The reconstruction of the medieval Hungarian standard of length based on the geodetic survey of contemporary buildings

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Abstract **— As it is known from documents of the Middle Ages there was an independent unit of length in Hungary between the 11th and 16th centuries – the etalon of that was kept in Székesfehérvár. The 1/16 part of the so-called king's length (royal fathom) was published in statute books. The etalon of this unit does not exist, only a cord was found the length of which is 3.126 metres. In this article we want to demonstrate that this etalon was used for building churches at that time because the measures of the buildings correspond to an ancient unit. Especially the round churches (rotundas) are interesting from this point of view. We precisely measured some Hungarian medieval round churches, but only three of them will be presented in this paper (Kallósd, Bagod and Ják). We used total stations without prisms, angle and distance measurement for detailed polar survey. The measures of these buildings can also be obtained by precise methods, for example the radius of the circle with adjustment. We redrew the floor plan of these buildings. The measures were first given in metres but later in the ancient unit of length, in Hungarian royal foot. This floor plan was used to recalculate the size of the royal foot in metres. We got 31.9 centimetres for royal foot from three buildings. It means that the Hungarian royal fathom (10 feet) equals 3.19 metres instead of the 'official' value of 3.126 metres. Our assumption and the reconstruction method to obtain conversion factors were thus confirmed.**

I. INTRODUCTION

As a result of standardisation, distance data, i.e. lengths are given uniformly in a metre-based system all over the world. Today, the definition of the metre, a unit of length as such, is traced back to the wavelength of light. This definition was recognised by the international association after a Hungarian physicist, Zoltán Bay. In the beginning, the etalon was made in the shape of a metal bar. This platinum-iridium bar with a special cross-section is currently kept in Sevres, France.

Before the metre system was introduced, the Vienna fathom had been the official unit of length in Hungary, like in all the countries within the Habsburg Empire. The etalon of the Vienna fathom can be seen both in Vienna and in Bratislava even today.

Certificates confirm that there used to be an independent Hungarian length measurement system in medieval Hungary. Its etalon hasn't survived. Moreover, the memory of its existence has since then disappeared from the common knowledge, too.

This article presents the geodetic measurements which led to our attempts to restore the medieval Hungarian standard unit of length. The fundamental idea behind our work is that large buildings were designed and constructed on the basis of architectural plans even in the Middle Ages, which must have been carried out with the aid of the then units of length. We can also reasonably assume that the size of buildings were mostly given in round multiples of the measurement. If we manage to determine the measurements of a wisely chosen building that has been preserved in its original form, we may get the original unit of length in a metric system. Round churches which were built in the 10th century in numerous settlements in the Carpathian Basin are particularly suitable for the subject of such geometric, floor plan analyses. In our article we try to reconstruct the length etalon by accurately determining several measurements of three round churches located in Hungary.

II. THE MEDIEVAL HUNGARIAN SYSTEM OF LENGTH

A. The names and conversion factors of the medieval units of length based on the archives

We know little about the history of the Hungarian units of length used in the Middle Ages. Their emergence must have been influenced by the Greek, Roman and eastern cultures. It is likely that these units of length emerged from the actual sizes of human nave parts, which their Hungarian and English names also suggest.

The smallest natural unit of length is the **finger** (Hungarian: 'ujj'), which corresponds to the width of an index finger or the overall width of 4 barley seeds placed widthways next to one another. It was referred to as 'daktylos' by the Greeks and 'digitus' by the Romans. The Greek finger measures 19.3 mm in today's metric system.

The **inch** (Hungarian: 'hüvelyk') corresponds to the width of a man's thumb. It was used all over Europe and it still exists in the systems of measurement of several countries. An inch is equal to 12 lines.

The **palm** (Hungarian: 'tenyér', Latin: 'palmus') is a unit of distance that corresponds to the width of 4 fingers.

The **foot** (Hungarian: 'láb', Latin: 'pes') is a unit that has Greco-Roman origins. It doesn't correspond to the average length of a human foot but 16 fingers or 12 inches. Its length varies from country to country between 27-35 cm, according to today's metric units.

