A Flexible Fuzzy Logic-based Risk Assessment Framework

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Abstract— **At present, risk assessment is an important research topic. The highest levels of this discipline deal with the management of economic crises, terrorist activities, as well as, environmental and climate changes. In addition, a huge development can be observed in IT (Information Technology) and medical-related applications. In many cases, both qualitative and quantitative factors appear among the risk factors and both of these should be handled in depth. Fuzzy logic-based inference systems are suitable for these kinds of tasks; moreover, they can manage the frequent subjectivity in the data and in the evaluation process. The author developed a fuzzy logic-based risk assessment framework, which can be adapted to various requirements, depending on circumstances. In the system, the number and type of risk factors can be varied and their membership functions can be tuned according to the individual factors' characteristics.**

I. INTRODUCTION

Risk assessment is a rapidly developing science, but there are several approaches to its definition, which factors are included and how these factors are managed. To unify these viewpoints an ISO standard was created [1], which defines the risk as the combination of the probability of an event and its consequences. Risk management involves the identification, measurement, and determination of their importance, and its aim is to minimize and control the risk level [2] [3].

To create an effective risk assessment model a well-considered plan is required to determine and handle the risk level reliably. An essential step of the model design is the determination of the relevant risk factors, their limits, the interactions between them and their effects. In practical applications these factors generally cannot be defined, because they can depend on the circumstances. For example, in the case of patient monitoring, the parameters, which should be measured and their limits can be different depending on the health condition of patients or the available devices [4] [5]. This feature makes it difficult, both in terms of the design and the implementation of the model. This is because fixed inputs cannot be applied in the system. In these cases, a flexible model is required, which can be adapted for the current circumstances. The author's framework allows one to vary the input parameters and their number. Consequently, if needed, more parameters can be taken into account and their type can also be fitted to the characteristics.

The other main question in the evaluation is the limits of the factors. In several cases, precise limits cannot be defined, i.e. the limit between the normal and abnormal level cannot be defined as an exact value. In these kinds of applications the soft computing-based techniques, such as fuzzy logic, can be useful tools. Fuzzy logic-based techniques can handle the smooth transition between the levels, including the uncertainty, imprecision and subjectivity in the data and in the evaluation process [6]. A further significant advantage of fuzzy logic is that it can handle both the qualitative and quantitative factors, which is also an important aspect of its usage.

In this paper a flexible fuzzy logic-based risk assessment framework is presented. This framework was designed and implemented by the author. This model fulfills the above mentioned requirements. It can handle the blurred boundaries and the number and type of the input parameters can be varied depending on the circumstances. The framework is of a general-purpose design and it can be specified for any given task, using a task-specific database.

II. FUZZY APPROACH

A. Basic concepts

The fuzzy approach was introduced by Lotfi A. Zadeh in the 1960s, it was a solution for the cases of previously, mathematically indescribable problems. It could handle the uncertainty, imprecision, and subjectivity in the data and in the evaluation process, in this way, the result becomes more realistic. This technique works well in situations, where insufficient data is available to create a statistical model or the cause-effect relation is not precise enough [7]. A further advantage of this approach is that it can handle both qualitative and quantitative parameters.

The fuzzy set theory can be derived from the conventional set theory, but the sets can be defined in this case by the generalization of the traditional characteristic function. It means that a value from the [0,1] interval is assigned to each element of the basic set (universe, $X \neq \varnothing$), which defines the membership degree of the element. The corresponding function is the *Membership* function, which defines a fuzzy set A, and can be described by (1).

$$\mu_A : X \rightarrow [0,1] \qquad (1)$$

In practical applications the shape of the membership functions is usually piecewise linear for ease of use. In the case of a triangular function the membership function can be defined by (2).

$$\mu_A(x) = \begin{cases} 0 & x \leq a \\ \dfrac{x-a}{b-a} & a \leq x \leq b \\ \dfrac{c-x}{c-b} & b \leq x \leq c \\ 0 & c \leq x \end{cases} \qquad (2)$$

where a, b, and c are the membership function parameters, which can be used to tune the membership function according to the specific requirements.

B. Fuzzy operators

The generalization of the conventional set operations in the fuzzy inference systems based on the operator-families was introduced by Schweizer and Sklar [8]. In this section the most important operators are presented.

