Semi-Analytical Approach for Adjusting GOCE SGG Observations

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***Abstract*—The Semi-Analytical approach has been implemented for adjusting GOCE SGG observations. A challenge of this is due to the band limited nature of the observed gravity gradients. The observed gravity gradients have been filtered by an IIR filter. The Semi-Analytical approach has subsequently been applied for analyzing the consequences of the filtering of GOCE gravity gradients in the spherical harmonic (or Legendre) domain. The results show that the relevant signal in the gravity gradients inside the MBW is equivalent to be above degree of 40.**

I. Introduction

Most recent gravity satellite missions starting with the XXI century have revolutionized space gravimetry, providing a unique knowledge of the long-wavelength and middle-wavelength part of the gravity field of the Earth [1]. The CHAMP satellite was capable to improve the "best-before-CHAMP" global gravity field model, the GRIM5-S1 model with one order magnitude up to degree and order 35 based on 6 months of measurements. Less than 2 years later for the GRACE satellites it took only 2 months to derive sufficient amount of observation to further improve the global gravity field model. The spatial resolution of the best GRACE models is about 250 km, according to its maximal degree and order of 150-180. The third dedicated gravity satellite mission was the GOCE, which has provided geoid accuracy of 1-2 cm with the resolution of about 160 km (maximal degree and order of 250) [2].

Even though the GOCE mission has already been ended (mission duration: 17 March 2009 to 11 November 2013) and excellent results have been delivered by the GOCE High Processing Facility (HPF), [3], there are still room for post-processing strategies to get different (desirably better) results from the same measurements.

There are three kinds of gravity field models delivered by the HPF following three different approaches [2].

The direct (DIR) approach obtains the gravity field model parameters by directly solving the inverse problem with the classical least squares method.

$$x = \left(A^T PA\right)^{-1}\left(A^T Pl\right) \quad (1)$$

In the equation P is the weight matrix, l is the observation vector, and x vector contains the unknowns, the spherical harmonic coefficients, $\overline{C}_{lm}$ and $\overline{S}_{lm}$. The partial derivatives with respect of the unknowns are derived analytically and used to construct the design matrix, A. The normal matrix,

$$N = A^T PA \quad (2)$$

is then determined and inverted by Cholesky decomposition, which is the crucial step of the approach. The DIR approach is efficient, though demanding in terms of CPU speed and computation time.

The Space-Wise (SPW) approach solves the observation equation with least squares collocation (LSC). The weakness of the LSC method is the large system of equations, which is avoided by an iterative, multi-step collocation procedure [4]. This approach involves (in an iterative sequence) a Wiener filtering along the orbit, a gridding of observation on the surface of a sphere at the mean satellite altitude, and a spherical harmonic analysis by numerical integration. This complicated processing sequence also means that no adequate variance/covariance information can be determined.

The Time-Wise (TIM) approach considers the observations simply as a time series along the orbit, and solves for the gravity field coefficients without involving a priori gravity field model into the solution. To replace the role of the a priori gravity field model, at the first stage a rough estimate is delivered by the Quick-Look Gravity Field Analysis (QL-GFA) [5]. The exact solution is then obtained by solving the full normal matrix with the preconditioned conjugate gradients method (PCGMA) [6].

Any of these solutions require notable CPU capacity. In fact, these methods are not developed for regular PCs. However, certain steps are can be performed under such conditions. This paper presents the first results of an own method for QL-GFA. The method is based on Semi-Analytical approach [7][8], similarly to the QL-GFA of the HPF [5]. For the filtering of the gravity gradients to the Measurement Bandwidth (MBW), an own filter has been developed [9]. The present investigation deals with GOCE Satellite Gravity Gradiometry (SGG) only, it does not deliver a full solution, this cannot be achieved without the inclusion of GOCE Satellite-to-Satellite Tracking (SST) solution as well in order to recover the long-wavelength gravity signal. Nevertheless, as the Semi-Analytical approach is applicable with the available computational facility, it has been chosen for adjustment in the present investigation.