The **span** (Hungarian 'arasz', Latin: 'spitama') has two types: the great span is the distance between a grown-up man's extended little finger and thumb, whereas the little span is the distance between the index finger and the thumb. The Hungarian span (great span) makes 10 fingers.

The **cubit** (Hungarian: 'rőf', Latin: 'sing') probably derives from the length of the forearm. It corresponds to 2 feet, 8 palms or 32 fingers in the Hungarian system.

The **step** is likely to originate from the average length of a step. It makes 3 feet in the Hungarian system.

The **fathom** (Hungarian: 'öl', Latin: 'orgia, cubitus') comes from the distance of a grown-up man's extended arms. The English and German or Austrian fathom measures 6 feet. However, the Hungarian royal fathom makes 10 feet, i.e. it is much longer than the aforementioned ones. The Hungarian fathom is equal to 5 cubits or 16 inches.

We know the conversion factors above thanks to the research by István Bogdán [1] and they are displayed in Table 1.

its actual size on the side of the page. Although this statute book was reprinted in Leipzig in 1488, then in 1490, and a copy has survived, the length of the royal span cannot be measured. Sadly, the top edge of the relevant page was eaten by mice, the bottom edge was cut off by the binder's knife. The remaining copies of the second edition suffered a similar loss as the end of the line representing the span was cut off while they were being bound.

Figure 1. The royal span in the Tripartitum published in Vienna in the year of 1628

	fathom	step	cubit	span	foot	palm	inch	finger
1 fathom	1	10/3	5	16	10	40	120	160
1 step		1	1,5	24/5	3	12	36	48
1 cubit			$\mathbf{1}$	16/5	$\overline{2}$	8	24	32
1 span				1	10/16	40/16	7,5	10
1 foot					1	$\overline{4}$	12	16
1 palm						1	3	$\overline{4}$
1 inch							$\mathbf{1}$	4/3
1 finger								1

TABLE I. THE MEDIEVAL HUNGARIAN UNITS OF LENGTH AND THEIR EXCHANGE FACTORS

It is also István Bogdán who collected the excerpts from medieval certificates and archival documents, which mention the use and the regulation of units of length [1]. These texts prove that back then there used to be a system of length in Hungary and it was applied, indeed. Some Latin examples include the certificate written by the Chapter of Pécsvárad in 1270 *('amplexus… cum mensura regia'*), the certificate written by the Chapter of Pécs in 1278 *('ulna seu mensura… regis et regni '*) or that of the Chapter of Székesfehérvár written in 1368 *('cubitus seu mensura regalis…'*). Occasionally, you can come across the Hungarian terms, such as in the certificate written by Palatine Miklós Garai in 1379 *('spatium longitudinalis regalis mensura vulgariter Kyralymertek voce reperissent…'*).

B. The metric length of the royal span

The Hungarian Royal units of length and area were first mentioned in King Matthias' statute book, and there were drawings as well because the royal span was displayed in

The later statute books, which are known as the Tripartitum by Werbőczy (Hungarian: Hármaskönyv) and of which 50 editions were made, the length of the royal span can be measured using a millimetre ruler. The various editions have been studied by many but their results differ significantly. It is little wonder, though. The paper could have become dry and the printing mould cannot have been perfect either. If we wanted to determine the length of the royal fathom from the size of the span above, we would get a value between 2.88 m and 3.07 m. We must come to the conclusion that this way the fathom cannot be determined precisely enough – it is for informational purposes only.

The royal span was released in the statute book due to the regulation (or standardisation, as we would now call it) of the area measurement. According to the legal text, the royal fathom is equal to the royal span times 16. The unit of area measurement is the royal jugerum-sized land (Hungarian: királyi hold), which is equal to the area of 12×72 royal fathom.

C. The metric length of the royal fathom

There are certificate data in Latin about measuring ropes (measuring cords) used in the Middle Ages to mark and survey areas, which were regarded as an official measure because they had to be transported in a sealed bag. We do not know how long a measuring rope was. We can assume that it was 12 fathoms (maybe 24) long. The width of a royal jugerum-sized land is 12 fathoms, so the rope had to be laid down only once to measure the width and six times to measure the length. The 12-fathom measuring rope is approximately 38 m long – this is similar to the modern-day measuring tapes, which are 20, 30 and 50 m long.