1) Fuzzy intersection (t-norm)

In fuzzy-based systems there are several t-norm operators, and the operator selection is application-dependent, but all of the t-norm operators should satisfy the following axiomatic-frame. Let $t:[0,1] \times [0,1] \rightarrow [0,1]$ be a function with the following properties [9]:

1. $t(a,1)=a \; \forall \; a \in [0,1]$ (boundary condition)
2. if $b \leq c$ then $t(a,b) \leq t(a,c) \; \forall \; a,b,c \in [0,1]$ (monotony)
3. $t(a,b)=t(b,a) \; \forall \; a,b \in [0,1]$ (commutativity)
4. $t(a,t(b,c))=t(t(a,b),c) \; \forall \; a,b,c \in [0,1]$ (associativity)

The above listed properties can be supplemented by additional restrictions for better practicability.

1. t is a continuous function
2. $t(a,a)<a$ (sub-idempotent), or $t(a,a)=a$ in the case of min t-norm (idempotent)
3. if $a_1<a_2$ and $b_1<b_2$ then $t(a_1,b_1)<t(a_2,b_2)$ (strict monotony) [9].

2) Fuzzy union (t-conorm)

In fuzzy-based systems there are several t-conorm operators, and the operator selection is application-dependent, but all of the t-conorm operators should satisfy the following axiomatic-frame. Let $t:[0,1] \times [0,1] \rightarrow [0,1]$ be a function with the following properties [9]:

1. $s(a,0)=a \; \forall \; a \in [0,1]$ (boundary condition)
2. if $b \leq c$ then $s(a,b) \leq s(a,c) \; \forall \; a,b,c \in [0,1]$ (monotony)
3. $s(a,b)=s(b,a) \; \forall \, a,b \in [0,1]$ (commutativity)
4. $s(a,s(b,c))=s(s(a,b),c) \; \forall \; a,b,c \in [0,1]$ (associativity)

The above properties can be supplemented by additional restrictions for better practicability:

1. s is a continuous function
2. $s(a,a)>a$ (super-idempotent), or $s(a,a)=a$ in the case of Zadeh union (idempotent)
3. if $a_1<a_2$ and $b_1<b_2$ then $s(a_1,b_1)<s(a_2,b_2)$ (strict monotony) [9].

3) Aggregation operators

Let $h: [0,1]^n \rightarrow [0,1]$ be an aggregation operator on n fuzzy sets ($n \geq 2$). If the arguments of the function are the fuzzy sets A_1, ..., A_n based on the basic set $X=X_1 \times X_2 \times ... \times X_n$ then h produce a fuzzy set for each $x \in X$ using the membership value of the arguments, i.e. $A(x_1, ... x_n)=h(A_1(x_1), ..., A_n(x_n))$. A well-defined aggregation operator should satisfy the following axiomatic conditions:

1. $h(0,...,0)=0$ and $h(1,...,1)=1$ (boundary condition)
2. h monotonically increasing in each arguments, i.e. in the case of any two n-tuple $\langle a_1,...,a_n \rangle$ and $\langle b_1,...,b_n \rangle$ where $a_i,b_i \in [0,1]$ and $a_i \leq b_i$ for all $i \in [1,n]$, then $h(a_1,...,a_n) \leq h(b_1,...,b_n)$
3. h is a continuous function.

The above listed properties can be supplemented by additional restrictions for better practicability:

1. h is a symmetric function in each arguments, i.e. $h(a_1,...,a_n)= h(a_{p(1)},...,a_{p(n)})$ where p is an optional permutation of $1,...,n$.
2. h is idempotent, i.e. $h(a,...,a)=a$ for all $a \in [0,1]$.

For all aggregation operators, which fulfill the above requirements, the following inequality is true: $min(a_1,...,a_n) \leq h(a_1,...,a_n) \leq max(a_1,...,a_n)$ in the case of each $\langle a_1,...,a_n \rangle \in [0,1]^n$ [9].

C. Fuzzy-based inference

In the inference systems, fuzzy sets are assigned to the input parameters, whose shapes can be different according to the characteristics of the examined factors. Depending on the inference type, the output can also be represented by a fuzzy set. In this case, the system behavior is much closer to human thinking, intuition can be built into the system, but the computational complexity is higher in this case. This kind of output representation is used in the conventional Mamdani-type and Mamdani-like systems, which uses *IF condition THEN consequent* type of natural language rules. Let the input parameters be $x_1, x_2,...,x_n,$ and the output parameter be y. In this case the system can be represented by the following type of rules:

$$IF \; x_1 \; is \; A_{1,i_1} \; and \; ... \; and \; x_n \; is \; A_{n,i_n} \; THEN \; y \; is \; Y_{i_1,...i_n} \qquad (3)$$

where $A_{k,ik}$ is the antecedent i_k belonging to the input k, $Y_{i1,...,in}$ is the fuzzy set belonging to the rule consequences, $i_j=1..n_j$, n_j is the number of the antecedent sets belonging to the input j. The rule-premises are obtained from all the possible combinations of the fuzzified inputs. In the following section the steps of the Mamdani-type inference system are presented, which are the fitting ones out of the observation and the antecedent sets; the firing strength calculation; implication; aggregation and if necessary, the defuzzification.

