# Accuracy analysis of Glacial Isostatic Adjustment models using for satellite gravimetry

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***Abstract*—Ice mass balance investigations in Antarctica with the use of GRACE monthly gravity models should take into account the GIA process of the Antarctic lithosphere. In the present study visual and statistical comparison of most recent and regularly used GIA models is performed. According to the results, the difference of the models is in the range of mm/yr of equivalent water column height. Since GRACE has detected Antarctic ice mass variations of 2-3 mm/yr, the choice of the GIA model may completely dominate the result. Therefore GIA models must be treated with reservation.**

## I. Introduction

### A. History of the Antarctic ice cover

It is known from geological (sediment records, permanent ice records) and geomagnetic evidences that starting in the Proterozoic eon (from 2500 to 542 million years ago (Ma)) the Earth regularly goes through glaciations interrupted by interglacial periods. There have been five major glacial periods so far, they are summarized in Table I.

The Antarctic continent (or the equivalent region within Gondwana before its breakup 160 to 23 Ma) has been affected by the ice ages to different extent. During the Andean-Saharan and Karoo ice ages Antarctica has been located at tropical latitudes, then it has started to move to the Southern pole [1] arriving to approximately its present position about 70 Ma (still merged with Australia). That time due to the global subtropical climate no ice formation could be developed. Gradual cooling has started 55 Ma, the first signs of Antarctic ice dates to 35 Ma (during this period Antarctica has split with Australia 40 Ma). Intensive ice formation periods can be detected from sediments about 13 Ma and also about 7-5 Ma. Antarctic ice has continuously developing until the Last Glacial Maximum (LGM) of the Quaternary glaciation occurring between approximately 110,000 to 12,000 years ago. Then it has started to melt, and it is in constant melting also in the present.

Obviously, the Antarctic ice has been developed independently from the glacial periods, c.f. Table I, approximately for 55 Ma, it has only been amplified by the Quaternary glaciation (in fact, glaciation on the Northern hemisphere has exclusively been formed by this glaciation event). Thus, Antarctic ice is the consequence of a regional features, which is can be explained by the opening of the Drake passage 20-25 Ma ago creating the Antarctic Circum Current (ACC), which blocks heat transport to the tropics, cooling the whole continent down [2]. As the ACC is the main factor of the Antarctic climate, present day climate changes due to global warming may essentially occur due to its changes.

### B. The effect of permanent ice loading

Fig. 1 shows the thickness of the ice cover over Antarctica from on the BEDMAP project [3]. Obviously, the thickness of the ice sheet is more than 4 km at the central part of East Antarctica, which is a huge amount of frozen fresh water. According to an estimate of [4] the present volume of the Antarctic ice is 27.000.000 $km^3$.

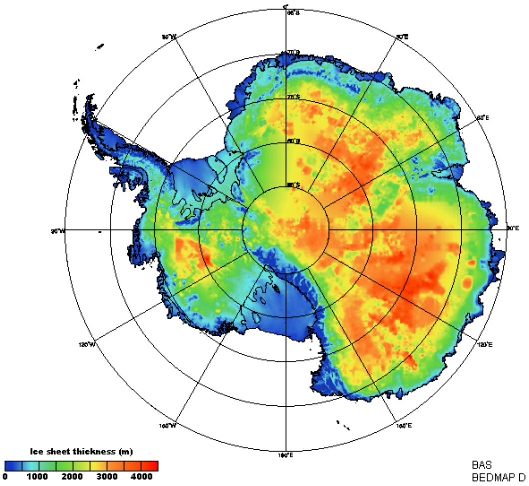


Figure 1. Thickness of the ice cover, source: [3]

TABLE I.
LIST OF MAJOR ICE AGES

| glaciation | eon | era | era | period |
|---|---|---|---|---|
| Huronian | Proterozoic | Paleoproterozoic | Siderian, Rhyacian | from 2400 Ma to 2100 Ma |
| Cryogenian | Proterozoic | Neoproterozoic | Cryogenian | from 720 Ma to 635 Ma |
| Andean-Saharan | Phanerozoic | Paleosoic | Ordovician, Silurian | from 450 Ma to 420 Ma |
| Karoo Ice Age | Phanerozoic | Paleosoic | Carboniferous, Permian | from 360 Ma to 260 Ma |
| Quaternary | Phanerozoic | Cenozoic | Quaternary | from 2.58 Ma to present |