The fact that the royal fathom used to have an etalon (standard measure), which was kept in Székesfehérvár, is known from a certificate that has survived in the archives at the Pannonhalma Archabbey. The Royal Basilica of Székesfehérvár (which has only few remains left to be seen) was the Hungarian kings' coronation and burial site from the time of King Stephen to the Ottoman rule (like Westminster in England, Saint Denis in France, Aachen in Germany and St Vid in the Czech Republic). The crown jewels, the treasury, the archives and the length etalon were all guarded in the provostry that belonged to the basilica.

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Figure 2. The wound cord and the drawn span in a report from the year of 1702. The place where it is kept: The Hungarian National Archives. Mark: MNL OL E 117 – Fasc. 14. – No. 1.

The above-mentioned certificate is about a land debate between Bakonybél Archabbey and squire of Bakonyszücs. If one of the partners does not regard the area measured by a measuring cord as legal, then they should go to Székesfehérvár (Alba Regalis) and fetch the standard measure of the royal fathom as a heredity of Saint Stephen. The Latin text: *'si mensuram ambiguitatis propulsivam et certam idem dominus abbas habere voluerit, ex tunc hominem suum cum homine eiusdem magistri in Albam Regalem pro aportanda mensura per Sanctum Stephanum regem derelictam et constitutam deberet destinare, alio autem modo nullam iteratam mensurationem acceptaret'.*

Unfortunately, no tangible memory of the etalon has survived, we know of one single copy to be precise, which turned up in the Hungarian central archives. This copy is a royal-fathom long measuring rope, which was attached to a report from 1702 year. Furthermore, a unit of length corresponding to one span was drawn in the report. The report was written about the survey of the lands that belonged to the two villages. The length of the measuring rope (the distance between the knots tied at the two endpoints of the rope) was measured in the Hungarian Metrology Office and it was said to be 3.126 m. This value is recognised as the metric length of the medieval royal fathom. If the metric value of smaller units is derived from this, we get the following: 1 foot $= 31.26$ cm; 1 span = 19.54 cm. The latter is in harmony with the distance drawn in the report, which was 19.6 cm. (If the length of the fathom is 3.20 m, the sixteenth of this is 20 cm. The official fathom-span ratio and the determined one are the same. This means that the extent to which the string shrank is equal to the extent to which the drawing on the paper shrank too.)

III. THE POSSIBILITY OF USING BUILDING MEASUREMENTS TO RESTORE THE UNIT OF LENGTH

A. Contemporary buildings as the possible guardians of the unit of length

Contemporary buildings (churches, castles, mansions) are objects usually made of symmetrical geometric shapes that have regular floor plans. We can rightly assume that the marking and the construction of these buildings required the use of plans or else they can't have been built in such quality.

Figure 3. Archaeological floor plan and editing of Prince Géza's church in Székesfehérvár [2]

Architectural design presumes the use of some scale, i.e. the correspondence between a drawing and a real (aerial) unit. It also presumes a system of length. Nowadays, the standard scale of architectural design when using a metric scale is 1:50 (1 mm on the drawing corresponds to 50 mm in reality) or 1:100 (1 mm on the paper represents 10 cm). At the time of the Vienna fathom the scale was 1:72 (1 Vienna inch corresponded to 72 inches in reality, i.e. 1 Vienna fathom or 6 feet). We do not know what the design scale was in medieval Hungary because there is no information about plans that have been preserved intact. It might have been 1:80 for instance, when 1 finger would correspond to 5 feet (0.5 fathom),

but it might have been 1:32, when 1 finger would represent 2 feet.

We can also assume that during the design and the construction the key measures of buildings were provided in round multiples of the unit of length. This simplifies work and is advantageous for practical reasons.