1) Fitting the observation and the antecedent set

The inputs can be both fuzzy set and crisp value. In this step the fitting degree of the input and the antecedent set should be defined, i.e. the extent in which the input parameters belong to the fuzzy sets, which are used to characterize them. In the case of triangular membership function it can be calculated by (2), and its result is generally within the [0,1] interval.
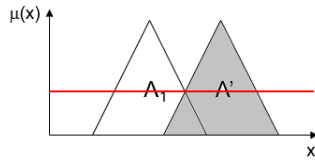
Figure 1.   Fitting degree in the case of fuzzy input

2)   Firing strength determination

The antecedent part of the rules usually contains more conditions, which can be connected by a fuzzy operator described in *Section II.B*. In the case of *AND* connection a t-norm, while in the case of *OR* connection a t-conorm is used.

3)   Mamdani-implication (compositional inference rule)

The aim of this step is to determine the rule output by fitting the consequent set and the firing strength belonging to the same rule, with a t-norm operator. The rule output is the result of this fitting. The most commonly used operators are the minimum and the product operators.
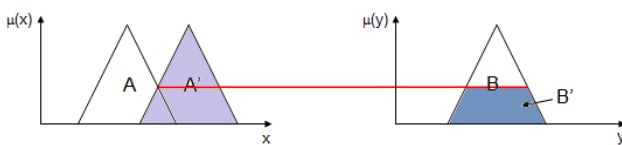

Figure 2.   Mamdani-implication (min t-norm)

4)   Aggregation of the rule consequences

After the implication the consequent set of each rule is obtained, but they should be summarized in a way to produce a single fuzzy set from them. This can be performed using an appropriately chosen aggregation operator, which fulfills the requirements in *Section II.B*.
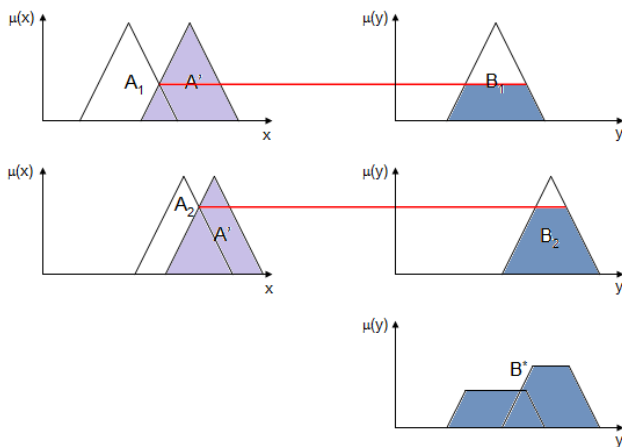

Figure 3.   Aggregation

5)   Defuzzification

After the aggregation the obtained set is a complex shaped fuzzy set, which can be handled by a human operator, but generally it should be represented by a crisp value. The aim of the defuzzification is to find the crisp value, which represents the fuzzy set the best way. There are several defuzzification methods, the method selection is application-dependent.
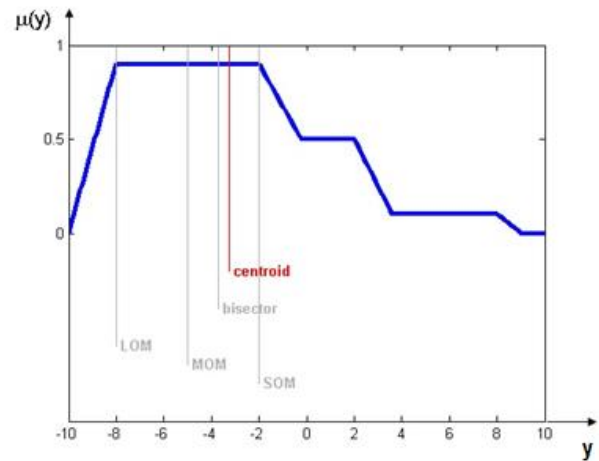

Figure 4.   Defuzzification [10]

III.   THE RISK ASSESSMENT FRAMEWORK

A.   Flexibility of the system

In the case of risk assessment, the determination of the relevant risk factors is essential, but these factors can be different under different circumstances. Furthermore, the limits of even the same parameters can also be different according to the current conditions. This was the main inspiration for the author to design a flexible risk assessment framework, where the input parameters can be varied to adapt the system to the specific requirements.

