplicity of the Lyapunov function technique it practical use is rather an "art" than a simple procedure that could easily be automated. Finding the appropriate Lyapunov function candidate and making the proper mathematical estimations that are needed for the proof of convergence needs great mathematical skills and practices, and these difficult proofs normally take pages of papers and generate complicated, nontrivial restrictions to be met.

As an alternative approach to adaptive control, the use of the Lyapunov function technique was found to be avoidable by the application of "Robust Fixed Point Transformations (RFPT)" [7]. This approach was successfully applied for the adaptive contr of Electrostatic Microactuators (EµA) in [8]. Later it became clear that the same method can be applied in a novel version of the MRAC scheme by replacing the Lyapunov function technique with RFPTs for "Single Input – Single Output (SISO)" systems (using the example of an EµA in [9]), and for "Multiple Input – Multiple Output (MIMO)" systems in [10]. This new adaptive approach in principle can compensate the effects of not modeled coupled dynamics and persistent external disturbances on which we can obtain information only by observing the motion of the controlled system.

The present paper is the extended version of [9]. While in [9] only the novel version was investigated via simulation, here we compare the operation of the novel method and that of a more traditional, Lyapunov function based implementation of the MRAC controllers.

In the sequel at first the details of the "traditional" solution are explained. Regarding the essence of the novel approach and the detailed model of the EµA (due to the lack of enough room in this paper) we only refer to [9]. The same model with the same actual and reference model parameters will be used in the here presented simulations. Following the simulation results concluding remarks will be provided.

2 A Possible "Traditional" and the Novel Implementation of MRAC

The "traditional MRAC philosophy" is wide framework that can be filled in various particular solutions. For comparison we choose a relatively simple implementation containing integrated feedback in the tracking error. Let the tracking error be denoted as $\mathbf{e} := \mathbf{q}^N - \mathbf{q}$ and let $\xi(t) := \int_0^t \mathbf{e}(\tau)d\tau$ ($\mathbf{q}^N$ denotes the nominal, $\mathbf{q}$ is the actual trajectory). The kinematically prescribed trajectory tracking can be defined by the constant positive definite matrix $\mathbf{\Lambda}$ and the „error metrics" of the VS/SM controllers as $\mathbf{S} := \left(\frac{d}{dt} + \mathbf{\Lambda}\right)^3 \xi(t) = 0$ leading to $\ddot{\mathbf{q}}^D = \ddot{\mathbf{q}}^N + \mathbf{\Lambda}^3\xi + 3\mathbf{\Lambda}^2\mathbf{e} + 3\mathbf{\Lambda}\dot{\mathbf{e}}$ as the desired joint acceleration. Let the reference model consist of the same analytical model as that of the actual system. The only difference between these models originates from the dynamic parameters. The "reference model" is described as $\mathbf{M}^{\mathrm{Ref}}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{B}^{\mathrm{Ref}}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{Q}^{\mathrm{Ref}}$ where $\mathbf{M}^{\mathrm{Ref}}$ corresponds to an "inertia" term (for Classical Mechanical systems it used to be symmetric positive definite), $\mathbf{B}^{\mathrm{Ref}}$ can describe other nonlinear couplings, Coriolis forcers, gravitational terms and the effects of friction, and $\mathbf{Q}^{\mathrm{Ref}}$ corresponds to the force/torque need of the reference model for the given joint coordinate acceleration in the givebn state. The "actual" system can be described in similar manner with the actual model values as $\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{B}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{Q}$. By „copying" the idea of the Adaptive Inverse Dynamics Controller let the exerted force/torque be $\mathbf{M}^{\mathrm{Ref}}(\mathbf{q})\ddot{\mathbf{q}}^D + \mathbf{B}^{\mathrm{Ref}}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{D} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{B}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{Q}$ in which $\mathbf{D}$ corresponds to an additive force to be determined by the MRAC controller. Via subtracting $\mathbf{M}^{\mathrm{Ref}}\ddot{\mathbf{q}}$ from both sides we can express the difference of the known *desired* and the measurable *actual* joint accelerations as $\ddot{\mathbf{q}}^D - \ddot{\mathbf{q}} = \left(\mathbf{M}^{\mathrm{Ref}}\right)^{-1}\left[\left(\mathbf{M} - \mathbf{M}^{\mathrm{Ref}}\right)\ddot{\mathbf{q}} + \left(\mathbf{B} - \mathbf{B}^{\mathrm{Ref}}\right) - \mathbf{D}\right]$. By the introduction of the arrays $\mathbf{x} := \left[\xi^T, \mathbf{e}^T, \dot{\mathbf{e}}^T\right]^T$ and $\dot{\mathbf{x}} := \left[\mathbf{e}^T, \dot{\mathbf{e}}^T, \ddot{\mathbf{e}}^T\right]^T$ the following equation of motion holds: $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{\Phi}$ wit the arrays