Basically, a notable difficulty of processing efficiently the GOCE gravity gradients, that they are band limited. It implies a need of filtering along the orbit in the time domain. The subsequent adjustment should also be elaborated for band limited data. In the present study a band-limited solution of the Semi-Analytical approach has been implemented and employed for adjusting GOCE SGG measurements.

Basic investigations have subsequently been performed to analyze the signal content of the band limited observations. Basically, it is investigated that how the spectral filtering is projected to the spherical harmonic domain. The task is to determine which spherical harmonics are affected by the spectral filtering, how the signal content of the observations is changed by degrees and orders of the spherical harmonics. By defining the signal content of the observed gravity gradients as a function of degree and order, degrees for need of additional gravity data or need of regularization in the processing sequence can be identified.

II. GOCE Gravity Gradients

The GOCE satellite [10] revolves on a nearly circular orbit with an inclination of 96.7 degree. The satellite is equipped with a Space Gravity Gradiometer in order to measure gravity gradients and also to determine the non-gravitational forces affecting the satellite [11]. Furthermore, it is equipped with GPS receivers in order to continuously detect the orbit and the orientation of the satellite. As the arm length of the gradiometer is only 50 cm, it is insensitive of the long-wavelength components of the gravity field. This later, however, can be determined from the GPS measurements.

Level 2 GOCE data for the period of 1 November 2009 to 30 April 2010 have been used in the present study. This is equivalent nearly to three full cycles (i.e. it takes 61 days for the GOCE satellite to observe the whole globe). For the SGG data the EGG_NOM_2 observations were used. These gradients are presented in gradiometer-fixed coordinate system (GRF). For the orbit the reduced-dynamic orbits from the SST_PSO_2 data sets have been used. Normally, use of kinematic orbit is preferred, since reduced-dynamic orbits are derived by numerical integration in a force model, which is "refreshed" by actual orbit information at stochastic pulses [12]. It means, reduced dynamic orbits are smooth, but involves a priori gravity field information into the solution. In the present case only SGG is investigated. As so, reduced-dynamic orbits are better, since the role of the orbit data is limited to positioning of the gradient data, which is quite smooth in space.

III. Methodology

After pre-processing, the observed data along the orbit is used for gravity inversion, determining spherical harmonic coefficients of the geopotential. This is done by the Semi-Analytical approach in the present study [7]. The Semi-Analytical approach yields a mathematically simplified tool only if the observations on the sphere are gridded, since these data can be converted into spectral domain by a 2D-FFT transformation.

The Semi-Analytical approach assumes the observation data to be available on the surface of a sphere in a grid. It means that the measurements first should be projected onto a mean sphere of the satellite altitude. This is done by expanding the observables into a Taylor series at the orbit [13]:

$$V_{ij}^{\,sphere}\left(\varphi,\lambda,r^{sphere}\right)=V_{ij}^{\,obs}\left(\varphi,\lambda,r\right)+ \\ +\frac{\partial V_{ij}^{\,obs}\left(\varphi,\lambda,r\right)}{\partial r}dr+ \\ +\frac{1}{2}\frac{\partial^2 V_{ij}^{\,obs}\left(\varphi,\lambda,r\right)}{\partial r^2}dr^2+\ldots \tag{3}$$

where V_{ij} is the gravity gradient, the subscript i and j refers to all possible combinations of Cartesian coordinates in any reference frame, $i\in[x,y,z]$ and $j\in[x,y,z]$, and φ, λ and r are spherical coordinates of the point of the SGG measurement. The superscript "*obs*" refers to the actual observation at (φ,λ,r), while superscript "*sphere*" refers to its counterpart on the surface of the mean sphere. Since the deviation of the GOCE orbit from a circular one is relatively small (some 10 kms), we can restrict the Taylor series to the first derivative only, which is technically equivalent to a linear extrapolation. The required third derivatives of the potential are derived by [14].

