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Advanced Fuzzy Rule-based Failure Mode and Effects Analysis

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ABSTRACT

In my thesis, I propose advanced methods to improve Fuzzy rule-based Failure Mode and Effect Analyses (F-FMEA) models. In chapter 1, the introduction of the study with aims, methods, and structure are described. Chapter 2 presents the theoretical background of scientific approaches used in the research work. Chapter 3 presents the fuzzy rule-based hierarchical FMEA (H-FMEA) model, which extends the standard risk analysis method. I demonstrate that fuzzy-based FMEA is advantageous since it can provide an efficient solution by merging numerical and linguistic factors while accounting for subjectivity. Simultaneously, the hierarchical structure proves to improve the efficiency of the assessment and the model's flexibility. A case study is devoted to illustrating the proposed method making a preliminary risk analysis of the wheel speed sensor's reliability and safety. In Chapter 4, I propose a novel methodological approach to implementing H-FMEA as a further development of an existing hierarchical case study. In the novel method membership functions are different depending on the level. Furthermore, I show if information transmitted across the levels is a fuzzy number instead of an exact value, then reliability and safety can be further improved. In Chapter 5, I use the F-FMEA technique to describe, classify and assess risk factors of big (Installation problem, short fatigue life) and small (Slack-running fit, early failure) that may materialize in the bearing manufacturing process. In the F-FMEA model, I investigate various defuzzification approaches and propose Summative-defuzzification methods that combine different fuzzy subsystems outcomes.

1 INTRODUCTION

Nowadays, the qualified functions of the engineering system play a more significant role in our daily lives than ever before. During the design and development of engineering systems and their components, safety and dependability become increasingly important [1]. The main reason for this is that when engineering goods and designed systems develop, they grow more complex.

Total quality and reliability awareness has nearly become a requirement for industries to keep up with the constantly developing and changing global competitive environment. In this context, the spread of high-quality and continuous improvement philosophy has paved the way for new strategies, techniques, and applications.

In the innovative automotive industry, the focus of companies has been on the quality and reliability of their products as a strong management strategy. The quality and reliability cannot be linked solely to the manufacturing process but to various methods and measures from the time the product is created until it is in the hands of the customer. As a result, the prevention of function failures is essential to the end product's quality since the goal is to produce long-lasting and user-friendly products that satisfy customers' expectations. Thus, implementing quality evaluation methodologies is a strategic tool that leads to more profitable products [2].

According to the researchers, continuous efforts must be made to prevent the causes of failures. Furthermore, it is inferred that preparing each action yields more significant quality results [3]. Hence, this dissertation's primary purpose is to identify and evaluate the failures in some critical parts of automotive components (hardware and software). Using some recent studies, I have introduced first Failure Mode and Effect Analysis (FMEA) and mainly focused on Fuzzy rule-based FMEA (F-FMEA) as the risk assessment method.

FMEA has proven to be an excellent systematic approach for assessing failures in a system, product, or process. During the product development cycle, specialists in the respective domains often perform it [4]. This approach can be used to improve the quality of new or current systems, products, or processes. Furthermore, studies show that successful FMEA implementations can improve a company's ability to compete on a worldwide scale [5][6]. This method is sometimes perceived as simple, but there are some flaws in obtaining adequate measurements against evaluations. As a result, a large number of scholars have developed a new risk assessment

methodology based on fuzzy sets and rule-based inferences. Furthermore, scientists have stated that the F-FMEA technique is an excellent foundation for obtaining reliable results [7]. In contrast to the language concepts employed in FMEA, the vulnerability of theoretical relations is transformed into numerical systems in fuzzy set theory [8].

Fuzzy sets and inference systems have made significant advances in every modern scientific research field. It has a wide range of theoretical and practical research applications, ranging from life sciences to physical sciences, engineering to health and sciences, computer science to arts and humanities. In recent years, the fuzzy sets have expanded into new types, and these extensions have been employed in numerous domains such as economics, energy, medicine, materials, and pharmaceutical science [9]. In addition, the considerable volume of Fuzzy logic has been reflected in several fields like automobile speed control [10], water filter automation [11], operating systems of automatic trains [12], and management of robotic manipulators [13].

The fuzzy logic approach has advantages and disadvantages compared to classical methods. The most crucial advantage of fuzzy logic is that it is very close to the human brain's functioning when comparing binary logic. It, therefore, appears to be a reflection of human thought. Fuzzy logic is used most successfully in uncertain and nonlinear systems. However, a specific method cannot use whether the system is uncertain or nonlinear. Because the membership functions are system-specific based on experiences, this adaptation process is very demanding and time-consuming. This means that enough data must be collected by experts and an appropriate rule base established to function optimally in any system. As a result, collecting data and establishing a rule base takes time through trial and error.

Currently, Fuzzy sets theory is a massive approach for gaining computer reasoning systems. Over the last four centuries, mathematical models have demonstrated their importance for understanding natural processes.

1.1 Research Objective

In my dissertation, I aim to develop mathematical modeling of the fuzzy inference process for risk assessment of engineering components required for automobiles.

From the introduction of analysis and background, the aims are:

- Addressing the Mamdani-type Fuzzy Inference Process (MFIP) concept to minimize overall losses, identify risk context and acceptability.
- To work out novel risk assessment methods by modeling MFIP for Failure Mode and Effect Analysis (FMEA).

1.2 Research Methodology

The primary research approach for this study is a review of the literature and elaboration. Thus, I first reviewed various past publications in my thesis to understand better the Failure Mode and Effects Analysis (FMEA), fuzzy sets theory, and risk assessment behavior and properties. Many works have been investigated before about the combination of FMEA with fuzzy logic. However, I have focused on projects that contain modern fuzzy sets applications based on risk analyses. Therefore, I present below some introductory studies that contributed to the progress of my dissertation.

By Pokorádi [14], "Modern equipment and systems should meet technical, safety and environmental protection requirements.", the author studied the fuzzy rule-based risk assessment method to manage a specific helicopter mission. By expert's (pilot's) reports, the author has determined the severity and probability of possible air-crashes. The article also reports the importance of expanding the fuzzy rule-based theory of use, research, and methodology and its practical use in modern Hungarian military science.

Zolotukhin and Gudmestad show how experts' information and assessment can be appropriately used to quantify possibilities for an accident in a risk analysis. They have used fuzzy set theory, which is a tool that is mathematically stringent and well established to quantify possibilities of accidental scenarios relying on the expert's assessment. The risk of lifting an offshore module onto a live platform and the risk of an offshore tow operation are both assessed in their research using the fuzzy sets approach [15].

Dombi and Tóth-Laufer noted that the applicability of traditional Mamdani control is limited by high-level computational requirements in real-time and adaptive systems such as medical-related applications. They have suggested improvements to the conventional Mamdani model, such as the Mamdani-like formed by a discretized output and the arithmetic-based model. They introduced technical adjustments to the Mamdani type controller based on the features of triangular and trapezoidal membership functions, which resulted in a significant reduction in computational requirements compared to the original technique formulated for general shape membership functions [16].

Aliye and Nilsu have proposed a quantitative approach, the proportional risk analysis methodology, integrated with the fuzzy logic operation for occupational health and safety in a case study conducted at a textile firm that makes towels and bathrobes. They have used three parameters for appropriate membership function: probability, frequency, and severity, which are fuzzified. IF-THEN rules-based fuzzy operations are represented for inference to determine the riskiness. After this process, the risk score has found for each defined event using the defuzzification process [17].

Kwai-Sang Chin et al. have presented a fuzzy knowledge-based assessment system at the conceptual design stage of product development, emphasizing the design of high-quality products. They proposed a fuzzy FMEA assessment method for the new product concept. By integrating multiple areas, they investigated to automate the planning and evaluation, so-called expert product development system. The proposed framework's functions aim to assist inexperienced users in performing FMEA analysis as considering alternative development design concepts in the dimensions of material and component preference for product process planning, robust design, and estimation of product and tooling costs. In design and planning applications, their prototype has proved to be beneficial in fuzzy set theory and knowledge-based systems. Moreover, a world top micro-motor manufacturer has supported the researchers' development effort on a permanent magnet direct current micro motor optimizing project [18].

1.3 Thesis Structure

The dissertation is organized as follows:

In **Chapter 2**, the importance of traditional FMEA, the basic concepts of fuzzy sets, and the fuzzy inference process are presented separately.

In **Chapter 3**, the hybrid qualitative methodology of FMEA and Fuzzy logic is tabulated in a hierarchical structured FMEA model for safety analysis of Anti-lock Braking System (ABS) components.

In **Chapter 4**, the level-specific evaluation-based hierarchical FMEA model is proposed to improve the hierarchical structured fuzzy model.

In **Chapter 5**, proposes the summative defuzzification methods for the bearing manufacturing process FMEA.

In **Chapter 6**, the findings are summarized.

In **Chapter 7**, references are provided.