To prove our previous assumption, we studied the measures of 27 medieval churches. They were measured on the basis of archaeological and architectural plans that had been made in the course of heritage preservation. We found that by converting the metric values according to *Table 1* we mostly got round numbers, which made us strongly believe that the former unit of length had been employed.

By way of illustration, we want to share the floor plan of one building and its measures expressed as royal feet with you. This very building was once situated at the highest point of Székesfehérvár and it was the oldest church in the city. It had been built by the first Hungarian king's father, Prince Géza, probably as a chapel. It used to be a church with four vaults, its remaining base walls were excavated by archaeologist Alán Kralovánszky only in 1971 [2]. He reconstructed the design and building process of the regular church and concluded that the outer radius of the vaults corresponded to exactly 1 royal fathom.

For further observations let us reverse the way of thinking detailed above. If the assumption that objects were constructed using the round (or half, maybe onefourth) multiples of the former unit of length based on plans turns out to be right, then the metric value of the contemporary unit of length could be calculated from the measures of the building based on an accurate survey (carried out in a metric system). It does matter, however, what sort of a building we choose and what method we employ to conduct the survey.

B. The significance of round churches in terms of size determination

Round churches are worth the attention for several reasons. In numerous countries all over Europe, especially in Central-Europe, the oldest churches of the 9th and 10th centuries were built to be circular and quite of few are still standing.

The circle is the simplest geometric shape. It was not only easy to draw with the aid of bows (rondure) on a piece of paper but it was possible to setting out during field work – you needed a string and two poles.

Round churches are advantageous in terms of size determination because in the simplest case there are at least two circles available to be studied – the circle of the outer wall and that of the inner wall. It is also very likely that the thickness of the wall is a multiple of the measure. If the thickness of the foundations and that of the wall are not identical, presumably the difference can be expressed as the former unit of length, too.

Nevertheless, the majority of round churches do not have a base that consists of two circles because the sanctuary and the nave are also rounded. Sanctuaries facing east can also be semicircular, horseshoe-shaped or they have corridor links.

We talk about round churches closing in a semi-circular sanctuary when the centre of the church's arc lies on the

inner arc of the nave (Fig. 4). On the floor plan of such regular churches the radius *(R)* of the sanctuary is generally half of the radius of the nave, the wall thickness (F) is identical. It is also worthwhile to measure the inner *(B)* and outer length *(K)* of the church. Since we get a distance that is longer than the unknown radius *(B=2,5R;* $K=2,5R+2F$, we can determine the radius more accurately. If we expect the *R* and *F* values to be round multiples of units smaller than the royal fathom (foot or span), the length of this smaller unit expressed as metres can be determined on the basis of *B* and *K.*

Figure 4. Semi-circular sanctuary closures

We talk about horseshoe-shaped sanctuary closures when the centre of the church's arc lies on the wall or the outer arc of the nave (Fig. 5). The wall thickness of the sanctuary is often half of what the nave possesses.

Figure 5. Horseshoe-shaped sanctuary closures

A sanctuary is regarded as elongated when a quadrilateral corridor link is found between the sanctuary and the nave.

A special group within round churches includes those with four vaults. We can determine not only several circles but also additional things because the location of the vault centres shows regularity.

C. The general technology of building survey – the role of surveying

Since we conducted the survey of numerous round churches as well as the determination of their measures recently, we can propose a technology how etalon reconstruction should be done by generalising our experience.

Figure 6. Good examples to right identificate ground wall points at Ják (rough walling) and Kallósd (brick walling) church