The next step of the processing is gridding the data into a regular grid by φ and λ. For the purpose 2D-spline interpolation is used based on tests of [15]. The projected and gridded gradients are already in an appropriate domain for a 2D-FFT, in which domain the adjustment is taking place. The conversion results in Fourier coefficients A_{mk} and B_{mk} [16]:

$$V_{ij}(u,\Lambda)=\sum_{m=0}^{L}\sum_{k=-L}^{L}A_{mk}^{ij}\cos(ku+m\Lambda)+ \\ +B_{mk}^{ij}\sin(ku+m\Lambda) \tag{4}$$

where u and Λ are torus coordinates, meaning to two perpendicular full angle coordinates; L is the maximal degree of the series expansion. The Fourier coefficients A_{mk}^{ij} and B_{mk}^{ij} are the so-called lumped-coefficients, which are linear combinations of the spherical harmonic coefficients, $\bar{C}_{lm}$ and $\bar{S}_{lm}$ [7].

$$\begin{bmatrix}A_{mk}^{ij}\\B_{mk}^{ij}\end{bmatrix}=\sum_{l=\max(|m|,|k|)}^{L}H_{lmk}^{ij}\begin{bmatrix}\bar{C}_{lm}\\\bar{S}_{lm}\end{bmatrix} \tag{5}$$

In (5) H_{lmk}^{ij} are the transfer coefficients, which are (according to [7])

$$H_{lmk}^{xx}=UD\left[-\left(k^2+l+1\right)\right]F_{lmk}(I) \tag{6a}$$

$$H_{lmk}^{yy}=UD\left[k^2-(l+1)^2\right]F_{lmk}(I) \tag{6b}$$

$$H_{lmk}^{zz}=UD\left[(l+1)(l+2)\right]F_{lmk}(I) \tag{6c}$$

$$H_{lmk}^{xy}=UD\left[ik\right]F_{lmk}^{*}(I) \tag{6d}$$

$$H_{lmk}^{xz}=UD\left[ik(l+2)\right]F_{lmk}(I) \tag{6e}$$

$$H_{lmk}^{yz}=UD\left[-(l+2)\right]F_{lmk}^{*}(I) \tag{6f}$$

where i is the imaginary unit, UD is the upward continuation term, i.e.

$$UD = \frac{GM}{R^3}\left(\frac{R}{r}\right)^{l+3} \quad (7)$$

In (7) G is the gravitational constant, M is the Earth's mass and R is the Earth's equatorial radius. Further variables in equation (6) are : F_{lmk} refers to the inclination function, I is the inclination, F^*_{lmk} is the cross-track inclination function, i.e.

$$F^*_{lmk}(I) = -\frac{\partial F_{lmk}(I)}{\partial \theta'} \quad (8)$$

with θ' being the co-latitude.

As the Semi-Analytical approach solves the gravity field model based on data projected onto a sphere, its results are approximate. However, by "recycling" the approximate solution into the adjustment, better solutions can be achieved in an iterative sense.

The Semi-Analytical approach has first been successfully applied on simulated data, e.g. [17]. Such simulations are assuming with white noise on the gravity gradient signal. However, the GOCE gravity gradiometry measurements are contaminated by colored noise [18]. In fact, the measurements are optimal in a limited MBW, in the 5 to 100 mHz frequencies [19]. Thus the measurements should be filtered to the MBW, coloring the signal and noise characteristics of the observations. Consequently, the Semi-Analytical approach should also be adopted for band limited gradients. Such a method has been developed by [5] for the GOCE HPF QL-GFA. While [5] derives solutions for SST-only, SGG-only and SGG+SST cases, in this study only the SGG-only case is investigated. In [5] the ARMA filter of [20] has been used, in this study an IIR filter has been applied [9].