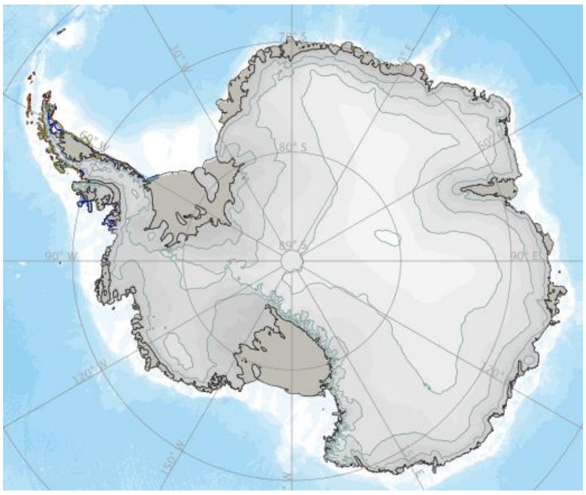

Figure 2. Surface topography

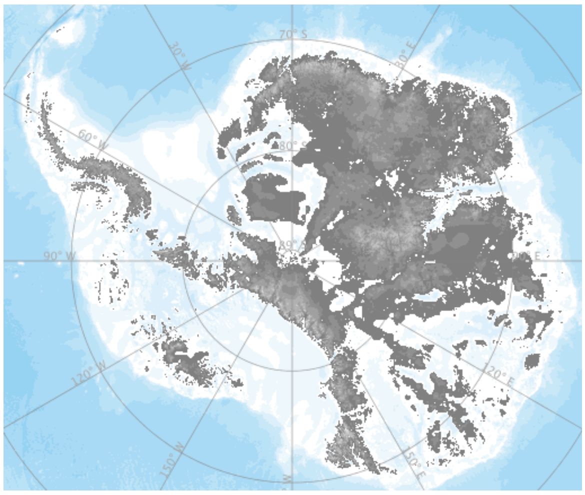

Figure 3. Bed topography

This incredible amount of ice mass pushes the upper rigid layer of the Earth, the lithosphere (composed of the earth crust and some part of the upper mantle), which is floating on an underlying viscous layer, the asthenosphere. The lithospheric plates on the asthenosphere are in equilibrium, each blocks of the lithosphere are dipping into the asthenosphere depending on their mass, on the viscosity and the elasticity of the underlying asthenosphere. The huge amount of ice load adds to the mass of the lithosphere, and depresses the lithospheric plate into the asthenosphere. Due to the additional mass load, several regions of the Antarctic continent lay below the sea level. Fig. 2 shows the Antarctic topography, while Fig. 3 displays the topography below the ice cover generated from BEDMAP data [3]. Obviously, most are of the West-Antarctic part is under the sea level.

However, situation shown on Fig. 1-3 is just a permanent situation, which may change drastically by time. As glacials and interglacials are changing, the ice content may change, changing the centre of mass of the lithospheric plate. The LGM has been "quickly' ended approximately 12,000 Ma ago, which means that (in geologic time scales) over a very short time a huge amount of ice mass has disappeared from the continent. The centre of mass of the lithospheric plate moved up relevantly, resulting in a real time isostatic uplift of the lithosphere. The viscosity of the asthenosphere is, however, essentially different from those fluids we consider as 'liquids' in daily life; for clarification the difference: while water has a viscosity of $8.94\times10^{-4}$ Pa·s, the viscosity of the asthenosphere is approximately $7\times10^{19}$ Pa·s [5], so 23 orders of magnitude difference in resistance to shear stress and tensile stress. This means that isostatic compensation takes notable time. In fact, isostatic compensation of the lithospheric plate due to the end of the LGM (this is the so-called Post-Glacial Rebound, abbreviated as PGR) at several locations of the Earth has not been completed yet. The apparently affected regions by PGR are Northern Eurasia (particularly Scandinavia), Northern America (surroundings of the Hudson Bay), Patagonia, and Antarctica.

When PGR is mentioned, it purely refers to the unloading after LGM. In reality change in ice mass content is more dynamic: even during the Quarternary glaciation there have been several glacial and interglacial periods, which may have been affected the amount of ice by unloading or uploading it. Each process has influenced the vertical motion of the lithosphere. Furthermore, recent ice mass changing processes also affects the motion. Also, the solid Earth also suffers deformations due to the loading, making the whole uploading and unloading phenomenon quite complex. The term used for this complex phenomenon is Glacial Isostatic Adjustment (GIA).