- 1.) Choosing the right building to be analysed. This means that it is recommended to choose such a contemporary building that has survived in its original form, the foundation walls are easily identifiable, and the building itself is symmetrical and geometrically regular. It is quite difficult to find a building that has been preserved in its original form because it could have been necessary to rebuild the building to a certain extent over the centuries. In this case, the individual building sections of the different construction periods must be separated as much as possible. The criterion of being identifiable is fulfilled if the foundations or the walls have been built of bricks or ashlar, for instance (Fig. 6). The identification of walls (surfaces, corners) made of rubbles is not clear, therefore such buildings are less ideal for our purposes. Asymmetrical and irregular buildings are unsuitable, too – the circles are ellipses, the columns are not the same, the column intervals are different or there is no other sign of regularity.
- 2.) Identifying the main lines and points of the building to be measured. The key question of all surveys is what we intend to measure. In our case it is enough to survey the elements that comprise the base geometry of the building. To do this, however, we need to study the construction history and the structure of the building or else we cannot choose the right points to be measured.
- 3.) Choosing the right measuring technology. There are different kinds of measuring equipment and technology which may meet our needs: measuring tapes, total stations, laser scanners, UAVs and photogrammetry, etc. For our surveying work we chose the technology of total stations – we presume its use from now on. It is a great advantage that inside and outside the building an accurate geodetic control point network can be created with the aid of direction and distance measurement, the scale of which (its metric system) is provided by the

frequency of the calibrated distance measuring instrument. It is also very practical that we can decide which points we wish to measure, i.e. we need to measure the points only which we find essential.

Figure 7. Free network around Kallósd church: sketch of control points, directions and distances

4.) The accurate measurement of the geodetic control point network. We need a continuous geodetic control point network based on direction and distance measurement both inside and outside the building (Fig. 7) and on more floors if necessary. Our aim is to provide a uniform, homogenous and accurate coordinate system. Accurate measurement means that we set up all the tripods with base plates, then we change the instrument and the prisms on the base plate to avoid positioning errors which can be dangerous due to the short distances (Fig. 8). As a result, we can ensure a network with deviations of one or two mm not only horizontally but also vertically. We must take a sufficient number of extra measurements (more than geometrically necessary).

Figure 8. Measuring control point network at Kallósd church: constrain centered setting up

5.) Precise measurement of detail points as polar points. Detail points (points for observation) are to be measured at the same time as the control points using the same measuring equipment. Since they are chiefly building corner points, column corner points and arc points, where the positioning of a prism is not possible centrally, it is better to measure all these points without a prism. A card should be placed on the point to be measured so that it is perpendicular to the direction line and the touchpoint (line) of the card and the building must be set by the measuring equipment (Fig. 9). If a building point is covered by something, we need to employ a method that relies on points outside. Wall surface points perpendicular to the direction line can obviously be measured without a card. Clearly identifiable points are to be measured from two or more station positions.

Figure 9. Measuring wall-points without prism, using card

- 6.) Calculating the coordinates of the geodetic control point network and the points for observation. The calculation of the coordinates and the altitude of the geodetic control points must be carried out with adjustment or else we cannot take into consideration all the measurements simultaneously and, therefore, the result will not be accurate. The coordinate deviations cannot exceed 3-5 mm. Not only the coordinates of the control points, but also those points for observation must be shown with an accuracy of some mm. The calculation is to be done in a separate system as a free network to avoid frame errors that can influence the result. If the building has a standard axis, it is better to turn the local system using a planar isometry so that one coordinate axis should be perpendicular to the main axis of the building. We also apply a planar transformation when we wish to export our points to another system (to an adjacent one) – to illustrate how the axis of a church matches a cardinal point, for instance. For such transformation purposes the control points around the building ought to be measured by GNSS technology.
- 7.) Calculating the standard floor plan measures of the building. This calculation is carried out based on the measured points for observation using methods of coordinate geometry with an accuracy of mm. Measures of length and width, wall thickness, sizes of columns, distances of column intervals and altitudes belong here. Providing the standard data of the circles in terms of round churches is considered to be a separate task. An adjusted (regression) circle must be fit to the measured points of the arcs according to the least squares method (Fig. 10). We will obtain not only the

coordinates of the centre *(C)* and the radius *(R)* of the circle but also the standard deviations (rms) errors of them. Depending on the residuals *(v)* and deviations (rms) we are able to decide whether the building suits our purposes or not. Furthermore, standard deviations can be useful when determining the weight of measures later on.