$$\mathbf{A} := \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ -\mathbf{\Lambda}^3 & -3\mathbf{\Lambda}^2 & -3\mathbf{\Lambda} \end{bmatrix}, \ \mathbf{\Phi} := \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \left(\mathbf{M}^{\mathrm{Ref}}\right)^{-1}\left[\left(\mathbf{M} - \mathbf{M}^{\mathrm{Ref}}\right)\ddot{\mathbf{q}} + \left(\mathbf{B} - \mathbf{B}^{\mathrm{Ref}}\right) - \mathbf{D}\right] \end{bmatrix} \quad (1)$$

For driving $\mathbf{x}$ to zero a simple Lyapunov function can be constructed by the use of a positive definite matrix $\mathbf{P}$ as $V := \mathbf{x}^T\mathbf{P}\mathbf{x}$ with the time-derivative

$$\dot{V} := \mathbf{x}^T \left(\mathbf{A}^T \mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{x} + 2\mathbf{x}^T \mathbf{P}\mathbf{\Phi} \tag{2}$$

that can be made negative in the following manner: by solving the Lyapunov equation the term quadratic in **x** can be made negative definite, and and attempt cen be made for making the remaining term zero. Since $\ddot{\mathbf{q}}^D - \ddot{\mathbf{q}}$ is known, the additive term can be considered as a known one that can be calculated from observed and known quantities. Furthermore, it is a sum of a *known* and an *unknown* term: if we define the array **w** as $\mathbf{w}^T := \mathbf{x}^T \mathbf{P}\left[\mathbf{0},\mathbf{0},\left(\mathbf{M}^{\mathrm{Ref}}\right)^{-1}\right]$ we have that

$$z := \mathbf{x}^T \mathbf{P}\left[\mathbf{0},\mathbf{0},\ddot{\mathbf{q}}^D - \ddot{\mathbf{q}}\right]^T = \mathbf{w}^T\left[\mathbf{0},\mathbf{0},\left(\mathbf{M} - \mathbf{M}^{\mathrm{Ref}}\right)\ddot{\mathbf{q}} + \mathbf{B} - \mathbf{B}^{\mathrm{Ref}}\right]^T - \mathbf{w}^T \mathbf{D} \tag{3}$$

where the value of the LHS is known, and the 1st term in the RHS is unknown. To make this term negative let us seek the additional generalized force **D** in the form of $\mathbf{D}=\alpha(t)\mathbf{w}$ in which $\alpha(t)>0$ is a scalar parameter. Since $\mathbf{w}^T\mathbf{w}\geq 0$ (3) yields information if $\alpha(t)$ needs some increase or it can stagnate. This consideration immediately yields a possible tuning for $\alpha(t)$ at follows: $\dot{\alpha} = \kappa[1 + sign(z)]z$ with a positive constant κ that influences the speed of the parameter tuning. It is eviednt that by properly chosen **P** and κ a decreasing Lyapunov function can be achieved that must yield an asymptotically stable control.