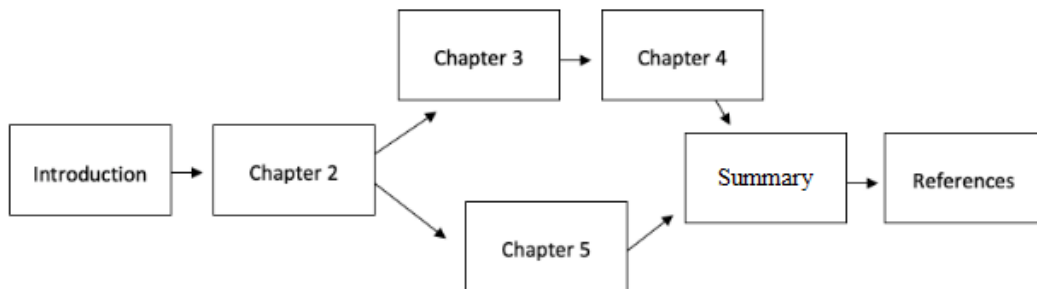


Figure 1.1 Flow chart of the dissertation structure

2 THEORETICAL BACKGROUND

Risk is the value determined according to the probability of damage that may occur in hazardous situations. The degree of risk is determined by the likelihood and severity of the hazard [19]. Risk refers to the uncertainty involved in the implemented activities. This unpredictability can have both positive and bad outcomes. The purpose of risk assessment is to control the effects of this uncertainty. The risk assessment will be more accessible when risk factors are analyzed beforehand. For this, it is necessary to identify and analyze the risk factors.

Risk assessment has been considered the essential topic in safety to make a reliable decision in modern automotive engineering. The increasing complexity of engineering systems will bring substantial uncertainties and imprecision associated with data in risk assessment problems [19].

The purpose of risk assessment is usually to determine the risk context and acceptability compared to similar risks. The foremost step in risk assessment is identifying and assessing the severity of the hazard. The review of severity should be based on the worst possible consequence that can reasonably be predicted when determining the severity of the danger in terms of its impact on the equipment or devices that are used. During the risk assessment, a fuzzy logic approach enables the probabilities of the investigated hazard and the calculated "sharp" severity to be found. Using fuzzy sets is best to deal with uncertainty for objective reliability quantitative risk assessment with a qualitative approach. This method is based on the experimental results and will minimize potential errors by giving the closest possible risk assessment with proper mathematical operations.

2.1 FUNCTIONAL SAFETY IN AUTOMOTIVE

Functional safety is a concept related to active systems and basics identifying hazards, risk assessment, and lowering the risk by using system engineering. The primary focus of functional safety is the systemic protection of electronic failures, including adding functionality to the system to properly manage safety. In particular, functional safety incorporates safety analyses implemented by the system, such as failure detection, physical or systemic redundancy, or transition to a safe state, which minimize the overall risk of malfunctions in the electronic system [20].

The V-shaped model for product development is widely used in the automotive industry. The product development cycle consists of specifying requirements, hardware design, software design, implementation, and validation testing. However, there is a safety-oriented process that runs parallel to the development of the product cycle. The software requirements integrate with functional safety analysis, concept development. The functional safety concept is designed to guarantee that the component operates safely under normal operating conditions and failure condition [21].

Since the 2000s, the International Electrotechnical Commission (IEC) has published the IEC 61508 series standards "Functional safety of electrical/electronic/programmable electronic safety-related systems". This standard has been adapted to suit the automotive applications with ISO 26262 and many other areas.

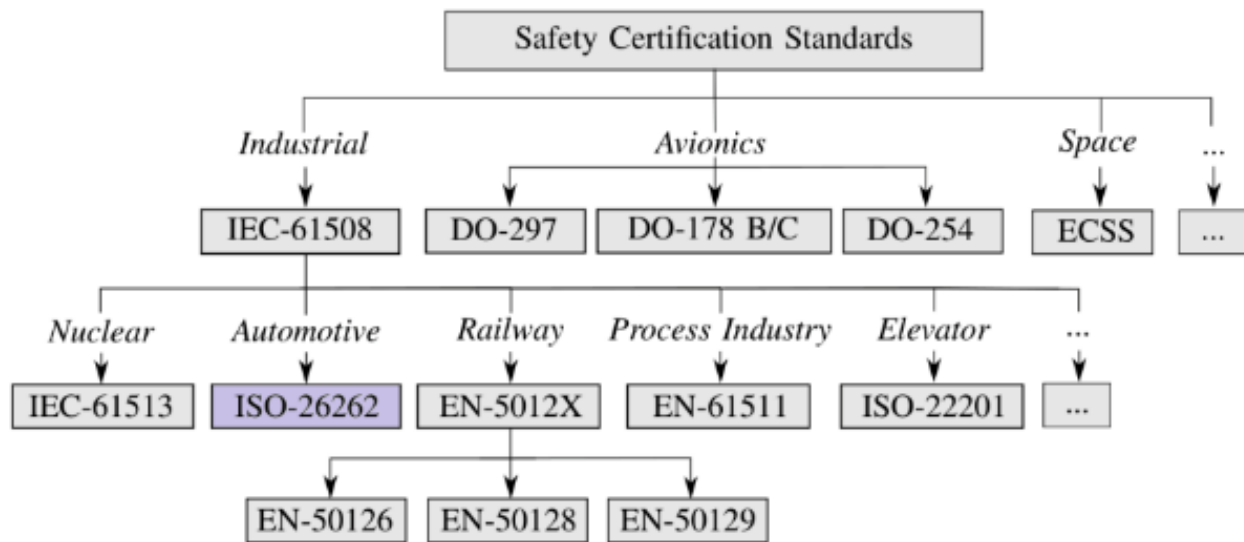


Figure 2.1 Sample of safety standards and functional safety standards derived from IEC 61508 (including ISO 26262) [22]

In 2011, the automotive industry was introduced to ISO 26262, represented by the International Organization for Standardization (ISO) to address the necessity of safety risk from the sensor to actuator by guiding requirements and processes failure [23]. “A system is a set of elements that relates at least a sensor, a controller and an actuator with one another” [24]. To end system failure, ISO-26262 defines procedures for managing deterministic design failures and unpredictable hardware failures. The ISO-26262 principle is that systematic failures can either be prevented by using restriction measures during the development process or controlled at runtime by safety mechanisms such as various redundancy.

The possible hazards are classified using the Automotive Safety Integrity Level (ASIL) system. The ASIL assists engineers in determining the criticality of specific components through the Hazard Analysis and Risk Assessment (HARA) procedure. To comply with ISO 26262, ASIL develops safety criteria based on the likelihood of acceptance of harm to automotive components. The system safety function is organized into four levels: A, B, C, and D. ASIL-A denotes the lowest level subsystem, such as rear lights. ASIL-D shows the highest level of automobile risks, such as anti-lock braking, airbags, and electronic power steering [25].

ISO 26262 standard uses system steps to functional safety management and product development regulation on the system. Hazard Analysis and Risk Assessment (HARA) determines ASIL and safety goals, after considering HARA and ASIL classification, then the hardware and software level requirements [26]. Based on the double V model, the ISO 26262 provides the structure with possible verification processes is shown in figure 2.2.

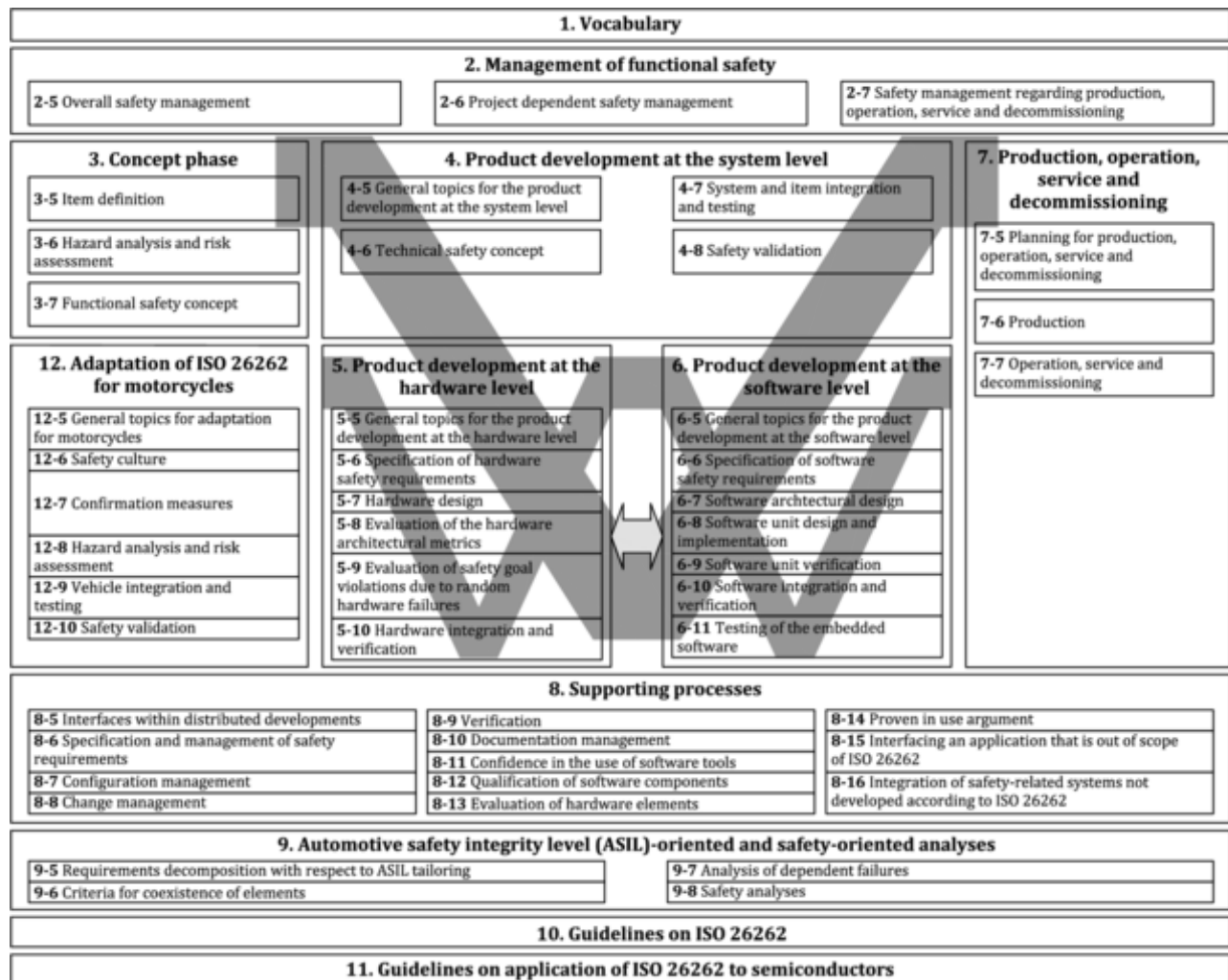


Figure 2.2 Automotive product development of double V-model [27]

The core process, the V model design and verification are integrated between parts 3 and 7.