The complexity of GIA process is challenging in theoretical level, while observing it is challenging in practical level: due to plate tectonics, vertical plate motions, and the ceaseless changes of the sea level, there is no fix point to compare to. Still, observations regarding to GIA process can be obtained by geodetic control networks (which are usually interpreted with certain assumptions and feasible for certain regions only). Instead of leaning on observations, GIA modeling is based on theoretical considerations. Assuming certain glaciation history for the whole globe (including an assumptions on ice mass distribution at a starting historical epoch), fixing certain Earth parameters, such as total mass, mean radius, angular velocity of the rotation, defining a mantle viscosity model, delineating the delineation and the ocean bed topography for every geological era, circumscribing all subsequent glacial events with climate parameters, then integrate all this information into a global flow model (the so-called 'Sea-Level Equation'), one can derive a GIA model for the whole Earth (a detailed description is provided by [6]. It is a very complex methodology, which depends notably on the preconceptions and arbitrary parameterization. Indeed, in some cases not a singular GIA model is determined by the same group of researchers, but a set of reliable scenarios, e.g. [7] has delivered 6 slightly different versions for their IJ05_R2 model.

At the end, the measure of the reliability of the GIA models derived in such a way is its validation with observed deformations by geodetic control measurement results, remote sensing satellites and present observations of climate change. As GIA models fits better at different regions, no globally most reliable model has been found by any validation so far. In this study, a statistical intercomparison of those GIA models is presented, which are most commonly used for Antarctic ice mass balance investigations.

The comparison is performed with the purpose of using the GIA models for satellite gravimetric investigations. As GRACE satellites enables investigation of mass variation in a monthly basis [8], elimination of GIA process from actual mass variations, such as ice mass variations, is unavoidable. Thus in this study GIA model are processed in a way to be appropriate for satellite gravimetric use.

## II. METHODOLOGY

Surface mass anomaly, $\Delta\sigma$ can be calculated by the formula derived by [9]:

$$\Delta\sigma(\vartheta,\lambda) = \frac{a\rho_E}{3}\sum_{l=0}^{L}\sum_{m=0}^{l}\frac{2l+1}{1+k_l}\bar{P}_{lm}(\cos\vartheta)\cdot$$

$$\cdot(\Delta C_{lm}\cos m\lambda + \Delta S_{lm}\sin m\lambda) \qquad (1).$$

In equation (1) $\vartheta$ and $\lambda$ are co-latitude and longitude, $a$ and $\rho_E$ are mean radius and average density of the Earth, $l$ and $m$ are degree and order of the spherical harmonics, $L$ is the maximal degree of the harmonic expansion, $k_l$ is the load Love number, $\bar{P}_{lm}$ is the Legendre function, $\Delta C_{lm}$ and $\Delta S_{lm}$ are Stokes coefficients describing relative variations of surface mass density. Surface mass anomaly describes mass variation over time e.g. in *Gt/yr* unit, which can be converted to equivalent vertical crust motion. The unit of surface mass anomaly is thus *mm/yr* as that of the vertical uplift [10].

Surface mass anomaly from each GIA models was calculated according to the methodology used by [11]. The calculation was performed in a *100 km* × *100 km* grid, altogether in 1333 pixels. The elastic loading was taken into account by the standard manner using elastic Love numbers, c.f. [9]. The calculated mass variations were smoothed by a Gaussian filter with *300 km* radius [12].

## III. COMPARISON OF GIA MODELS

In order to quantify the uncertainties GIA models, statistics of the following models are derived: ICE-6G model [13], W12 model [14], IJ05 Revision 2 models [7] and the combined GIA model used in [15]. In order to differentiate the IJ05 Revision 2 models, the key parameters are hidden in the model name: the first number after the R2 tag refers to the assumed lithospheric thickness (65 km is more consistent with the West Antarctica rift system, while 115 km with the East Antarctic craton), afterwards different values for $\eta_{UM}$ viscosity at the upper mantle, finally for $\eta_{LM}$ viscosity at the lower mantle – both values should be implemented in $10^{21}$ Pa·s unit.