In the framework the number of the input factors and their membership function parameters can also be varied. For example, in the case of a sport monitoring system, where the physiological parameters are measured and they are the basis of the risk assessment, the parameters which should be monitored depend on the health condition of the patient, the medical recommendations, and the selected sport. In this way the number and type of the measured parameters can be different for different people. Moreover, the parameters are different even in the case of the same person, and for different sport activities. In some cases the available devices can also limit the measurable factors.

Not only the relevant factors, but also their limits can be different in different cases. Referring again to sport monitoring, if the same factors are to be measured for different people, the normal and abnormal values can be completely different depending on their health state, or medical recommendations. Moreover, in the case of the same person, the normal limits differ depending on the chosen sport type.

In the framework it is available to define personal profiles, where the measurable factors and their limits can be fixed depending on the personal characteristics. To ensure this ability of the system, a well-structured database design is indispensible. In the case of sport monitoring, the input parameters and their limits can be defined person- and sport-specifically, which means that in each profile the possible sport activity should be fixed

and the parameters can be assigned to these activities, which should be measured and also their limits in the case of the given sport type. The detailed database design of the sport activity monitoring framework can be found in [11].

In addition to determining the appropriate input parameters and their limits, the evaluation rules generation is also a key question in the case of risk management. The antecedent parts of the rules are defined based on the input membership functions, because this part is generated by the combination of them. However, the consequent part of these rules can also be different. Consequently, this also had to be solved in the framework. In the author's model it is possible to adjust this part of the rules according to the specific-characteristics, taking into account the medical recommendations.

B.   Modified inference

The basis of the fuzzy inference in the framework is the conventional Mamdani-type inference, which was introduced in *Section II.C*. A great disadvantage of this kind of evaluation, that it has a high computational demand, which makes it inadequate to be used in real-time and adaptive systems. However, the author mainly deals with real time systems, consequently, this framework should also be used for these kinds of problems, where reaction time is essential, in some cases the long response time can cause serious problems. For this reason the original steps of the fuzzy inference are modified, the defuzzification is performed for each rules, and the obtained results are aggregated. In this case the functions, which define the fuzzy sets, and should be defuzzified, are simple shaped, and piecewise linear. Consequently, the operation needs of the defuzzification are negligible compared to the defuzzification in the last step, when a complex shaped function should be defuzzified. The correctness and operation-needs decrease of the modified evaluation was proven [12].

The steps of the modified evaluation and the used operators are as follows:

1. *Fuzzification*: The current input parameters depend on the specific-requirements, their number and the parameters of their membership functions can be varied. During the fuzzification trapezoidal membership functions are used, defined by (4).

$$
\mu_A(x) = \begin{cases} 0 & x \le a_i \\ \dfrac{x - a_i}{b_i - a_i} & a_i \le x \le b_i \\ 1 & b_i \le x \le c_i \\ \dfrac{d_i - x}{d_i - c_i} & c_i \le x \le d_i \\ 0 & d_i \le x \end{cases} \tag{4}
$$

where a_i, b_i, c_i, d_i, are the membership function parameters, which can be used to tune the membership functions specifically as it was described in *Section III.A*.

2. *Firing strength calculation*: product t-norm was used to connect the different conditions in the rule premise.

$$
w_i = \prod_{j=1}^{m} \mu_j(x) \tag{5}
$$

where m is the number of input parameters.

3. *Mamdani-implication*: In this step the product operator was used again for the obtained firing strength (w_i) and the rule consequents (μ_{Y_i}).

$$
y_{Y_i} = w_i \mu_{Y_i} \tag{6}
$$

4. *Defuzzification*: This method is performed for each consequent part separately for simple shaped membership functions, consequently, instead of the COG method, its simplified version can be used, which is defined for trapezoidal membership functions by (7).

$$
f_i = \frac{h}{3}\left(\frac{d_i^2 + c_i^2 - b_i^2 - a_i^2 + c_i d_i - a_i b_i}{d_i + c_i - b_i - a_i} \right) \tag{7}
$$

where h is the supremum of the membership function obtained from the previous step, and a_i, b_i, c_i, d_i are its parameters.