An item must meet the subcomponents requirements for both hardware safety integrity and systematic safety integrity levels to reach a given ASIL. The appropriate ASIL is determined through HARA performed at the vehicle level. For example, the sensors are crucial as critical item components to contribute to the overall ASIL grade.

The HARA process's first step is done systematically using qualitative methods like brainstorming, Failure Mode and Effect Analysis (FMEA), field studies, etc. Hazards are determined by considering the different situations and behavior of the vehicle. All performance conditions and consequences are taken into account when determining hazardous situations. Suppose the identified hazardous situation is outside the scope of ISO 26262. In that case, it is considered essential to highlight the need to control this hazard and inform the person responsible for handling it [21].

The product development phase begins with the initialization of the product development at the system level, part 4 [28], including the hardware level and the software level. According to this phase, the system developers can design the ideal system and define the technical system requirements and technical safety requirements. Then, it will lead to the item integration and testing, safety validation, functional safety assessment, and system release for production is defined.

The overall system, including the technical safety concept, is designed to confirm the Functional Safety Requirement (FSR)s and Technical Safety Requirement (TSR)s specification. The specification of technical safety requirements aims to develop the technical safety requirements, refine the preliminary architectural design's functional safety concept and verify that the TSRs are appropriate to the FSRs. Moreover, the applicable item safety requirements and system-level safety technical requirements are reduced to allocating hardware and software items [29]. Specification of the TSRs points to safety mechanisms like measures to detect or prevent inside the system's failures or external device associated with the designed system, complete a safe state from a hazardous condition, and execute a warning and degradation concept [21]. During design, it is essential to have two-way traceability between System design and TSR specification.

After defining system structural design constraints, the next stage is to establish measures for avoiding systematic failures by analyzing the internal and external causes using deductive and inductive analysis methods. In contrast, the deductive approach tests theory, an inductive approach deals with generating new hypotheses from data. After this analysis, TSRs are dedicated to hardware, software, or both, and hardware and software interfaces are tested. Consequently, the system design and technical safety concept are validated for integrity and conformity with earlier safety conditions [21].

During the system design, different elements (subsystems) make up the general system. These elements must be suitably integrated for the system when developed and then tested for compatibility with the structural design and system interfaces between each combined item. This phase, which includes hardware and software assembly and testing, aims to take them separately and finally integrate them as a whole and complete the test. Each safety requirement must be tested for compliance with its specific ASIL level and must verify the system design. This step-by-step process followed by under ISO 26262, the hardware-software integration, specific tests will accurately prove its compatibility [23].

In the previous step, the system was integrated and designed to be consistent with safety requirements. The purpose of this step is to provide evidence that the safety goals are entirely achieved at the vehicle level and that safety concepts are proper for the item's functional safety [24]. The plan of validation includes:

- The configuration of the item.
- The test case's characteristics and acceptance criteria.
- The necessary environmental conditions.

Moreover, this step considers the probability of validating, which is evaluated in this research.

2.2 FAILURE MODE AND EFFECT ANALYSIS (FMEA) APPROACH

Failure Mode and Effect Analysis (FMEA) is a frequently used method in advanced manufacturing systems such as automotive and aerospace, producing safety-critical products and including advanced electronic and mechanical equipment based on system analysis. FMEA is a powerful technique for predicting and preventing failures, based on analyzing the effects on the end-user of the problems that may occur due to the emergence of errors from the user's perspective [30].

The United States military first developed the FMEA method on November 9th, 1949. Military procedure MIL-P-1629 entitled was evaluated to achieve a Failure Mode, Effect, and Criticality Analysis (FMECA). The method aimed to determine the impact of system and equipment failures as a reliability assessment. Failures were addressed based on personnel/equipment safety and their effects on military mission success. As a result, they have reduced the sources of variation in ammunition production and related potential failures [31].

The National Aeronautics and Space Administration (NASA) accepted the FMEA methodology as an essential project planning method in early 1960, after seeing that the military's usage of FMEAs reduced project risk. FMEAs have proven critical to the success of NASA's Apollo missions and have been used extensively by the civil aviation industry to evaluate aircraft safety [32].

In the mid-1970s, Ford Motor Company engineering met FMEA in the automotive industry and led this path with the Ford Pinto model as an internal response to safety and public relations issues [33]. After that, other automotive producers in the US and Europe soon followed Ford's lead. The French Renault and Citroen automotive companies, call this method AMDEC (Analyse des Modes de Défaillances, de leurs Effets et de leur Criticité), which stands for Failure Mode, Effect, and Criticality Analysis [34]. In 1993, the Automotive Industry Action Group (AIAG) and the American Society for Quality Control (ASQC) established the industry wide FMEA standard [35].

Today, the FMEA method is widely used in many industries, including automotive, semiconductor processing, plastics, healthcare, food services, software, aeronautics, etc. Many standards and guidelines have been published that cover the scope and general procedure for leading an FMEA [36]. The following are relevant more common:

- MIL-STD-1629A, Procedures for Performing a FMECA (In 1994, declared for cancelation, but in the military and other applications are still in use) [37].
- IEC 60812, "Analysis techniques for system reliability- Procedure for failure mode and effects analysis (FMEA) " [38],
- Society of Automotive Engineers (SAE) J1739, " Potential Failure Mode and Effects Analysis in Design (Design FMEA), Potential Failure Mode and Effects Analysis in Manufacturing and Assembly Processes (Process FMEA) [16] " [39],
- Alignment Automotive Industry Action Group (AIAG) and German Association of the Automotive Industry (VDA), " New global FMEA standard [40]"

The FMEA risk analysis technique, recommended by international standards, goes through a systematic process to identify failures, identify possible causes, and detect failure effects to perform its intended function [41].

In today's technology, engineering systems have a very complex structure; FMEA assesses potential failures arising from a system, design, process, or service and continuously reduces these failures (risk, problems, inaccuracies, etc.) [42].

The purposes of using the FMEA technique can be summarized as follows [41]:

- Identify and assess the potential failure modes and causes related to the design and manufacture of a product
- Recognize actions that could eliminate or reduce the case of the possible failure occurring
- procedure documentation.

2.2.1 Types of FMEAs

FMEA has four fundamental types (see Figure 2.4), are the following: System FMEA, Design FMEA, Process FMEA, and Service FMEA [41].

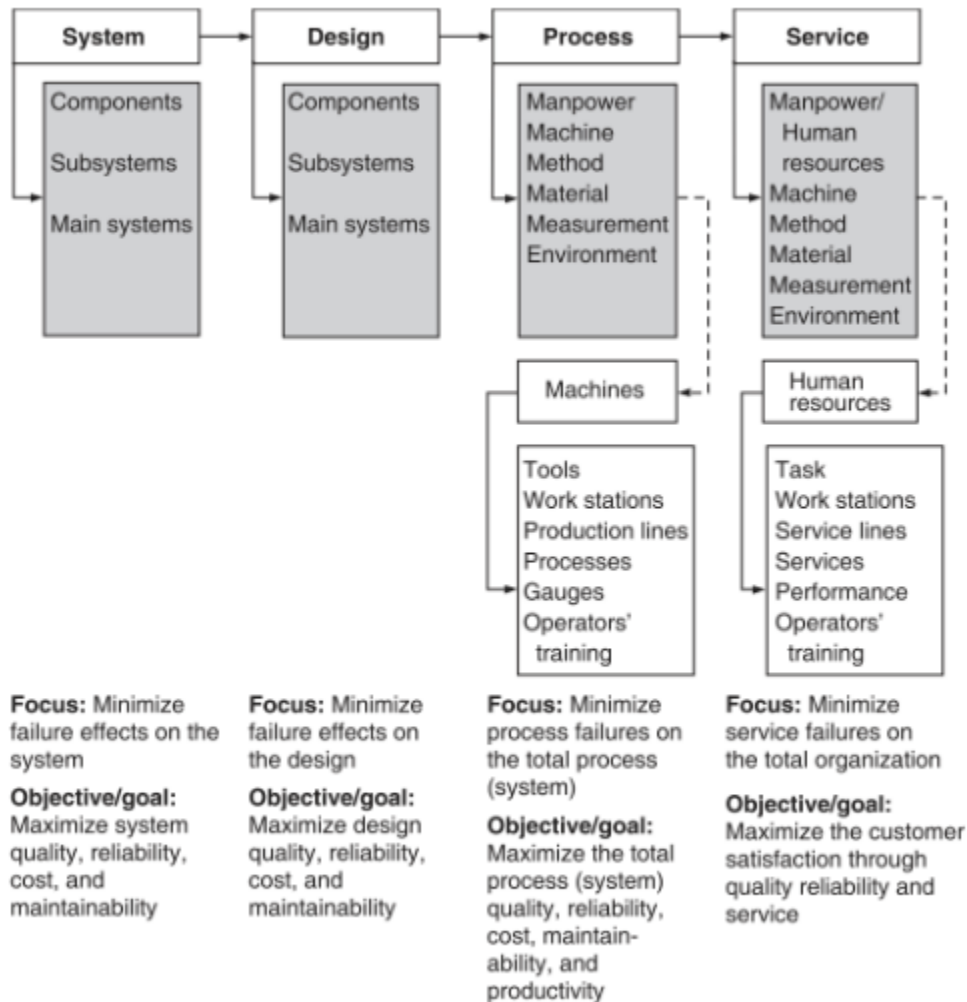


Figure 2.3 FMEA Types [19]

