Relative Orientation in Photogrammetry

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***Abstract*— In photogrammetry to form a 3D model we need at least a stereo pair of images taken from two different stations. By this way we get two photos having their own perspective centres. In order to observe the stereo model we have to calculate the position of images in space in the moment of exposure. There are several solutions for this problem, although we use most frequently the relative orientation approach. By this task we try to calculate the position of images comparing to each other in a common spatial coordinate system. In other words we can form the task as a geometric problem: we are looking for the position of two planes (images) in the space and we have to describe the relation of these two planes with the minimally necessary parameters. Present paper summarizes the most common computational method emphasizing the problems and the limitations for applying it for photogrammetric tasks.**

***Keywords*— Relative orientation, Stereo pair, Least square method**

I. Introduction

A stereo pair is oriented relatively if the projection rays of corresponding points intersect each other in the model space. In other words the pair of rays should be on the same plane otherwise we experience a vertical parallax when observing the stereo model for example with anaglyph glasses. This condition is called coplanarity (Figure 1). If we want to describe the coplanarity condition we need a spatial coordinate system bound to our stereo model. In this coordinate system we have to describe the position of image planes. Present paper summarizes the different approaches for relative orientation. The common point in all methods that we need five parameters to determine the orientation of images.

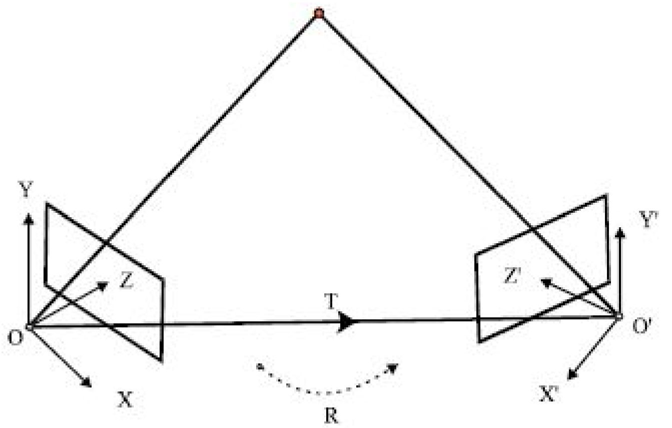

Fig. 1 Coplanarity condition

II. Coordinate systems and parameters

The relative orientation actually is not a transformation problem, which means our goal is not to assure a pathway between different coordinate systems. Instead, we want to define the relative position of the two images in the model coordinate system to serve as a spatial reference system. All subsequent processing steps are performed in the model coordinate system, as the projection rays of common points are defined in this system as illustrated in Figure 2 [1].

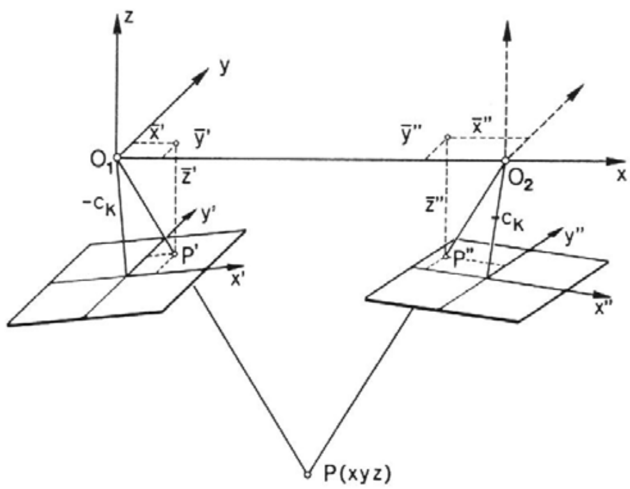

Fig. 2 Coordinate system for relative orientation

A. Different approaches

The question may arise whether is it necessary to rely on this coordinate system, to be more precise, is it necessary to know the model coordinates of a field point? In addition, we need to form the spatial coordinates $\bar{x}', \bar{y}', \bar{z}', \bar{x}'', \bar{y}'', \bar{z}''$ of image points P', P'' located in the image plane but considered as model points. It would seem much preferable, if the image coordinates x', y', x'', z'' could be used immediately to get the terrain coordinates. Indeed collinear equations create a direct link between the image and the terrain point.

The reason why the relative orientation is needed is the stereoscopic viewing and pointing, as a measurement method. Earlier in the era of analogue instruments the stereo-image pairs were considered as real planes in the space and they were moved and rotated around three coordinate axis inside the evaluation instrument (Figure 3).

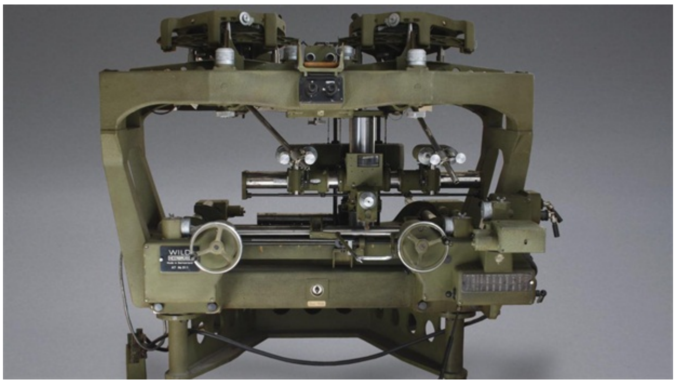

Fig. 3 Analogue photogrammetric evaluation instrument

The relative orientation is actually an attempt to restore the situation in which the images were situated relatively to one another at the moment of image recording. If we skip this step, the stereoscopic image observation would not be possible because of the conjunctive projection rays would not be in the same plane, which means they would not intersect each other.