In gravity field analysis from satellite missions, the normal matrix, i.e. equation (2) is often ill-posed. For the Semi-Analytical approach it is always crucial whether the iterative solutions converge to an optimal solution, or diverge. It depends only on the actual noise content of the observations, on lack of data at the polar region (referred to as polar gap), and on the unavoidable need of downward continuation. The ill-posed problem can be solved be regularizing the normal matrix [16]:

$$N = A^T PA + \alpha^2 R \quad (9)$$

In (9) matrix R contains the inverse *a priori* variance of the measurements, and α is an arbitrary parameter, which controls the weight of the regularization term in the normal matrix. For the present method a set of α parameters has been tested, and an optimal value for the each gradient and each run has been selected. These values have been chosen by comparing the signal content to that of EGM08 coefficients [21].

IV. Signal content of SGG observations

As SGG-only adjustment has been performed, overall comparison with other GOCE gravity field solutions makes no sense. In fact, SGG-only solution cannot deliver adequate model due to its lack of long-wavelength information.

Within the frame of this study the SGG-only gravity field solutions were used for characterization of the SGG observations. The observed data, representing time series of gravity gradient data along the orbit is known to be band limited. The adjusted data presents information on the signal content in the spherical harmonic (or Legendre) domain. An order of magnitude comparison of the adjusted data with a reference gravity model describes the actual information content of the observations in the spatial (spherical harmonic, or Legendre) domain. Furthermore, comparison of the adjusted signal may be compared to a solution, where no filtering is performed. The latter solution is basically characterized by the noise. The comparison may conclude on the effect of the filtering. Further details of the test are presented in [22] and [23].

All in all, the test was defined by performing two independent runs of the Semi-Analytical adjustment:

- Containing the whole signal
- MBW-filtered signal

These two are compared afterwards. A usual technique for comparison of sets of spherical harmonic coefficients is the degree variance. By definition, degree variance describes the signal content of the coefficients per degree as

$$\sigma_l = \sqrt{\sum_m \left(\bar{C}^2_{lm} + \bar{S}^2_{lm}\right)} \quad (10)$$

Estimate of the signal content of the spherical harmonic coefficients is presented both using degree variances and degree by degree comparison. The degree variances are presented on Fig. 1 to 6 for the six independent gravity gradients. All the figures are scaled logarithmically. In these figures the blue line is Kaula's Rule of Thumb [24], which is a rough estimate of the signal content of gravity per degree. The black curve shows the gradients synthesized from the EGM08 model [21] to serve as a reference curve. The red line shows the non-filtered solution, while the green is the MBW filtered one.

There are two obvious features seen on these figures: MBW-filtering has mainly affected the long-wavelength signal up to degree of 40, showing a sharp rise at this degree, except for the V_{xz} gradient, which shows a gradual increase of the power from this degree and gains full power around degree 70. The other feature is that both solutions fail to keep the information content of the signal on the short-wavelength. This is probably a consequence of the Semi-Analytical approach, an unavoidable effect due to the vertical projection by equation (3) and the subsequent interpolations on the sphere.

Results of the degree by degree comparison are not shown, only for the V_{zz} component in Fig. 7 (all the other components can be found in [22]). By the degree by degree analysis it is obvious that no signal on the longest wavelength of the MBW-filtered data was found, as it is the expected consequence of the filtering. Another general feature of the MBW-filtered coefficients is that they show more signal on the middle- and short-wavelength than the non-filtered coefficients. This shows the efficiency of the filtering, since huge noise content of the long-wavelength