2.2.1.1 System FMEA

System FMEA is usually performed through conceptual design, detailed design and development, and testing and evaluation. At this stage, design is the evaluation process involving the application of various technologies and methods to produce an effective system output. This result will then be used as input for the design FMEA, which becomes an input for the process/assembly part and/or the service FMEA.

Selection of appropriate technologies may involve using an existing system(s), currently known or proposed standard approaches, directed research results, or a combination of all of these.

An acceptable System FMEA is basically accomplished through product development, research and development, or a combination of these assets throughout the systems engineering process.

Analyzing the System and its subsystems focuses on identifying potential error types among system functions arising from the System's deficiencies. Its goal is to increase the System's quality, reliability, and maintainability. System FMEA assists in determining the most suitable design alternative by examining the following system stages and their relationships in detail.

- A system is a combination of sub-systems (materials, tools, facilities, resources, software, etc.) where the task is required to achieve a mission or function
- A subsystem combines assembly's functions to obtain the specific activities necessary to accomplish a mission
- Assembly, the combination of sub-assemblies
- Sub-assembly, assembly of components
- Component, a variety of parts
- Part, lowest manufacturable part
- Interactions are interaction points between system elements required to produce desired and fundamental effects (i.e., energy and information transfer interaction)

System FMEA, when implemented effectively, will be able to identify the list of potential activities that will eliminate error types and safety issues and reduce errors, and the benefits are listed below [41].

- Assists in the selection of the best system design alternative
- Assists in determining redundancy
- Assists in establishing the foundation for system-level diagnostic processes
- Increases the possibility that prospective failures will be taken into account
- Determinates potential system failures and their relations with other systems or subsystems

2.2.1.2 Design FMEA

Design FMEA concentrates on product design, generally at the subsystem or level of the component. The priority is on design-related shortcomings, emphasizing that the design and development process is safe and reliable throughout the equipment's useful life. The subsystem or component itself, the interfaces between bordering components are included in the Design FMEA scope. Typically, design FMEA considers that the product will be manufactured based on specifications [36].

An effective design FMEA is basically implemented throughout the systems engineering process, product development, research and development, marketing, manufacturing, or combining all these assets.

Preventing defects or malfunctioning product features is a crucial part of a quality design effort. Suppose we want to win new customers and satisfy the existing customers in the best way when the current product features are known. In that case, these should be expanded in terms of design in new-generation products or replacement products.

The benefits of using a design FMEA can be summarized as following [41]:

- Sets a priority for actions to improve the design
- Documents the reasons behind modifications
- Assists in identifying the key or significant aspects
- Assists in the assessment of design needs and alternatives
- Assists in the identification and elimination of potential safety issues
- Assists in the early detection of product failure during the product development process.

Since the manufacturing process control methods will not eliminate the negativities in the design during the manufacture of the product, the problems that will come out of trusting the manufacturing process should be tried to prevent in the Design FMEA phase at the beginning. There are two approaches in the Design FMEA technique. In the first approach, the system or product is considered fully and analyzed to the lowest unit. The second, which is also accepted in practice, starts from the lowest level units of the systems such as parts and components, passes through the stages such as sub-assembly, sub-system, and progresses to the last level of the system or product. In other words, the product is divided into sections, its elements for easy inspection;

engineering design data is reviewed. Each assembly element's functions, impacts, and relationships are assessed. Block diagrams are employed, and the assembly elements their functions are listed in the block diagram.

Potential failures of the elements are determined by detecting operational and environmental fault mechanisms that may affect the product. Some elements can have more than one potential fault. Therefore, the consequences of possible failures in the next assembly operation or the final product are analyzed. The failure probability of product elements is estimated based on experience, and the overall line failure probability is calculated using reliability theory by these data.

2.2.1.3 Process FMEA

Process FMEA is applied to examine the assembly processes and manufacturing. It determines any potential faults caused by manufacturing/assembly procedures, components of machinery, production methods. Traditionally, process FMEA starts when the design FMEA report is released [43].

Design FMEA and Process FMEA aim to eliminate errors before they occur, whereas they are applied to different phases of the product or system [44]. The application process differs only in terms of the information they use. In order to obtain very positive results from both FMEA types, the written documents should not be forgotten after the analysis is completed, and the results of the implementation of the recommended corrective actions should be monitored [45].

The benefits of using a Process FMEA can be summarized as following [41]:

- Assists in developing control plans and specifies the significant characteristics
- Specifies process problems and proposes a plan of action
- Determines the order of priority for corrective actions
- Helps in the examination of the manufacturing or assembly process
- Documents the reasons behind modifications

Process FMEA should include the following manufacturing processes:

- All new products/parts
- Modified products/parts
- Products/parts known to have applied new manufacturing technologies

Process FMEA should be done when production equipment has been identified but not yet manufactured; like this, better reliability in fulfilling product specifications.

Process FMEA should begin with a flowchart of the process. This flowchart should identify the product characteristics produced in each operation. Determination of some effects and estimation of some intensity values can be obtained from the responsible design engineer or, if available, from the relevant Design FMEA study. If there are many consecutive processes in the process and they have different possible error types, it may be appropriate to list them as separate processes. All the process steps required for the proper production or assembly of the product/part are examined in the best detail by drawing a flow chart.

2.2.1.4 Service FMEA

Service FMEA is applied to examine services before they deliver to the client. A service FMEA concentrates on the causes of failure modes (tasks, mistakes, errors) by system or process defects [41].

Service FMEA allows for the prioritization of development initiatives as well as the recording of change explanations. It offers benefits such as efficient workflow, system, and process analysis, recognizing work faults and critical tasks, and developing control strategies [46].

The benefits of using a Service FMEA can be summarized as following [41]:

- Helps in task flow analysis
- Assists in the system and/or process analysis
- Specifies task deficiencies
- Helps in the control plans development and specifies significant tasks
- Sets a priority for actions to be taken for improvement.
- Documents the reasons behind modifications

2.2.2 Risk Prioritization Ranking

In the system, random and natural events that may occur during the process that may cause damage should be rated and prioritized. The parts within the whole function are handled separately, and potential failure events are detected; thus, the failure modes are determined.

Using rating scales, the classical FMEA ranks each failure mode based on its Severity (S), Occurrence (O), and Detectability (D). These specific risk possibilities are assessed based on their Risk Priority Number (RPN), which follows

$$RPN_i = S_i \cdot O_i \cdot D_i \quad (2.1)$$

a formula to calculate the failure mode critically [24].

Based on the criteria from a severity case, the S relates to the most significant effect for a specific failure mode. It is a relative rating within the scope of the given FMEA and is established without consideration of the likelihood of occurrence or detection.

O is the frequency or the fact that something is happening. The rankings are based on the likelihood of the failure reason occurring.

D evaluates the change in the observed failure. It is based on the chances that the failure will be detected before the customer finds it.

RPN is an excellent tool for prioritizing focused improvement efforts by calculating the RPN for failure mode and effect.

In addition, the Relative RPN calculation can be calculated as follows

$$Rel_RPN_i = \frac{RPN_i}{\sum_{j=1}^m RPN_j} \quad (2.2)$$

where RPN_j is the sum of the RPN_i results of the i^{th} sub-conclusion and m is the number of sub-conclusion.

The value of the criteria ranking can be anything. There is no standard for such value; nonetheless, there are two persistent rankings utilized in all industries today. The scale of the first ranking is based on a 1 to 5, and the second is a 1 to 10. The ranking from 1 to 5 is narrowed but provides convenience and ease of interpretation. Because it reflects a uniform distribution, it does not offer

certainty (accuracy) for a given quantization. The ranking 1 to 10 is commonly used and extremely suggested as it provides ease of performance, accuracy, and precision in quantifying the rank [41].