Figure 10. Symbolizing the adjusted circle

- 8.) Constructing the floor plan. The floor plan of the building is now ready to be constructed based on the points that have been measured and the calculated measurement data $-$ in a metric system, of course. The points (in the same vertical plane) that are on one line can be drawn as a regression line. Standard measures are given with an accuracy of some mm.
- 9.) Matching standard building measurements to the former units of length. It is best to do this in an Excel table. The first questions we wish to answer is: Was the unit of length at the time of the construction expressed as royal foot or royal span? We get the desired information after dividing the standard, metric building measurements by the 'official' metric length of the foot (0.3126 m) and that of the span (0.1954 m). For some units of length we get round (or half) pieces which are regarded as preliminary values.
- 10.) Constructing the floor plan using the former unit of length. We try to make a floor plan that is similar to what the original could have been where standard measures were probably given in round (or half) multiples of the foot or span. It is really time-consuming and we may not succeed at once.
- 11.) Reconstructing the former unit of length. We need to make a table which includes both the standard distance data of the building given in metres and the unit of length in pieces. The metric value of the former unit, usually expressed as cm, is obtained from the quotient of the two data. The metric values of the unit will certainly not be the same. We recommend such a weighted average as an end result where we take into consideration how well the two endpoints of the measure observed were identifiable. For example, the deviations of the radii are helpful in this case.

The technology described above has been invented after surveying several buildings and processing their measures.

We can highly recommend it for similar purposes. In our view, appropriate results can only be obtained using accurate surveying methods. This is how surveying contributes to reconstructing the length etalon.

IV. THE RECONSTRUCTION OF THE UNIT OF LENGTH BASED ON THE ACCURATE SIZE DETERMINATION OF FIVE ROUND CHURCHES

In this chapter we present our practical surveying work and some results of it. The workflow of surveying method was the same we mentioned in Chapter III. Five Hungarian round churches were surveyed and analysed to reconstruct the ancient unit of length

A. The Saint Anne round church in Kallósd

Kallósd is a small bag village in Zala county. Its parish church was built around 1270 in Romanesque style.

Figure 11. Round church of Kallósd

The inhabitants of the village were forced to leave the church by reason of the Turkish occupation in the 17th century. The population returned in 1711. They started to clean the thicket around the church and renovated the building in 1740. Because of the growing number of inhabitants they had to build a hallway to the church in the 19th century which was demolished during the renovation between 1989 and 1993 in order to preserve the round church in its original shape.

Figure 12. Specialities of Kallósd: sitting bays and lizenas

The walls are built from brick. it means the identification of walls and measuring points are ideal for our purposes. There are seven sitting bays (sedilia) inside the nave, the sizes of them are also interesting for us. The other specialities are the small columns outside the nave wall a so called lizenas. 9 of them are on the northern part (left to entrance) and 3 of them are on the southern part (right to entrance). The results are seen in the Table II.

Figure 13. Floor plan and notations Figure 14. Floor plan of Kallósd in former unit (in foot)

THE SIZES OF KALLÓSD CHURCH										
	Description of sizes		Distance (meter)	RMS (meter)	Pieces.	foot (cm)	weight			
1	Inner radius of nave (from 20 points)	R ₂	2,671	0,003	8,5	31,42	3			
2	Outer radius of nave (from 20 points)	$\mathbf{R}1$	3,937	0,003	12,5	31,50	3			
5	Inner radius of sanctuary (from 6 points)	r2	0.980	0,012	3	32,67	1			
$\overline{4}$	Outer radius of sanctuary (from 8 points)	r1	1,627	0,006	5	32,54	2			
3	Outer length of the church $(2R1+r1)$	K	9,501	0,008	30	31,67	2			
6	Inner length of the church $(2R2 + F + r2)$	B	7,588	0.019	24	31,62	$\mathbf{2}$			
7	Thickness of nave wall (R1-R2)	F	1,266	0.005	4	31,65	2			
8	Thickness of sanctuary wall (r1-r2)	f	0,647	0,014	\overline{c}	32,35	1			
9	Lizena width (12)		0,155	0,002	0,5	31,00	1			
10	Distance between lizenas (10)		1,267	0,003	4	31,68	1			
11	Column width at sitting bays (5)	$\mathbf c$	0,314	0,002		31,40				
12	Radial size of columns (4×8)	a, b	0,204	0,002	0,625	32,64				
13	Width of sitting bays (7)		0.99	0,004	3,125	31,68				
14	Radial size of lizenas (2×12)		0.101	0.004	0.3125	32,32	1			

TABLE II.