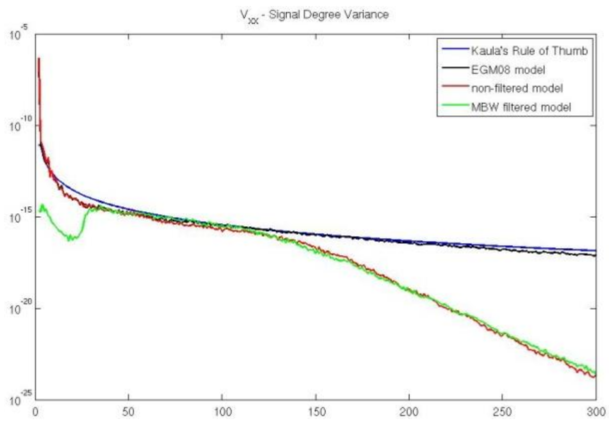

Figure 1. Degree variance of gravity models based on non-filtered and MBW-filtered GOCE data for the V_{xx} gradient.

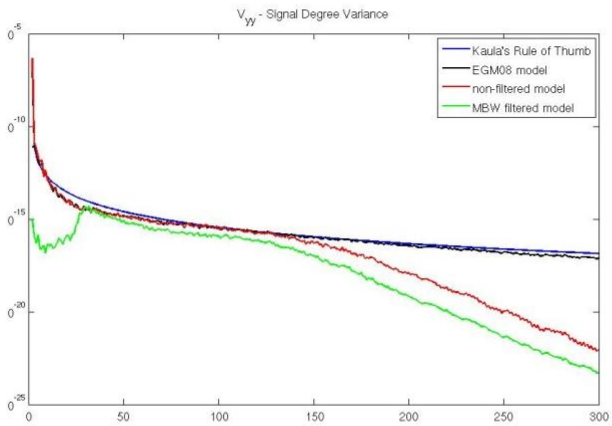

Figure 2. Degree variance of gravity models based on non-filtered and MBW-filtered GOCE data for the V_{yy} gradient.

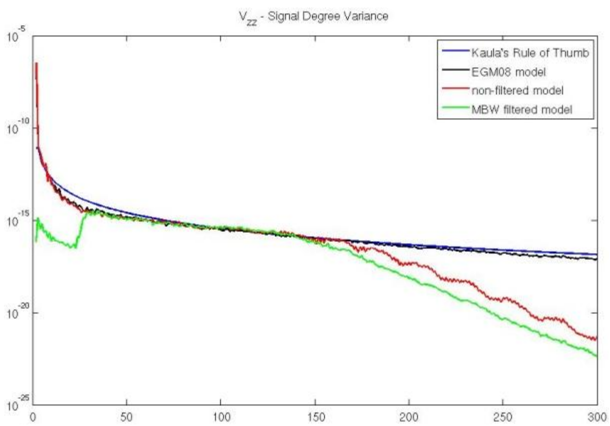

Figure 3. Degree variance of gravity models based on non-filtered and MBW-filtered GOCE data for the V_{zz} gradient.

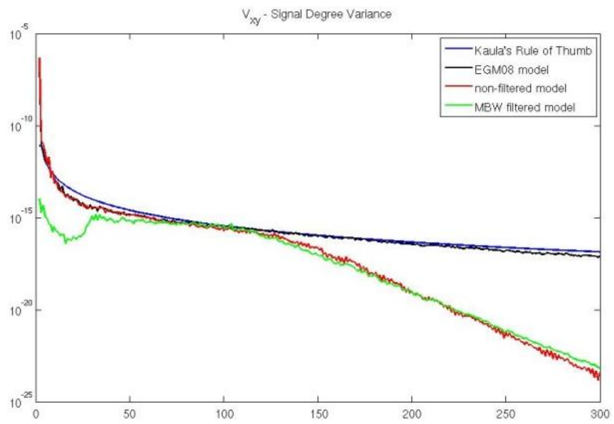

Figure 4. Degree variance of gravity models based on non-filtered and MBW-filtered GOCE data for the V_{xy} gradient.