In 2018, the Alignment of VDA and AIAG FMEA working group [47] introduced the "Design FMEA Action Priority (*AP*)" (see table 2.1) and "Process FMEA Action Priority (*AP*)" (see table 2.2). The *AP* number is a further development of the *RPN*. Contrary to *RPN*, the *AP* number considers different weights of factors *S*, *O*, and *D* (see table 2.3). The final *AP*, which can be the High (*H*), Medium (*M*), and Low (*L*), are formed as a function of the values of the individual components (*S*, *O*, *D*) between a particular range. The "Monitoring and System Response" section is also displayed, taking into account the system's response and compliance with safety requirements. The evaluation catalog only evaluates compliance with safety requirements rated H-high or L-low.

<i>S</i>	<i>O</i>	<i>D</i>	<i>AP</i>	Justification for Action Priority – DFMEA
9-10	6-10	1-10	H	High priority due to safety and/or regulatory effect that have a high or very high occurrence rating
9-10	4-5	7-10	H	High priority due to safety and/or regulatory effect that have a moderate occurrence rating and high detection rating
5-8	4-5	5-6	H	High priority due to the loss or degradation of an essential or convenience vehicle function that has a moderate occurrence rating and moderate detection rating
5-8	4-5	1-4	M	Medium priority due to the loss or degradation of an essential or convenience vehicle function that has a moderate occurrence and low detection rating
2-4	4-5	5-6	M	Medium priority due to perceived quality (appearance, sound, haptics) with a moderate occurrence and moderate detection rating
2-4	4-5	1-4	L	Low priority due to perceived quality (appearance, sound, haptics) with a moderate occurrence and low detection rating
1	1-10	1-10	L	Low priority due to no discernible effect

Table 2.1 Design FMEA Action Priority [47]

S	O	D	AP	Justification for Action Priority – DFMEA
9-10	6-10	2-10	H	High priority due to safety and/or regulatory effect that have a high or very high occurrence rating
9-10	4-5	7-10	H	High priority due to safety and/or regulatory effect that have a moderate occurrence rating and high detection rating
5-8	4-5	5-6	H	High priority due to the loss or degradation of an essential or convenience vehicle function that has a moderate occurrence rating and moderate detection rating
5-8	4-5	2-4	M	Medium priority due to the loss or degradation of an essential or convenience vehicle function that has a moderate occurrence and low detection rating
2-4	4-5	2-4	L	Low priority due to perceived quality (appearance, sound, haptics) or a manufacturing disruption with a moderate occurrence and moderate detection rating
2-10	1	1	L	Low priority due to the failure being virtually eliminated through prevention controls
1	1-10	1-10	L	Low priority due to no discernible effect
2-10	1	2-10	Error	O=1 implausible without D=1
2-10	2-10	1	Error	D=1 implausible without O=1

Table 2.2 Process FMEA Action Priority [47]

Action Priority	Action Expectation
High	The team must either identify an appropriate action to improve prevention and / or detection controls or justify and document why current controls are adequate
Medium	The team should identify appropriate action to improve prevention and / or detection controls, or, at the discretion of company, justify and document why controls are adequate.
Low	The team could identify actions to improve prevention or detection controls.
It is recommended that potential Severity 9-10 failure effects with Action Priority High and medium, at minimum, be reviewed by management including any actions that were taken.	
This is not the prioritization of High, Medium, or Low risk. It is the prioritization of the need for actions to reduce risk.	

Table 2.3 Action Priority Evaluation [47]

Knowing the risk of functions qualifies to identify exposures and weaknesses of a system that compromise a system's ability to maintain a steady state. It is advisable to analyze and test these to see how robust and reliable these critical points work.

The guidelines used in automotive are very successful in identifying and avoiding malfunctions. To bring this achievement to the next level, I will present Fuzzy logic and then hybridize it with FMEA in the case studies of the following chapters.

2.3 FUZZY APPROACH

Fuzzy logic is defined as the beginning of artificial intelligence and can imitate dynamically human behavior. The concept of fuzzy logic and related fuzzy set theory was first introduced and published in 1965 by Lotfi A. Zadeh to handle mathematical concepts that are difficult to quantify [48].

The fuzzy controller provides the transformation by qualitative rules into the corresponding quantitative mapping, while fuzzy knowledge compression transforms the quantitative data into qualitative practices, like Boolean algebra. However, In fuzzy logic, the approximate values are used instead of absolute ones. The peculiarity of traditional formal computation, modelling, and reasoning tools are mostly deterministic, crisp, and precise. The Crisp mean dichotomous that is, classical set theorem that Boolean logic yes or no rather than more or less type. In dual logic case, a statement can be literary either true or false, and numerical either 0 or 1, nothing between. This approach serves more realistic solutions to daily life problems because it has a more flexible structure [49].

Fuzzy logic applications once were thought to be an obscure mathematical curiosity, but today it can be found in many engineering applications and scientific studies. Numerous applications have been used to control knowledge-based systems such as multi-objective optimization of power systems, air conditional systems, washing machines, vacuum cleaners, anti-lock braking systems for automobiles, subway control systems, models for process risk assessment, medical diagnosis, stock training, treatments plan, etc. Accordingly, in numerous fields successfully used such as control systems engineering, power engineering, image processing, industrial automation, robotics, optimization, etc. This branch of mathematics has taken its place as the beginning of artificial intelligence [50][51].

Mathematical models, along with fuzzy sets, have proven how useful they are for understanding natural phenomena. Thousands of researchers are working extremely hard to solve many engineering problems. Till today, numerous fuzzy logic related applications have been developed. Since engineering systems become complex day by day; These applications will be continuously developed based on experience.

In recent years, the structure of fuzzy logic has contributed to many research topics. The fuzzy sets theorem provides realistic solutions in many scientific fields as an application of artificial intelligence technology.

In 2015, “*Fuzzy control of anti-lock braking system and active suspension in a vehicle*” by A. Berouaken and R. Boulahia [52], the authors have proposed a robust system fuzzy logic-based control for the Anti-Lock Braking System (ABS) in conjunction with an active suspension. They have developed a two-degree of freedom quarter automobile system, along with models for the ABS and a hydraulic active suspension. It has simulated the proposed control scheme using a MATLAB tool and significantly improved ABS performance during the active suspension.

In 2016, “*On the design of a fuzzy logic-based control system for freeze-drying processes*” by Davide Fissore [53], the author optimized a drug freeze-drying time process by designing a fuzzy logic-based control system. The system aims to keep temperature and threshold values as close as possible. The author created fuzzy sets and rules obtained through process simulation to balance product temperature and the threshold value. As a result, the influence of the input variables' initial value and the control range is considered, thus resulting in the control system's optimum configuration.

In 2017, “*A fuzzy set-based method to identify the car position in a road lane at intersections by smartphone GPS data*” by Mario Marinelli et al.[54], the authors presented a fuzzy set-based method for vehicle positioning on-road lanes near intersections using GPS data from smartphones. It is not easy to identify the position of the vehicle within the road lanes. Therefore, they used a fuzzy set to consider the uncertainty embedded in GPS data. Moreover, to obtain a novel supervised clustering technique, they included a Genetic Algorithm to regulate the fuzzy parameters.

In 2018, “*Adaptive fuzzy logic control of fuel-cell-battery hybrid systems for electric vehicles*” by Jian Chen et al. [55], the authors introduced an adaptive control approach with fuzzy logic

parameter tuning for electric vehicle energy management using a hybrid system fuel-cell battery. They aimed to improve the power flow between the fuel cell and battery in real-time without the need to predict demand behavior. To achieve the control objectives, two-parameter update laws have been developed for the battery's immeasurable internal resistance and the current of the ideally controlled current source—fuzzy logic parameter tuning integrated with the adaptive controller to provide performance in different driving conditions.

In 2018, “*A supervisory online tuned fuzzy logic-based sliding mode control for robotics: an application to surgical robots*” by Mohd Salim et al. [56], the authors worked on the control methodology for surgical robots based on sliding mode controlled by using fuzzy logic control. They used 2 DOF surgical robot manipulators formulated for high-speed trajectory tracking and typical during surgery. System stability has studied using the Lyapunov theorem. All numerical simulations have completed using the MATLAB tool.

In 2019, “*Position control of a quadcopter drone using evolutionary algorithms-based self-tuning for first-order Takagi–Sugeno–Kang fuzzy logic autopilots*” by Edwar Yazid et al. [57], the author addressed a self-tuning quadcopter drone trajectory tracking control based on evolutionary algorithms. They controlled the uncertainties in the flight environment by using first-order Takagi–Sugeno type fuzzy logic. They tackle three major optimization algorithms with the latest technology - Genetic Algorithm, Particle Swarm Optimization, and Artificial Bee Colony to challenge automatic tuning.

In 2020, “*A fuzzy-logic Internet of Things (IoT) lighting and shading control system for smart buildings*” by Giacomo Chiesa et al. [58], the authors presented a working prototype of a low-cost IoT system that controls daylight and artificial light balance in a dynamic shading system. They have developed a control application for smart buildings to allow user interaction based on seasonal automatic modes or manual functionalities. The required lighting can change while moving according to the seasonal profile bioclimatic design of the shading system. They have achieved a fast response, low computational requirements, and immediate reaction to environmental changes by fuzzy logic control.