As in most GRACE applications a Gaussian smoothing is applied during the processing, in this analysis only the smoothed versions of the different models are compared. The smoothed models are displayed on Fig. 4-12. Looking at these figures, obvious spatial differences can be found. In case most of these models, there are two local maxima of the GIA velocity in West Antarctica, however their exact location is variable by model. Obviously, the second peak is much smaller in case of the IJ05 R2 models. Another regular difference among these models is the emphasis of the East Antarctic coastal area: while ICE6G doesn't show here relevant uplift, all other models find these regions in definite rise. Particular difference can be found in Enderby Land (located along the coast at the upper right corner of these figures).

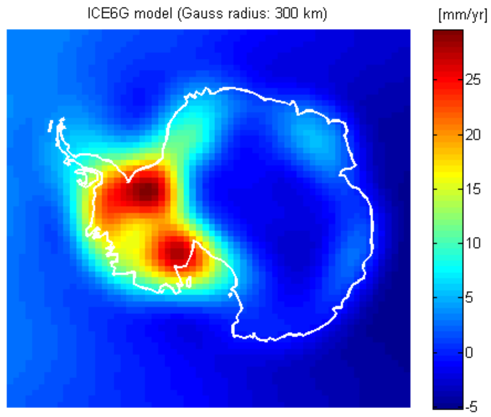


Figure 4. Linear trends calculated from ICE6G model with Gaussian filter of 300 km applied.

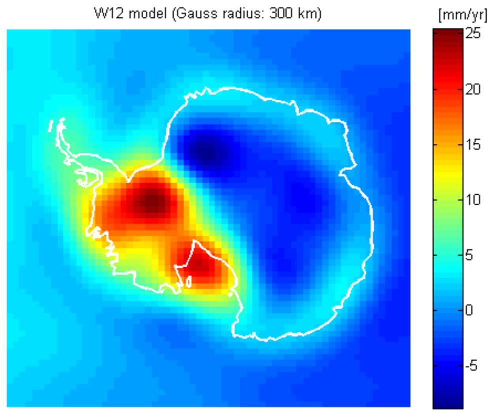


Figure5. Linear trends calculated from W12 model with Gaussian filter of 300 km applied.

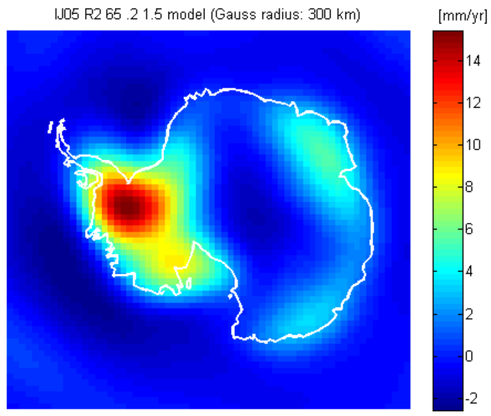


Figure 6. Linear trends calculated from IJ05 Revision 2 version 1 model with Gaussian filter of 300 km applied.

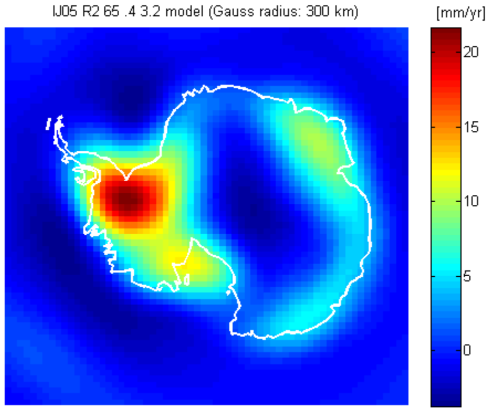


Figure 7. Linear trends calculated from IJ05 Revision 2 version 2 model with Gaussian filter of 300 km applied.

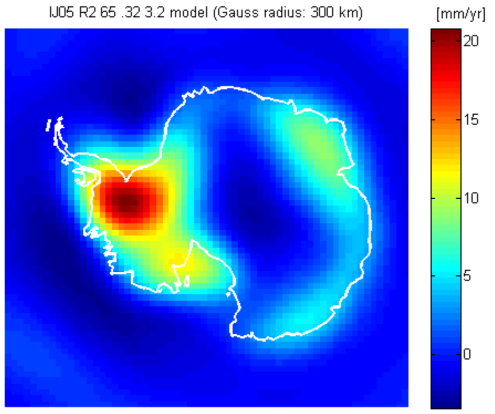


Figure 8. Linear trends calculated from IJ05 Revision 2 version 3 model with Gaussian filter of 300 km applied.