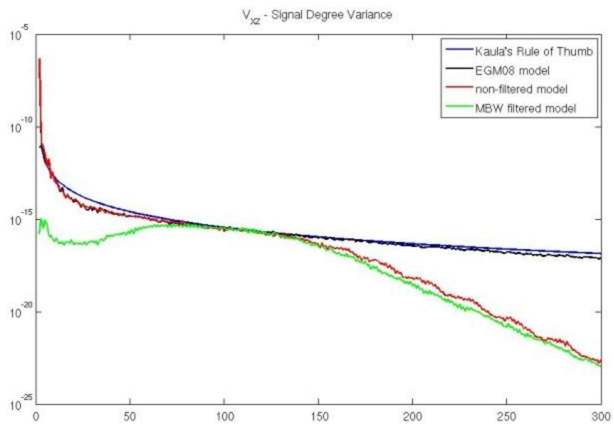

Figure 5. Degree variance of gravity models based on non-filtered and MBW-filtered GOCE data for the V_{xz} gradient.

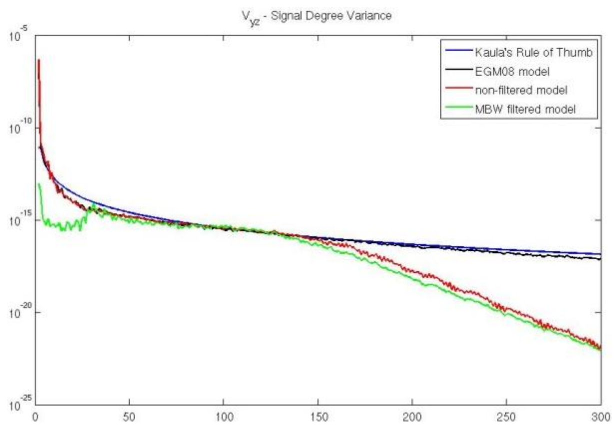

Figure 6. Degree variance of gravity models based on non-filtered and MBW-filtered GOCE data for the V_{yz} gradient.

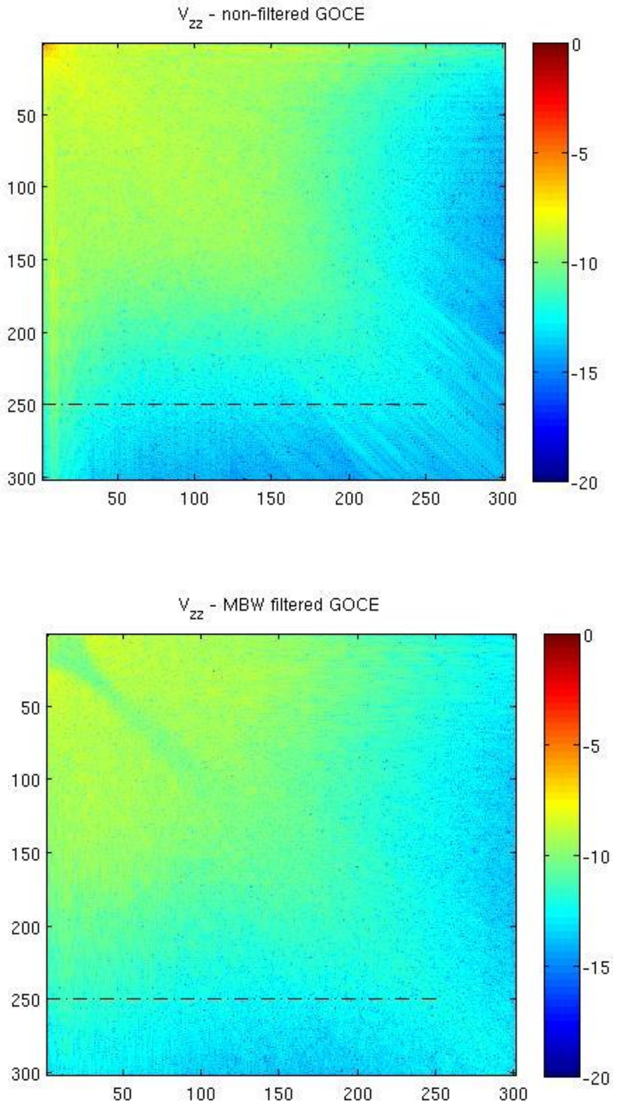

Figure 7. V_{zz} coefficients from non-filtered observations (up), from MBW-observations (bottom). Their corresponding degree variance can be found in Fig. 3. The dashed-dot line shows the section used for Fig 8.

signal in the non-filtered case leaks into the MBW suppressing the useful signal, which is enhanced efficiently by the filtering.