In 2020, “*Forecasting of COVID-19 time series for countries in the world based on a hybrid approach combining the fractal dimension and fuzzy logic*” by Oscar Castillo and Patricia Melin [59], the authors presented Fuzzy Logic and Fractal theory hybrid approach for estimating

confirmed cases and deaths of the countries based on their COVID-19 time series complexity. They were used fractal dimension concept to measure the dynamics complexity in the world's existing time series. Fuzzy Logic was used to determine the uncertainty in the estimation process. Public data sets of 10 countries were used to create a fuzzy model with time series for a specific period. The fuzzy fractal model was then tested by forecasting other time series in window periods of 10 days to verify the proposed approach's effectiveness.

2.3.1 Basic Concept

2.3.1.1 Fuzzy Sets Theory

Firstly, it is necessary to understand the membership function terminologies for the fuzzy set operations working concept [60]. The Height, Core, and Support of the Fuzzy Set are expressed in figure 2.4.

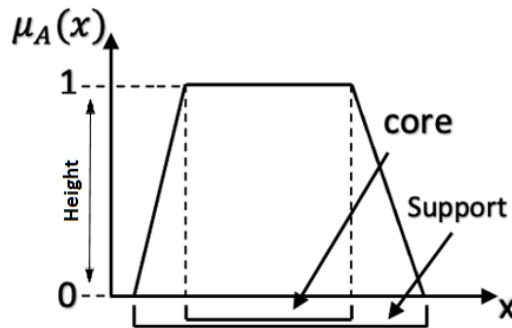


Figure 2.4 Membership function of fuzzy set A

Let X be a non-empty classical set, that is $X \neq \emptyset$. A fuzzy set A of X is defined by its membership function.

$$\mu_A : X \rightarrow [0,1] \quad (2.3)$$

where $\mu_A(x)$ denotes the degree of membership of x in A .

The maximum membership value of a fuzzy set A in X is called the Height of A and defined by [61]

$$Height(A) = \sup_{x \in X} \mu_A(x) \quad (2.4)$$

Furthermore, if there exists $x^* \in X$ such that $\mu_A(x^*) = 1$ then A is called normal.

The Core of a fuzzy set is the crisp subset of elements with membership value equal to 1.

$$core(A) = \{x \mid \mu_A(x) = 1\} \quad (2.5)$$

The Support of a fuzzy set is described as the crisp subset of elements concerning universal set X having positive membership function and given as [62].

$$support(A) = \{x \mid \mu_A(x) > 0\} \quad (2.6)$$

In fuzzy sets, the most frequently encountered membership functions are normal and convex. However, due to many operations on fuzzy sets, hence operations on membership functions, non-normal and non-convex fuzzy sets are produced. They are usually characterized in terms of one-dimensional universes. However, multidimensional (or n-dimensional) can undoubtedly explain them as well. The curves of the membership functions become single-faceted in two dimensions and hypersurfaces in three or more dimensions [63].

As an attempt to mathematically represent linguistic expression, fuzzy logic makes use of the membership function and the operators that work in it. In fuzzy logic, where membership functions and operators are closely related, the main goal is to find membership functions on a theoretical basis, which are easy to calculate and can only be described with meaningful parameters [64].

There are various Membership Function (MF) curves available such as triangle, trapezoidal, and Gaussian, which are the most commonly used ones. In this thesis, MF curves of trapezoidal, R and L take into account.

R Membership Functions

The figure 2.5 shows two parameters a and b , which are used to describe R type membership function of set A ,

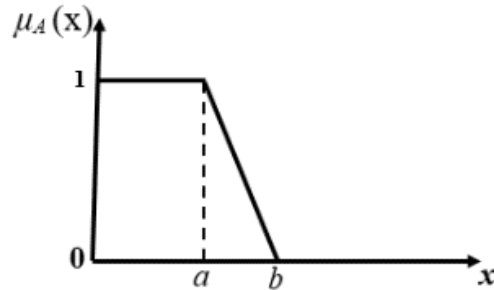


Figure 2.5 Membership function of a fuzzy set of type R

and mathematically can be formulated as following

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \leq a \\ \frac{b-x}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{if } x \geq b \end{cases} \quad (2.7)$$

where a, b are the membership function parameters; $a \neq b$.

Trapezoidal Membership Functions

Four parameters form the core and support parts of the trapezoidal membership function. Trapezoidal numbers can be defined by four different variables a, b, c, d shown as figure 2.6.

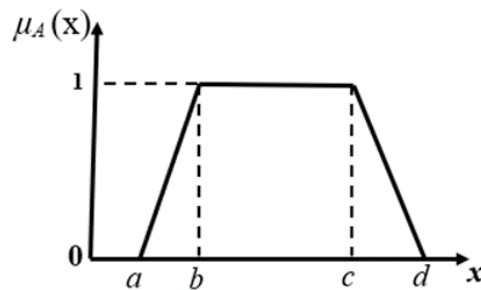


Figure 2.6 Membership function of trapezoidal fuzzy number

Respectively, a is the lower limit of the support, b is the lower limit of the core, c is the upper limit of the core, and d is the upper limit of the support. See also the more detailed illustration of the trapezoidal membership function (Fig. 2.6).

Trapezoidal fuzzy MFs will be defined by equation following

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \leq a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ 1 & \text{if } b \leq x \leq c \\ \frac{d-x}{d-c} & \text{if } c \leq x \leq d \\ 0 & \text{if } d \leq x \end{cases} \quad (2.8)$$

where a, b, c , and d are the membership function parameters; $a \neq b$ and $c \neq d$.

L Membership Functions

The figure 2.7 shows L membership function, two parameters, a and b , are used to describe the mathematical representation of the membership function of A ,

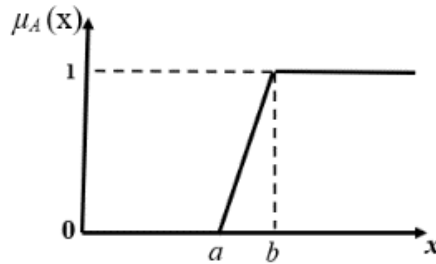


Figure 2.7 Membership function of a fuzzy set of type L

and formulated by following equation as,

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \leq a \\ \frac{x-a}{b-a} & \text{if } a < x \leq b \\ 1 & \text{if } x > b \end{cases} \quad (2.9)$$

where a, b are the membership function parameters; $a \neq b$.

2.3.1.2 Fuzzy operators

Intersection (T-norm) and Union (S-norm) are operations that generalize the binary conjunction and disjunction to fuzzy logic. The semantics of fuzzy mathematical logic is a natural understanding of the conjunction and disjunction, and they are used to integrate criteria in multi-criteria decision-making.

In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition - an element either belongs or does not belong to the set. This can be clearly seen when looking at the usual set algebraic operations such as intersection (eq. 2.10) and union (eq. 2.12). The conditions characterize the sets of crisp A , B as,

$$x \in A \cap B \Leftrightarrow x \in A \wedge x \in B \quad (2.10)$$

$$x \in A \cup B \Leftrightarrow x \in A \vee x \in B \quad (2.11)$$

Fuzzy logic is structured on fuzzy sets and subsets. In fuzzy sets, the degree of specified membership ranges is accepted between 0 and 1. By Zadeh [48], the membership degrees of the set intersection (eq. 2.12) and union (eq. 2.13) operations for defined as,

$$\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\} \quad (2.12)$$

$$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\} \quad (2.13)$$

also, shown other operations for fuzzy sets, called “algebraic”, for example, an algebraic product and an algebraic sum defined by the following equations.

$$\mu_{AB}(x) = \mu_A(x) \cdot \mu_B(x) \quad (2.14)$$

$$\mu_{A+B}(x) = \min\{\mu_A(x) + \mu_B(x), 1\} \quad (2.15)$$

Fuzzy sets were designed as a mathematical tool for modelling uncertain concepts. The membership function of fuzzy sets, ordinary fuzzy sets are usually certain.

2.3.2 Approximate Reasoning

Generally, inference systems use the *IF-THEN* rule structure to determine the connection between the universe of the condition (X) and the universe of consequence (Y). Zadeh proposed approximate reasoning rules in fuzzy-based inference systems based on fuzzy inputs, fuzzy premises and consequences, and fuzzy outputs that may be implemented using operators defined on fuzzy sets [65].

In the case of single input single output systems, fuzzy sets (A , B) can define linguistic variables that may be used in the form of the *IF* condition *THEN* consequence-type rules as follows:

$$IF\ x\ is\ A_i\ THEN\ y\ is\ B_i \quad (2.16)$$

where $x \in X$, $y \in Y$, and $i = 1, 2, \dots, n$, n denotes the number of the rules.

In the case of multi-input, let the parameter of input be, $x_1 \in X_1, x_2 \in X_2, \dots, x_n \in X_n$ and the parameter of the single output be $y \in Y$, the evaluation may be presented using the following type of rule:

$$IF\ x_1\ is\ A_{1,i_1}\ AND\ \dots\ AND\ x_n\ is\ A_{n,i_n}\ THEN\ y\ is\ B_{i_1, \dots, i_n} \quad (2.17)$$

where n is the number of the inputs belonging to set j , A_{j,i_j} is the fuzzy set belonging to the input i_j , $i_j = 1, \dots, n_j$, n_j .

2.3.2.1 Implication and composition

In logical systems, *IF-THEN* type rules can be modeled by implication, and the conclusion is reached using inference rules such as Modus ponens. The Generalized Modus Ponens scheme used for Mamdani type fuzzy-based inference systems is as follows:

$$\text{Rule} \quad \quad \quad \textit{IF } x \textit{ is } A \textit{ THEN } y \textit{ is } B \quad \quad \quad (2.18)$$

$$\text{Observation} \quad \quad x \textit{ is } A' \quad \quad \quad (2.19)$$

$$\text{Consequence} \quad \quad y \textit{ is } B' \quad \quad \quad (2.20)$$

where the predicted output B' corresponds to the rule consequence B to the degree that the premise A correlates to the system input A' operating on the rule system.