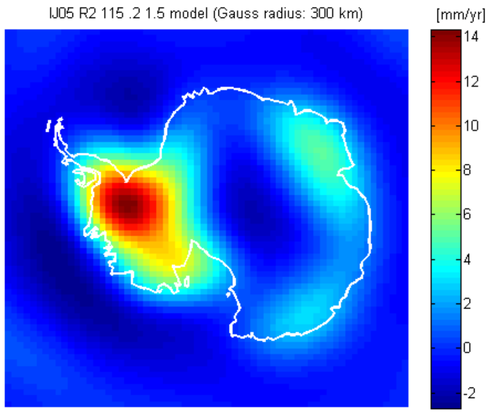


Figure 9. Linear trends calculated from IJ05 Revision 2 version 4 model with Gaussian filter of 300 km applied.

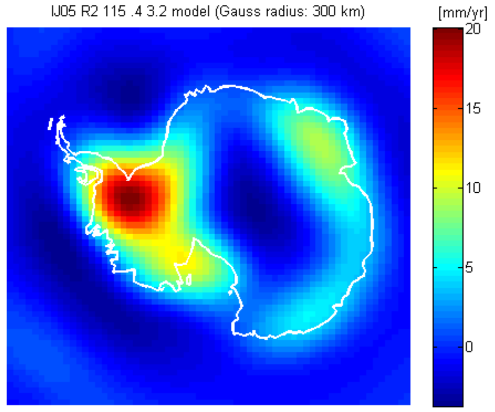


Figure 10. Linear trends calculated from IJ05 Revision 2 version 5 model with Gaussian filter of 300 km applied.

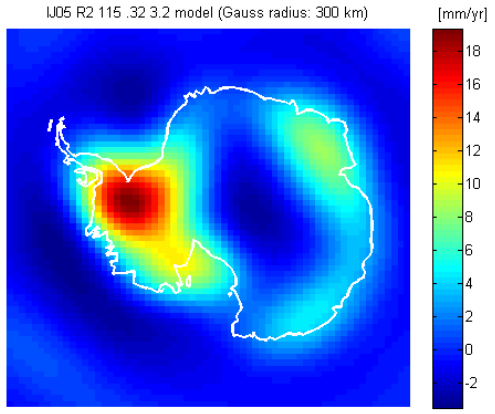


Figure 11. Linear trends calculated from IJ05 Revision 2 version 6 model with Gaussian filter of 300 km applied.

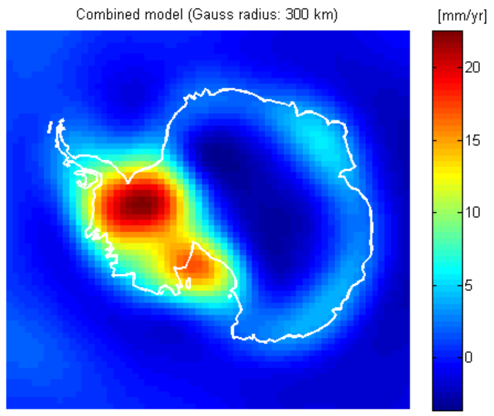


Figure 12. Linear trends calculated from combined model of [15] with Gaussian filter of 300 km applied.

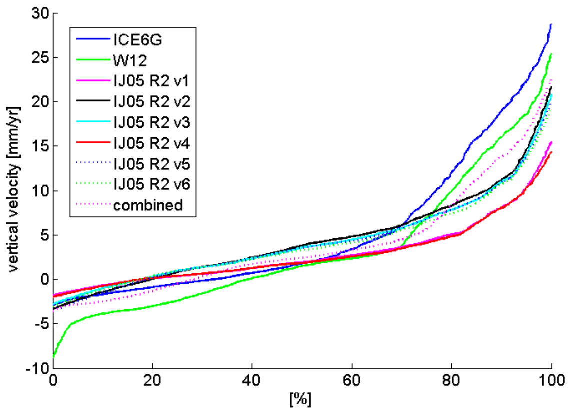


Figure 13. Velocity values derived from different GIA models for the area of Antarctica arranged in increasing order.