5. *Aggregation*: In the last step the obtained crisp values are aggregated by a weighted sum, where the firing strength of the rules are used as weighting factor. Its result is the system output for the given input values.

$$
O = \sum_{i=1}^{n} \overline{w}_i f_i = \frac{\displaystyle\sum_{i=1}^{n} w_i f_i}{\displaystyle\sum_{i=1}^{n} w_i} \tag{8}
$$

IV.   CASE STUDY

In this section, the usage of the framework is illustrated through the author's sport activity risk assessment model. The application of the system for this purpose is ensured by a well-structured database, which contains the personal profiles of the users. This database stores the personal characteristics and different sport types that can be assigned to each user (Table 1). The number and type of the factors, which should be monitored, can be very diverse [13], and they are patient- and sport-specifically defined in the database (Table 2). It means that different factors and limits can be assigned to different patients, and in the case of the same patient the limits can also be different for different sport activities (Tables 3, 4). Consequently, the risk levels for the same measured values can be different depending on the specific limits (Table 5). The detailed database design of the sport activity monitoring framework can be found in [11].

TABLE I.
BASIC INFORMATION (SSN: SOCIAL SECURITY NUMBER)

SSN	Age	Sex	State	Sport
152735261	28	Male	Beginner	Cycling
152735261	28	Male	Beginner	Running
014351673	55	Male	Athlete	Running
123354725	35	Female	Rehabilitation	Walking
123354725	35	Female	Rehabilitation	Running

TABLE II. PARAMETERS SHOULD BE MEASURED PATIENT- AND SPORT-SPECIFICALLY (HR: HEART RATE, SBP: SYSTOLIC BLOOD PRESSURE, DBP: DIASTOLIC BLOOD PRESSURE, RR: RESPIRATION RATE)

SSN	Sport	Parameters
152735261	Cycling	HR, SBP, DBP
152735261	Running	HR, SBP, DBP
014351673	Running	HR, RR
123354725	Walking	HR, RR
123354725	Running	HR, RR, SBP

TABLE III. MEMBERSHIP FUNCTION PROPERTIES

Parameter	MFname	MFnumber
HR	(target, mhigh, vhigh)	3
SBP	(low, normal, increased, abnormal)	4
DBP	(lower, average, arisen, irregular)	4
RR	(normal, higher, RRvhigh)	3

TABLE IV. MEMBERSHIP FUNCTION PARAMETERS (SBP)

Low		Normal			
c	d	a	b	c	d
125	147	125	147	160	185
148	168	148	168	191	203
158	169	158	169	193	204
130	140	130	140	167	177
150	160	150	160	187	197

Increased				Abnormal	
a	b	c	d	a	b
160	185	199	210	199	210
191	203	215	226	215	226
193	204	210	222	210	222
167	177	183	193	183	193
187	197	203	213	203	213

TABLE V. RISK LEVELS FOR USER 152735261

SBP	DBP	HR	Sport type	Risk level
140	70	130	Cycling	Safe
140	70	130	Running	Safe
180	90	170	Cycling	Dangerous
180	90	170	Running	Medium

V. CONCLUSIONS

In risk assessment, the determination of the relevant risk factors has a key importance, in order to obtain a reliable result. However, in many practical applications, they cannot be generally defined, i.e. the parameter selection can depend on the current circumstances or the specific-characteristics. These criteria justify the usage of a flexible model, which can adaptively handle the various and disparate inputs. Furthermore, the limits of these factors can also be different according to the specific-characteristics. Usually, these limits cannot be defined as exact values. Due to these requirements, the author has designed and implemented a fuzzy logic-based framework, which can handle the imprecision and uncertainty in the data and in the evaluation process, while varying the input parameters with the circumstances. In the system, the evaluation rules can also be modified according to specific needs.

The usage and relevance of this flexible fuzzy logic-based risk assessment framework was presented as a case study, which calculates the risk level of various sport activities, where the input parameters are the measured physiological values. The number and types of input can be varied in a patient- and/or sport-specific way, i.e. different inputs can be used in the case of different people, or for the same person for different sports. The limits for the inputs can also be adjusted with the same patient- and/or sport-specific method. Finally, the rules in the system can also be adjusted.

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