The based-on t-norm, the scheme of Generalized Modus Ponens inference rule (sup – t compositional rule of inference) has the following mathematical model:

$$B'(y) = \sup_{x \in X} (t(A'(x), (A \rightarrow B)(x, y))) \quad (2.21)$$

$$B'(y) = \sup_{x \in X} (t(A'(x), A(x) \rightarrow B(y))) \quad (2.22)$$

where in *IF x is A THEN y is B* model, implication of $(A \rightarrow B)(x, y)$ is a min predictor operator.

Mamdani modeled the *AND* relationship instead of the implication by simplifying the relationship between the fuzzy rule premise and the rule consequence.

Mamdani simplified the link between the fuzzy rule premise and the rule consequence to model the *AND* relationship rather than the implication. Naturally, this implication model does not entirely satisfy the conditions for implication as an operation of a logical process. Naturally, this implication model does not entirely satisfy the conditions for implication as an operation of a logical process. Nonetheless, its use in control systems and other applications has been ubiquitous and proved beneficial.

As a result of the above, the Mamdani-type inference system generalized model is as follows:

$$B'(y) = \sup_{x \in X} \left(t \left(A'(x), t \left(A(x), B(y) \right) \right) \right) \quad (2.23)$$

where t denotes the t-norm with the associated qualities.

When the t-norm condition is taken into account, the expression may be written as

$$B'(y) = \sup_{x \in X} \left(t \left(t \left(A(x), A'(x) \right), B(y) \right) \right) \quad (2.24)$$

If the left of the t-norm is continuous, then as

$$B'(y) = t \left(\sup_{x \in X} \left(t \left(A(x), A'(x) \right) \right), B(y) \right) \quad (2.25)$$

where $B'(y) = \sup_{x \in X} \left(t \left(A(x), A'(x) \right) \right)$ corresponds to the rule's firing strength.

2.3.3 Mamdani-type Fuzzy Inference Model

A control system requires a stable execution of mathematical modeling that considers input and output relationships. Mathematics operations are utilized to depict a complicated constructed system qualitatively and quantitatively. The output of any system might be unclear due to the complexity of its structure. In a complex system, fuzzy logic approach with qualitative interpretation can model the uncertainty by formulating experiences and conditions. As a result, fuzzy logic is intuitive, based on knowledge. It will be concluded in a qualitatively appropriate system to achieve acceptable results.

The fuzzy inference process has information on how to optimally manage the system in the form of a set of rules [66]. Fuzzy rules collect linguistic statements that describe how the Fuzzy Inference System should decide to classify an input or control output.

A control engineer or an expert can do the process of this system structure. The artificial decision-maker must collect information about behaving in a closed-loop system to design a fuzzy controller.

In the fuzzy evaluation, natural language rules are used, whose structure in the Mamdani inference system is as bellow with *AND/OR* connections.

Let the inputs be x_1, x_2, \dots, x_n , the output is y , and the structure of the rules are

$$IF x_1 \text{ is } A_{1,i_1} \text{ AND } \dots \text{ AND } x_n \text{ is } A_{n,i_n} \text{ THEN } y \text{ is } B_{i_1, \dots, i_n} \quad (2.26)$$

where A_{k,i_k} is the fuzzy set i_k belonging to the input k , B_{i_1, \dots, i_n} is the fuzzy rule consequence, $i_j = 1..n_j, n_j$ is the number of the antecedent sets belonging to the input j .

2.3.3.1 Fuzzification

A crisp value is transformed into a fuzzy one in the fuzzification process, i.e., a fuzzy set is created for this purpose. In most cases, singleton type fuzzification is used, but other types are also possible. A fuzzified value can be determined using equation (2.27) in the case of trapezoidal membership functions and crisp inputs, as shown in figure 2.8.

$$\mu_{A_i}(x) = \begin{cases} 0 & \text{if } x \leq a_i \\ \frac{x-a_i}{b_i-a_i} & \text{if } a_i \leq x \leq b_i \\ 1 & \text{if } b_i \leq x \leq c_i \\ \frac{d_i-x}{d_i-c_i} & \text{if } c_i \leq x \leq d_i \\ 0 & \text{if } d_i \leq x \end{cases} \quad (2.27)$$

where a_i, b_i, c_i , and d_i are the parameters of the membership function; $a_i \neq b_i$ and $c_i \neq d_i$, $i=1, \dots, n$, number of the MFs.

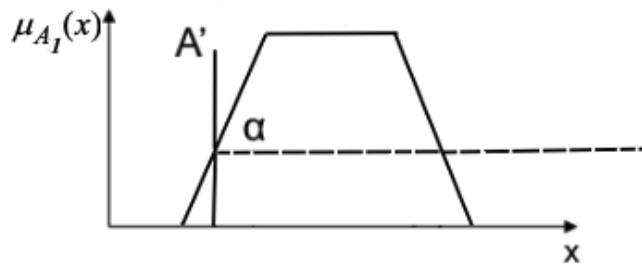


Figure 2.8 Fuzzification for Trapezoidal membership function

2.3.3.2 The Firing Strength

The first step is called the firing strength calculation process; the input data must connect based on the rule antecedent by *AND* connections with their operators. Example of figure example 2.9. describes the minimum operator for *AND* connection. The operators of firing strength calculation can be formulated as

$$w_i = \min(\mu_{A_{i,j}}(x)) \quad (2.28)$$

Where $x \in X$, $\mu_k(x)$ is the fuzzified value of the antecedent i of input j .

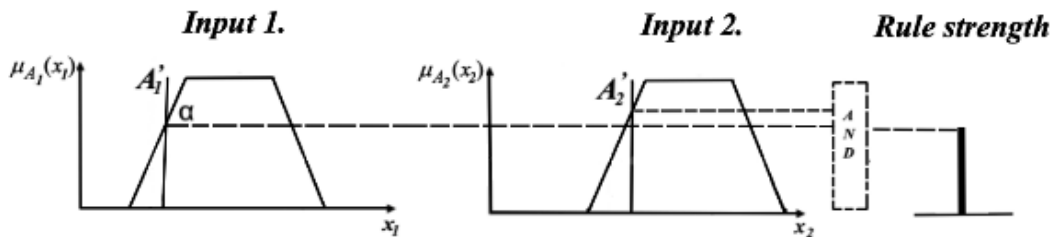


Figure 2.9 Calculation of firing strength using minimum operator

2.3.3.3 Implication

The second step is the implication process that projects the result of data combination for determined each rule. In figure 2.10, the development of the minimum operator illustrated, and that can be calculated with the formula following

$$y_{B_i} = \min(w_i, \mu_{B_i}(x)) \quad (2.29)$$

where w_i is the firing strength of rule i and $\mu_{B_i}(x)$ is the consequent set belonging to rule i .

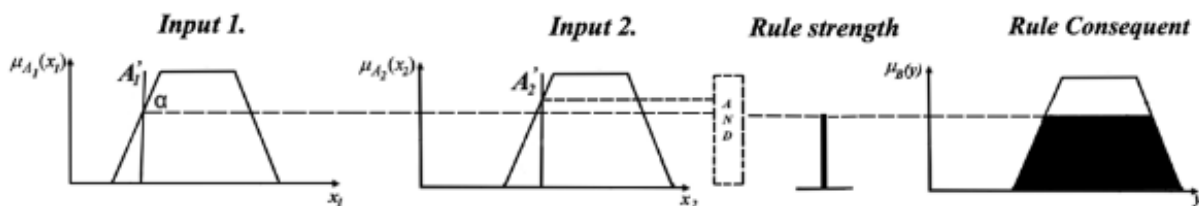


Figure 2.10 Fuzzy implication with AND connection using minimum operator

2.3.3.4 Aggregation

The last step is the aggregation process that is summing obtained all rule consequent sets from the implication process. The maximum operator applied in figure 2.11, and that can be calculated by formula following

$$y = \max(y_{B_i}) \quad (2.30)$$

where y_{B_i} is the sub-conclusion for rule i .