TABLE II.
AREAL MEAN AND RMS OF GIA MODELS

| | GIA model | mean [mm/yr] | RMS [mm/yr] |
|---|---|---|---|
| 1 | ICE6G | 5.18 | ± 7.89 |
| 2 | W12 | 3.32 | ± 7.47 |
| 3 | IJ05 R2_65_.2_1.5 | 2.84 | ± 3.52 |
| 4 | IJ05 R2_65_. 4_3.2 | 4.44 | ± 5.07 |
| 5 | IJ05 R2_65_.32_3.2 | 4.31 | ± 4.78 |
| 6 | IJ05 R2_115_.2_1.5 | 2.74 | ± 3.42 |
| 7 | IJ05 R2_115_. 4_3.2 | 4.16 | ± 4.83 |
| 8 | IJ05 R2_115_.32_3.2 | 4.06 | ± 4.59 |
| 9 | combined GIA | 3.88 | ± 5.97 |

Further comparison on the range of GIA velocity values of these models is provided by Fig. 13. This figure has been derived by arranging all GIA velocities over the continent into a vector, then sorting them in increasing order. The figure shows in percentage that how much of area uplifts with a certain (or less) velocity. According to the figure, ICE6G model shows generally the largest extent of vertical uplift, while IJ05 R2 models are the least dynamic showing the smallest values and slightest variability.

The statistical description of the content of these models is provided in Table II. The areal mean of the data represents the typical GIA trends of the continent according to the models. The RMS column of Table II refers to the quadratic mean of the differences with respect to the areal mean. It describes the variability of the GIA trends over the Antarctic continent. According to Table II, no consensus on the *mm/yr* level can be detected, neither in the average velocity nor in its spatial variability.

The comparison of the models to each other is summarized in Table III. The difference in every grid point was calculated, then the typical difference was estimated by areal mean. The number of the models, i.e. the first row and the first column, refers to the numbers defined in the first columns of Table II. The similarity of the IJ05 R2 models to each other, i.e. no. 3 to 8 models, is obvious: they difference is less than *2 mm/yr*. The combined model, no. 9, includes one of the IJ05 R2 models and the W12a model as well, thus it differs slightly from both; the difference is in the *2-3 mm/yr* interval. The ICE6G model is not included in the combined model, but its processor, ICE5G was involved. Thus the difference of the combined and ICE6G models is also not so much, only *3.44 mm/yr*. Finally, comparison of GIA models with independent origins, i.e. ICE6G, IJ05 R2 and W12a with each other are discussed. These differences slightly exceed the *4-5 mm/yr* interval. This is the essential property, since these models can be assumed to be uncorrelated.

## IV. CONCLUSIONS

State-of-the-art GIA models have been compared with each other visually and with simple statistics. As it turned out, the differences of independent GIA models are in the range of *4-5 mm/yr*, which thus can be considered as an empirical error estimate of GIA process modeling. This is a huge difference, since GRACE has detected the areal mean of Antarctic ice mass variations to be ±*2-3 mm/yr* [11], thus these estimates may completely been dominated by the choice of the used GIA model. According to [16], all modelling errors of Antarctic ice mass balance may reach the ±*11 mm/yr* level, in which the uncertainties of the GIA model takes a notable share. Therefore, GIA models must be treated with reservation to make use of the accuracy of the GRACE models. But whatever result is provided by a GRACE-borne Antartic ice mass balance study, it should be considered to be relevantly influenced by the choice of the GIA model.

TABLE III.
STATISTICS OF GIA MODELS

| No. | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 1 | 3.82 | 5.13 | 4.63 | 4.62 | 5.18 | 4.67 | 4.66 | 3.44 |
| 2 | | 5.04 | 4.78 | 4.74 | 5.04 | 4.77 | 4.73 | 2.30 |
| 3 | | | 1.76 | 1.38 | 0.28 | 1.52 | 1.21 | 2.97 |
| 4 | | | | 0.40 | 1.90 | 0.39 | 0.67 | 2.50 |
| 5 | | | | | 1.53 | 0.32 | 0.35 | 2.45 |
| 6 | | | | | | 1.62 | 1.31 | 3.01 |
| 7 | | | | | | | 0.35 | 2.49 |
| 8 | | | | | | | | 2.46 |

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