In Fig. 1 to 6 in some cases the non-filtered coefficients (red curve) overcome the filtered coefficients (green curve), which is due to the averaging nature of the degree variance; at any order an extreme value can push apparently the signal content estimate above. Exhaustively looking into the signal content of the V_{zz} gradient degree by degree in Fig. 7, the non-filtered signal shows large near-zonal values, while no particular emphasis on these coefficients is detected in the case of the MBW-filtered coefficients. This can be illustrated with displaying the order dependence of the V_{zz} gradient at a high degree, such as 250 degree (see dashed-dot line sections on Fig. 7). Fig. 8 shows the coefficients for both cases being juxtaposed to each other. It is obvious that all power of the non-filtered coefficients (blue line) comes from the low orders, but in

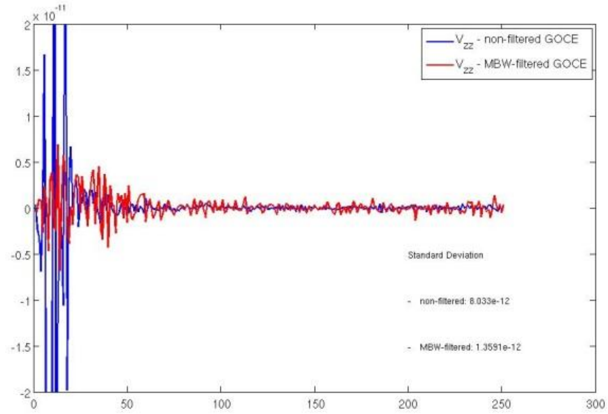

Figure 8. Cosine coefficients of degree 250 determined for V_{zz} using non-filtered observations (blue), and MBW-filtered observations (red).

general at the most part of the spectrum the MBW-filtered (red line) signal dominates. Their signal content has been estimated by their STD, which is in contrast to the general dominance of the MBW-filtered coefficients. The STD has been found to be $8.0*10^{-12}$ and $1.36*10^{-12}$ for the non-filtered and the MBW-filtered coefficients, respectively.

V. Summary

The main achievement of this paper is the implementation of the Semi-Analytical approach for adjusting GOCE SGG observations. The present method has been made use of the IIR filter developed by [9]. As the present study has dealt with SGG-only solution, and the long-wavelength gravity field has not been recovered from SST, the results cannot fully be validated. Instead, analysis of the signal content in the Legendre domain has been performed.

As it was demonstrated, the filtering of the observations along the orbit is mapped *systematically* to the spherical harmonic solution of the gravity field, and demolishes long-wavelength information up to degree 40 (with the exception of the V_{xz} gradient, which is gaining the full power around degree 70). At the other end of the MBW, no actual degree of signal loss could be detected. The signal is gradually vanishing until the high degrees. This is the consequence of the Semi-Analytical approach. Basically, the signal content may significantly be influenced by the noise content of the long-wavelength observations, c.f. unlike demolishing of the non-filtered middle- and short-wavelength signal in Fig. 8. The increased noise content is mainly involved in the near-zonals. According to that, for SGG-only solutions regularization of the (near-)zonals are suggested, and also for the short-wavelength coefficients, starting from degree and order of approximately 150. This result is in accordance with the parameterization of the TIM models, where (near-)zonals and all coefficients above degree and order of 181 are regularized to the Kaula's curve [2].

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