B. The round church of Bagod

Figure 15. Bagod church outside and inside todays

Bagod is a settlement in Zala county, earlier on its territory 3 independent villages were found: Bagod, Vitenyéd and Szentpál. Nowadays Szentpál is a bag village, in the cemetery of this village situated the earliest rotunda It was built at the end of 13th century, but in 18th century it was totally rebuilt and enlarge. The original nave become as a sanctuary and new rectangular nave was built.

During 20th century the church was deserted, the roof was destroyed. Between 1999 and 2001 the church was totally renovated

From our point of view only the today's sanctuary (the original nave) is interesting. There are four circles which could measure well: the inner and outer wall, the foundation and the circle of sitting bays. The results are seen in the Table III.

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TABLE III.

Figure 16. The floor plan and notations of Bagod church

Figure 17. The floor plan in original unit (in foot)ch

C. The Saint James church of Ják with four vaults

Figure 18. The St James round chapel (left) and the Benedictian monastery church in Ják (foto:Civertan)

Two churches were built in the middle of $13th$ century in Ják settlement (Vas county): one as benedictian monastery church and one as presbyterian chapel. The first one is the famous monument of Hungarian medieval architecture, the second one is the small Saint James round chapel with four vaults. serving as a church of the village in the Middle Ages.

As it has been proven by the excavation in 1997 the St James chapel have been built on a rotunda foundation also. The curves of this earliest rotunda is now seen on the brick floor.

Figure 19. Ják church outside and measuring inside

Description	Centre		\mathbf{x}	Radius	r	RMS v	RMS x	RMS r
$1st$ arc, inner wall	K1	499,850	202,025	r1	1,492	0.010	0.019	0.009
$1st$ arc, outer wall	(Northen) arc)	499,852	202,009	$\mathbf{R}1$	2,620	0.002	0.007	0.002
$1st$ arc, foundation		499,848	202,014	RL1	2,815	0.002	0.005	0.001
$2nd$ arc, inner wall	K2	502,046	200,183	r2	1,501	0.012	0.002	0.005
$2nd$ arc, outer wall	(Eastern) arc)	502,049	200,210	R ₂	2,603	0.007	0.002	0.002
$2nd$ arc, foundation		502,076	200,217	RL2	2,799	0.011	0.004	0.002
$3rd$ arc, inner wall	K3	500,208	198,014	r3	1,487	0.003	0.007	0.003
$3rd$ arc, outer wall	(Southern) arc)	500,197	198,015	R3	2,607	0.003	0.009	0.003
$3rd$ arc, foundation		500,219	197,988	RL3	2,795	0.005	0.011	0.002
$4st$ arc, inner wall	K4	498,078	199,844	r4	1,515	0.032	0.008	0.030
$4st$ arc, outer wall	(Western) arc)	498,050	199,843	R4	2,608	0.035	0.006	0.012
$4st$ arc, foundation		498,055	199,861	RL4	2,803	0.027	0.007	0.005

TABLE IV.

Figure 20. The sketch of control point micronet around the church

Figure 21. The ground plan of Ják church and notations

We set up a micro-net around the chapel with 5 control points (Fig. 20) and measured all detail points (in every meter sequentially). We identified 3 circles (arcs) in each vault: the inner and outer wall points and the outer foundation points (12 arcs altogether). After it we calculated the center point coordinates and radiuses of these arcs (Table IV). The standard deviations of these parameters are below 1 centimetre only one exception is the western arc. The reason is that on this vault we could measure few points because of the entrance door.

The centres (K1, K2, K3, K4) of different adjusted circles are mainly the same. These centres are corner points of square. The interesting thing is that K1-K4 centres and the wall endpoints (F1-F4) are located on the same circle.