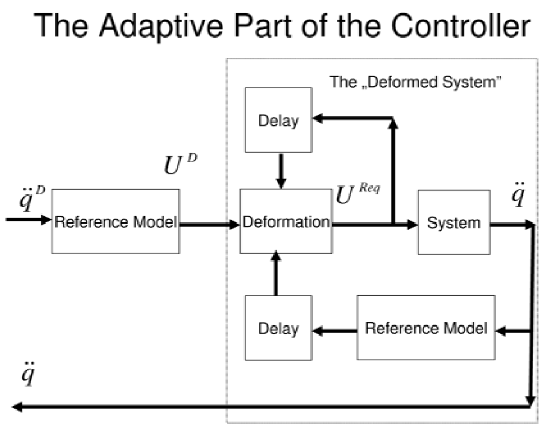

Figure 1
The novel MRAC structure

For the purpose of comparison in the present paper the novel MRAC structure is only briefly outlined in Fig. 1. The kinematical trajectory tracking can be the same PID controller as in the case of the traditional solution. However, the force/voltage to be exerted on the actual system is first calculated by the use of the reference model then it is further “deformed” by the RFPT transformer. Fo comparing the desired and realized responses the measured acceleartion of the actual system is transformed back to the reference model to make comparison. This comparison works in a causal, iterative manner so the scheme in Fig. 1 contains two delay blocks that makes it possible to use the actual and the past values in the comparison. In both blocks the delay time must be constant and equal to the cycle time of the controllers.

In the sequel simulation results are presented for this Lyapunov function based method and the novel one using the models, parameters, and the same novel method detailed in [9].

3 Simulation Results

The EμA corresponds to the special case in which **D** has a single component and in the place of the “generalized forces” **Q** we have control volatge U. The forthcoming reults in Fig. 2 belong to the traditional solution with the parameter settings as follows: $\alpha_{ini}=2\times10^{-4}$, $\mathbf{\Lambda}=500\times\mathbf{I}$, $\mathbf{S}=10\times\mathbf{I}$ in the equation determining **P** as $\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}=-\mathbf{S}$, $\kappa=2500$.

In the trajectories and the phase trajectories little improvement of the tracking precision can be observed while α evidently is increased in certain phases in which its increase is needed for making the Lyapounov function negative. It is important to note that the accelartion of the nominal motion, the “desired” acceleration that contains kinematically determined PID corrections, and the “realized” accelerations till significantly differ from each other that means that the adaptation is not very efficient. It must be noted that some increase in α_{ini} and in κ leads to numerical instabilities (they are coded in the singular model of the EμA), so the resulst cannot very signifianly be improved.

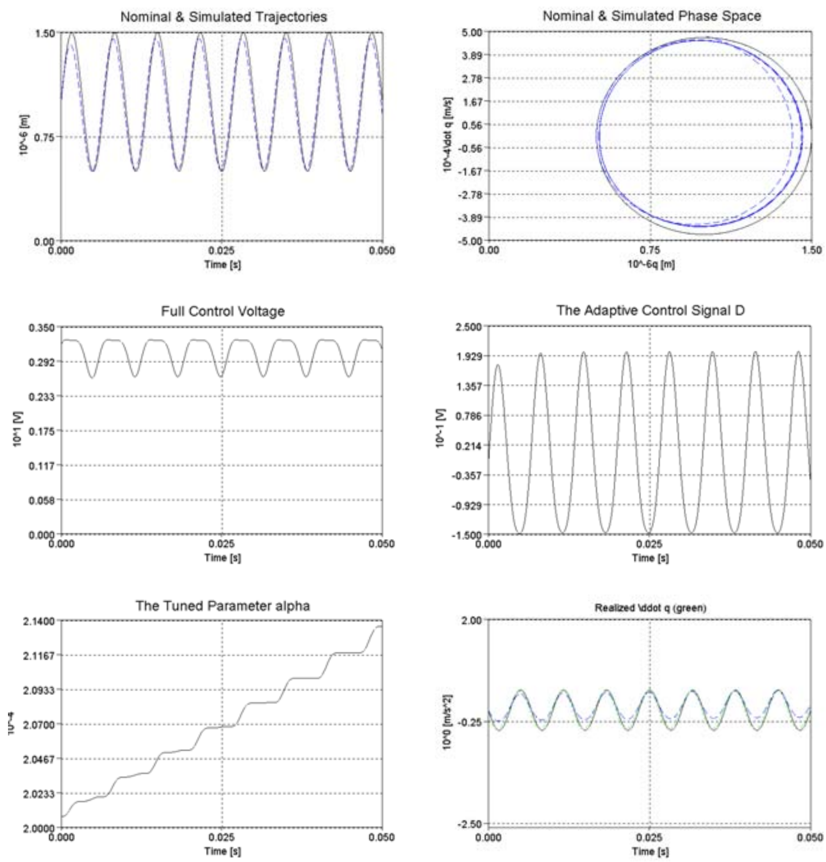

Figure 2

Typical results for the traditional approach: trajectory tracking and phase trajectory tracking (1^{st} row); the exerted full control volatage and its adaptive part (2^{nd} row); the tuned parameter α and the nominal acceleartion (black solid line), the desired acceleration (blue dashed line), and the realized acceleration (green line with dense dashes) (3^{rd} row)