Later there was a need for the stereoscopic observation even for the analytical stereo comparators and digital photogrammetric workstation; and therefore, the relative orientation is still part of the orientation process (Figure 4).

Fig. 4 Digital Photogrammetric Workstation (DPW)

However, the collinear equations mathematically and accurately describe the strict connection between the corresponding image and field points, so we prefer to calculate the field coordinates of points with this method. As a consequence, the model coordinates are no longer needed, as the terrain coordinates are directly calculated from image coordinates. In practical terms, the relative orientation should be carried out to perform the stereoscopic viewing. Of course, there are several digital evaluation programs which offer the conventional orientation routes, after which the field coordinates are calculated with the spatial similarity transformation based on model coordinates, but it is actually not necessary and it is getting increasingly neglected.

Basically we distinguish two types of relative orientation, depending on the set of parameters. In the first variant just the right image is moved and rotated, in the second variant both images are rotated. In the digital photogrammetric evaluation both modes may be selected since the images are not rotated and shifted relative to each other in reality, the orientation elements are calculated only. The resulting orientation elements can be used to calculate the theoretical y''^0 at a given px parallax. Next, the right image is moved in y-direction on the screen, while the measured image coordinate y'' within a certain margin of error does not correspond to the theoretical value of y''^0, so we enable a smooth stereoscopic observation. The theoretical vertical parallax is calculated by:

$$y''^0 = \frac{\begin{vmatrix} b_x & b_y & b_z \\ \bar{x}''^0 & \bar{y}''^0 & \bar{z}''^0 \\ \bar{x}' & \bar{y}' & \bar{z}' \end{vmatrix}}{\begin{vmatrix} b_x & b_y & b_z \\ \bar{x}' & \bar{y}' & \bar{z}' \\ r''_{12} & r''_{22} & r''_{32} \end{vmatrix}}$$

<u>Symbols:</u>

b_x, b_y, b_z : Base vector components, in other words, the vector between O_1O_2

$\bar{x}', \bar{y}', \bar{z}', \bar{x}'', \bar{y}'', \bar{z}''$: Model coordinates of image points P', P''

$r''_{12}, r''_{22}, r''_{32}$: Elements of the rotation matrix (direction cosines)

$\bar{x}''^0, \bar{y}''^0, \bar{z}''^0$: Theoretical model coordinates of P'' (we assume $y'' = 0$)

B. Solution and optimization

The initial conditions equation of the relative orientation is deduced from a simple but ingenious geometrical consideration. Intersecting rays of a model point are connected with four notable points in the model coordinate system. These points are O_1, O_2 as the perspective centres, and the image points of P', P'' corresponding to the model point P . All these points are determined in the model coordinate system. (Figure 2).

As a final result of the relative orientation the projection rays aligned to the aforementioned four points should be on the same plane, then the volume of the tetrahedron they form becomes zero. This coplanar condition can be formulated by a determinant [1], [2].

$$\Delta = \begin{vmatrix} \bar{x}_{O_1} & \bar{y}_{O_1} & \bar{z}_{o_1} & 1 \\ \bar{x}_{O_2} & \bar{y}_{O_2} & \bar{z}_{O_2} & 1 \\ \bar{x}' & \bar{y}' & \bar{z}' & 1 \\ \bar{x}'' & \bar{y}'' & \bar{z}'' & 1 \end{vmatrix}$$

We remark that the exact equation contains a multiplier of 1/6, but if we omitting this factor, it will not affect the result, since the volume should be zero in any way.

The image rotation angles $\omega', \varphi', \kappa', \varphi'', \kappa''$ as orientation elements are determining the orientation of planes and they form the rotation matrix elements for the images as direction cosines, which are built into the coplanar equation condition as trigonometric functions. So unfortunately, after the determinant is extracted, the resulting equation will not be a linear equation considering the orientation elements. The number of unknowns is 5, in order to solve the equalizations with the adjustment method of least squares there must be at least 6 stereoscopic points to be measured by the arrangement proposed by Gruber. Thus, according to Taylor series polynomial we solve the equations with a gradual approximation according to the method of least squares. We have only a few options to optimize the mathematical model. The only thing that could accelerate the iterative process is applying higher degree elements of the Taylor polynomial derivatives, although it is doubtful whether the greater computing need results more rapid iteration process.

The real change would be if we could describe an iteration-free process with the same number of unknowns. There are solutions by this approach mainly in the field of pattern recognition and robot vision based on artificial intelligence [3], [4]. In general, these solutions are derived with Gröbner-basis and there is no need to set up initial values for unknown parameters. This is, however, beyond the scope of the present paper.

C. Initial values

Since the conditional equations are generated by polynomial series (Taylor series), we need to know the initial values of unknown variables (orientation parameters). Normally it is not a problem to determine these initial values if the images have small rotation angles. In special cases, some terrestrial and aerial images can have large rotation angles, in which cases during the imaging we have to assure and record the good approximations of the orientation elements.

III. Conclusions

As a summary we can say that the relative orientation is a necessary process but it would be better to avoid, since later we use only the collinear equations without any model coordinates. We have to go through the relative orientation process only for calculation of y''^0 as the theoretical vertical parallax.

Certainly, there are cases when we don't need geodetic coordinates and to know the model coordinates is sufficient. These tasks can occur in the terrestrial photogrammetry when we need the digital terrain model covering only a small area where the knowledge of scale factor is enough and we don't need to place the model into the geodetic coordinate system. The exact scale factor can be calculated by the reference lengths observable on images.

Another interesting field of research can be if we register the model coordinates although we don't need them for mapping but we could use them for gross-error detection.

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