Figure 22. The sizes of Ják chapel in old Hungarian spans

What is the radius size of this circle? If we analyse and examine the size we will find that the size of radius is exactly 10 span. So it means that the sizes of tis chapel are not determined on foot but in span. All other radiuses we can determine in integral number of spans, for example

	Description of sizes		Distance	RMS	Pieces.	span	weight			
			(meter)	(meter)		(cm)				
1	$1st$ arc,) radius of inner wall (from 8 points	r1	1,492	0,009	7,5	19.89	$\mathbf{1}$			
\overline{c}	$1st$ arc, radius of outer wall (from 12 points)	$\mathbf{R}1$	2,620	0,002	13	20,15	$\overline{2}$			
5	$1st$ arc, radius of foundation (from 16 points)	RL1	2,815	0,001	14	20,11	$\overline{2}$			
$\overline{4}$	$2nd$ arc, radius of inner wall (from 6 points)	r2	1,501	0.005	7,5	20,01	$\mathbf{1}$			
3	$2nd$ arc, radius of outer wall (from 16 points)	R ₂	2,603	0,002	13	20,02	\overline{c}			
6	$2nd$ arc, radius of foundation (from 16 points)	RL2	2,799	0.002	14	19.99	$\overline{2}$			
7	$3rd$ arc, radius of inner wall (from 6 points)	r3	1,487	0,003	7,5	19,83	$\mathbf{1}$			
8	$3rd$ arc, radius of outer wall (from 13 points)	R3	2,607	0,003	13	20,05	\overline{c}			
9	$3rd$ arc, radius of foundation (from 17 points)	RL3	2,795	0,002	14	19,96	$\overline{2}$			
10	4 st arc, radius of inner wall (from 6 points	r4	1,515	0,030	7,5	20,20	$\mathbf{1}$			
11	$4st$ arc, radius of outer wall (from 7 points)	R ₄	2,608	0,012	13	20,06	\overline{c}			
12	$4st$ arc, radius of foundation (from 11 points)		2,803	0.005	14	20,02	$\overline{2}$			
13	Distance between K1-K2 points		2,856	0,007	14,14	20,19	$\mathbf{1}$			
14	Distance between K2-K3 points		2,872	0,005	14,14	20,31	$\mathbf{1}$			
15	Distance between K3-K4 points		2,830	0,014	14,14	20,01	$\mathbf{1}$			
16	Distance between K4-K1 points		2,810	0,013	14,14	19,87	$\mathbf{1}$			
17	Distance between F1-F2 points		2,822		14,14	19,96	0,5			
18	Distance between F2-F3 points		2,814	from floor plan	14,14	19,90	0,5			
19	Distance between F3-F4 points		2,799		14,14	19,79	0,5			
20	Distance between F4-F1 points		2,822	14,14		19,96	0,5			
21	Total outer length (E-W)		9,613		48	20,03	$\overline{2}$			
22	Total outer length (S-N)		9,633		48	20,07	$\overline{2}$			
23	lizena width (6 pieces)		0,506	0,013	2,5	20,24	$\mathbf{1}$			

TABLE V. THE SIZES OF JÁK CHAPEL

the radius of foundation arc is 14 span, the radius of outer wall is 13 span, the radius of inner wall is 7 and half span.

The other interesting thing is that the total length of this chapel is 48 span it means exactly 3 old Hungarian fathom (Table V).

V. CONCLUSIONS

TABLE VI. THE RECONSTRUCTED HUNGARIAN FOOT UNIT FROM ALL SIZES OF THREE BUILDINGS

We nicely measured the sizes of three round churches from Medieval Ages, redrew the floor plan of these buildings. The measures were first given in metres but later in the ancient unit of length. These floor plans were used to recalculate the size of the royal foot in metres. We used all the sizes and calculated the weighted average for all three buildings (Table VÍ).

At the end, as the average size of ancient Hungarian royal foot we got 31.91 centimetres. So we reconstruct the earlier original unit of length in a metric system. It means that the Hungarian royal fathom (10 feet) equals 3.19 metres instead of the 'official' value of 3.126 metres. Our assumption and the reconstruction method to obtain conversion factors were thus confirmed.

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