To reveal the significance of the adaptivity results are given fdor the non-adaptive common PID controler using the imprecise model (i.e. working with $\mathbf{D}\equiv\mathbf{0}$) in Fig. 3. It is evident that the traditional adaptive approach results in quite significant improvement.

It is worth noting that the need of solving the Lyapunov equation imposes certain limitations to the applicabilty of the present approach. A considerable increase in the absolute value of $\mathbf{\Lambda}$ could significantly increase the tracking precision but very big $\mathbf{\Lambda}$ makes the numerical solution of the Lyapunov equation difficult because matrix $\mathbf{A}$ contains its 2^{nd} and 3^{rd} powers, too. Therefore in the simulations we were not able to significantly increase $\mathbf{\Lambda}$.

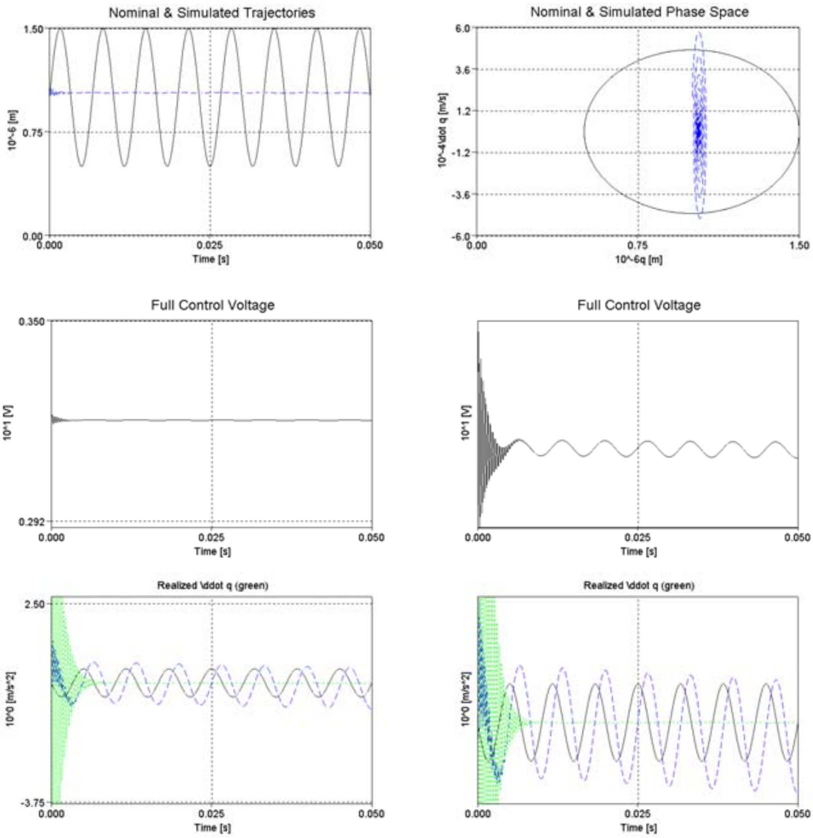

Figure 3

Typical results for the non-adaptive approach using simple PID feedback: trajectory tracking and phase trajectory tracking (1st row); the exerted full control volatage and its zoomed excerpt (2nd row); the desired acceleration (blue dashed line), and the realized acceleration (green line with dense dashes) (3rd row)

The simulations were made via simple Euler integration by a common SCILAB program.

For comparing result the SCILAB-SCICOS implementation of the novel RFPT-based method was used exactly as in [9]. The appropriate results are given in Fig. 4 revealing that this approach is quite efficient, precise than the Lyapunov function based approach. In this simulation the same PID tracking was prescribed with $\mathbf{\Lambda}=10^4\times\mathbf{I}$, which in this case did not cause numerical problems and it resulted in very precise tracking.

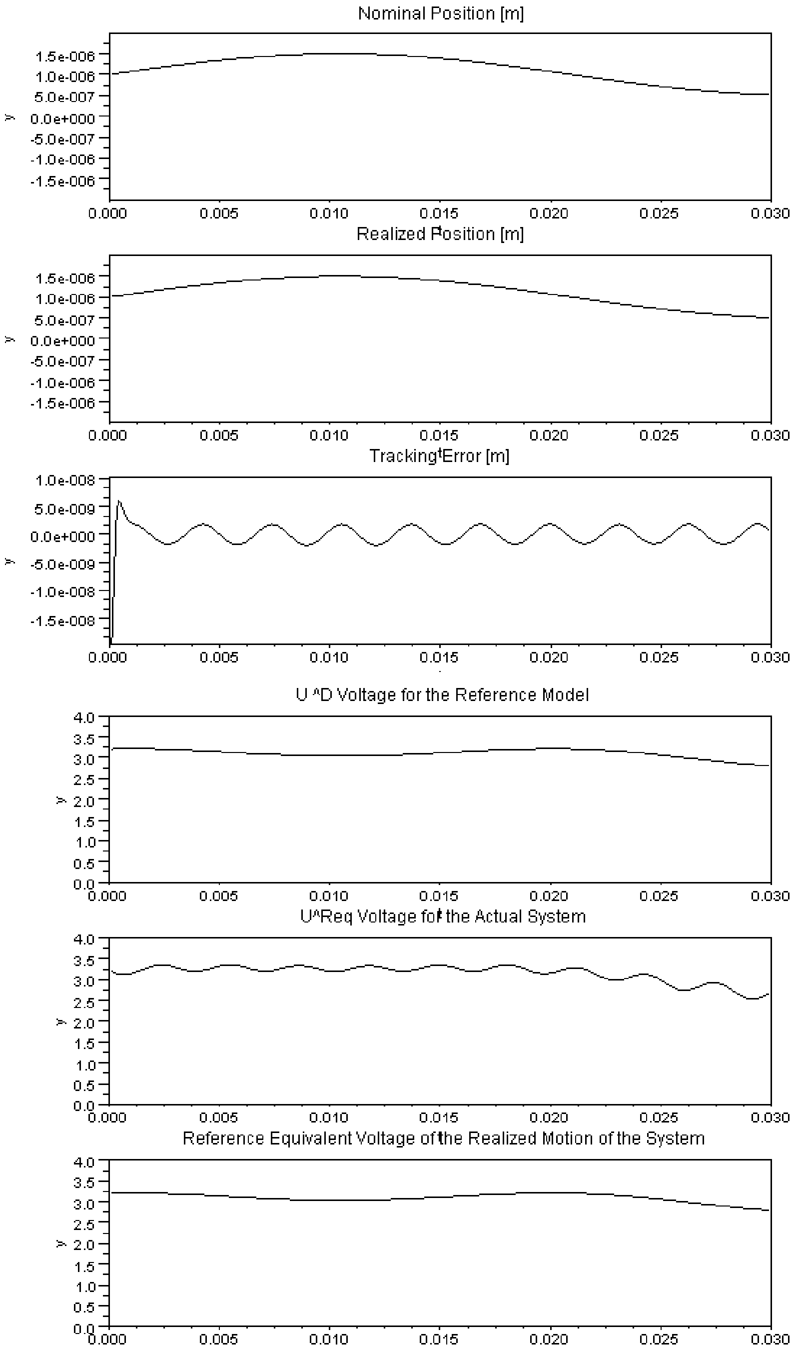

Figure 4

The trajectory tracking (upper 3 charts) and the control volatges (lower 3 charts) of the novel approach

Conclusions

In this paper the opeartion of two different MRAC approaches were comaperd to aech other using the example of the control of an EμA.

It was shown that due to its mathematical structure the traditional approach was sensitive to the use too big PID control coefficients that limits its precision. Furthermore, due to the Lyapunov function based construction of the tradiotional controller the parameter tuning is too complicated and needs too much calculations. No similar computational burden occurs in the use of the novel technique, so it can be much faster than the traditional adaptive MRAC controller.

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