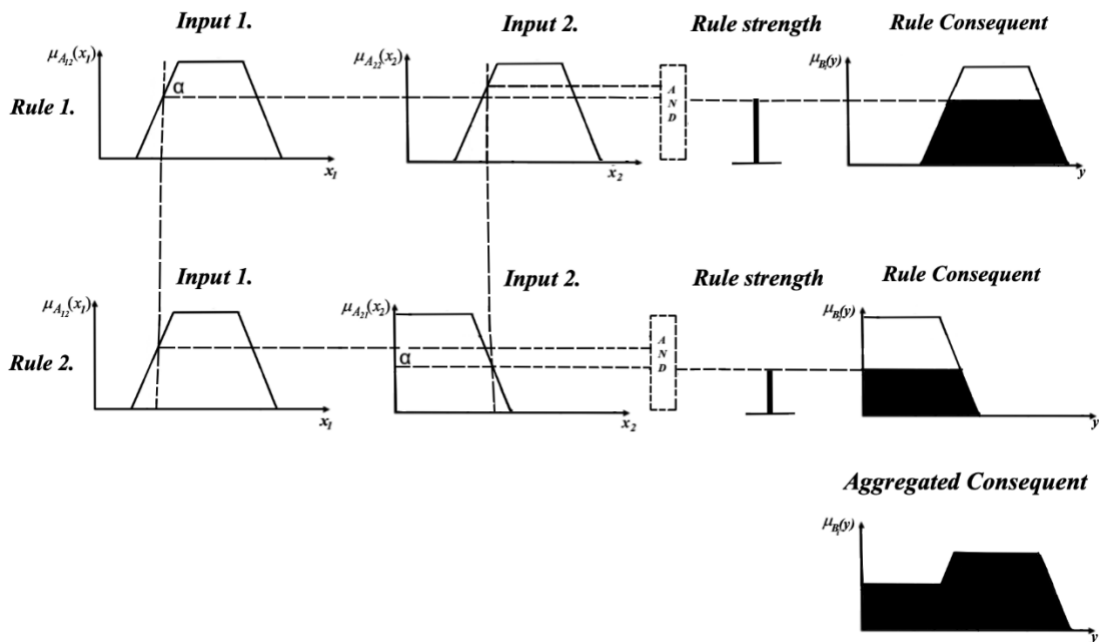


Figure 2.11 Composition process

The last step of the general fuzzy inference model is the defuzzification process, described next.

2.3.3.5 Defuzzification

The defuzzification enters the circuit as the last stage of the fuzzy inference process. If necessary, the aggregated rule results obtained from the composition process can be converted to a crisp value, which best characterizes the system result.

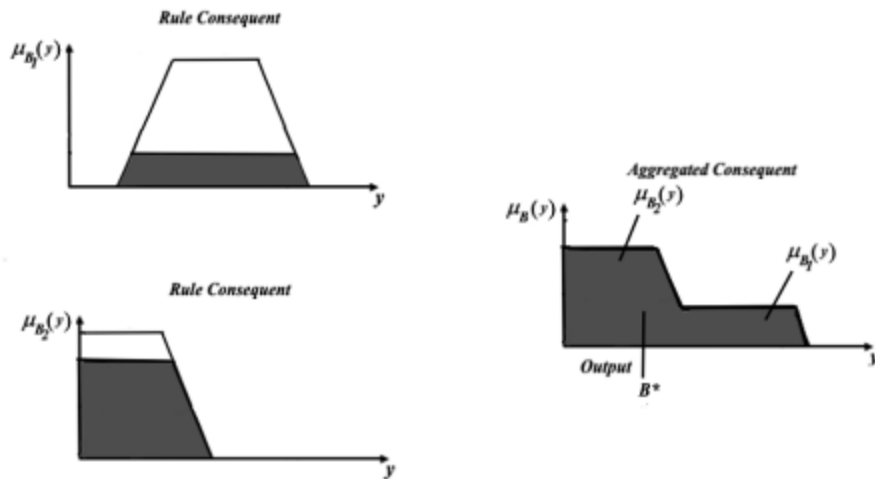


Figure 2.1 Defuzzification process

Many techniques have been developed to complete to success for this mathematical process. The specific application properties will determine which form of defuzzification method can be adapted. Nonetheless, there is no well-organized procedure to decide which way is entirely suitable for any given application [67]. The most widely used are Centroid, Bisector, and Maximum defuzzification methods explained in the next section.

Centroid (Center of Gravity) Method

The Center of Gravity (COG) method is the most frequently used defuzzification method. It defines the center of gravity under the aggregated complex-shape, considering overlapping areas more times and provides a crisp value [68]. The method advantage is easy to calculate for triangular and trapezoidal functions, leading to continuous behavior when direct control is concerned. Generally, a complex-shaped part should be defuzzified, complicated in a calculation, and derivation is very slow. The Figure 2.13. describes centroid defuzzification method and the equation defines the crisp result following,

$$B_{COG} = \frac{\sum_{i=1}^n \int_{-\infty}^{\infty} \mu_i(y) y dy}{\sum_{i=1}^n \int_{-\infty}^{\infty} \mu_i(y) dy} \quad (2.31)$$

where n is the number of sub-conclusions, μ_i is truth value of the i^{th} sub-conclusion.

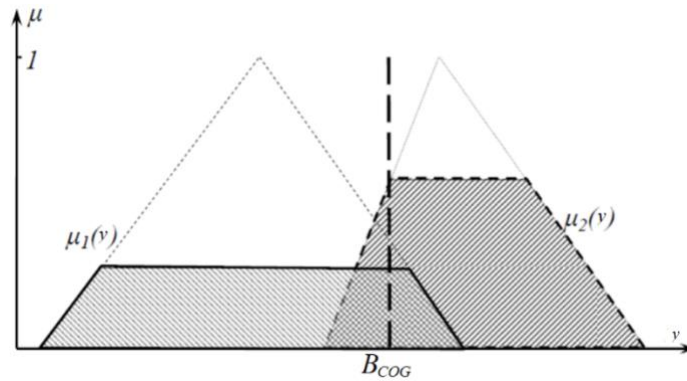


Figure 2.13 Centroid Defuzzification method

Center of Area Method

The Center of Area (COA) method is comparable to the Center of Gravity method. The difference between the two ways is that the center of gravity method considers the overlapped areas of sub-conclusions multiple times. In contrast, the center of area method only considers them only once. As compared with the Center of Gravity method, the major disadvantage of COA is that it is complicated to calculate it in complex shape partial conclusions [69]. In Figure 2.14., the example illustrated for the Centroid method and the equation defines the crisp result following,

$$B_{COA} = \frac{\int_{-\infty}^{\infty} \mu_{\Sigma}(y)ydy}{\int_{-\infty}^{\infty} \mu_{\Sigma}(y)dy} \quad (2.32)$$

where μ_{Σ} is the maximum height of the conjunct set of sub-conclusions.

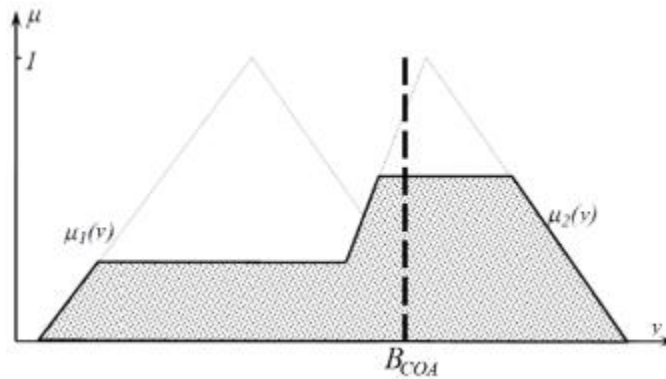


Figure 2.14 Center of Area Defuzzification method

Smallest of Maximum (SOM) Method

This method chooses the first of the highest value that domain under the curve of the aggregated complex shape [69], The Figure 2.15. illustrations the Smallest of maximum method example, and that can be calculated as the following equation.

$$B_{SOM} = \text{Min}(y \mid \mu(y) = \text{Height}(B)) \quad (3.33)$$

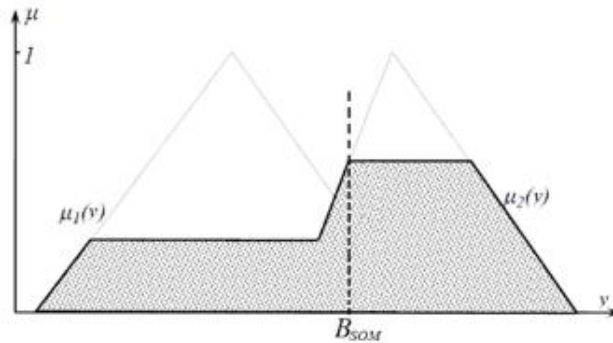


Figure 2.15 Smallest of Maximum Defuzzification method

Largest of Maximum (LOM) Method

This method selects the last of the highest value that domain under the curve of the aggregated complex shape [69], The Figure 2.16. demonstrations the Largest of maximum method example, and that can be calculated as the following equation.

$$B_{LOM} = \text{Max}(y \mid \mu(y) = \text{Height}(B)) \quad (3.34)$$

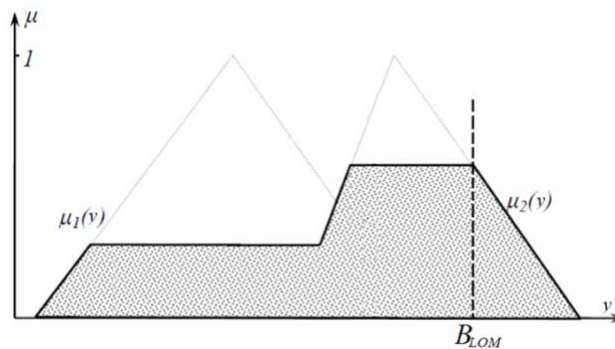


Figure 2.16 Largest of Maximum Defuzzification method

Mean of Maximum (MOM) Method

The MOM method calculates the average of those output values with the highest membership degrees [69]. The Figure 2.17. shows the Mean of maximum method example, and that can be calculated as the following equation.

$$B_{MOM} = \frac{B_{SOM} + B_{LOM}}{2} \quad (2.35)$$

where, B_{SOM} represents smallest of maximum and B_{LOM} largest of maximum.

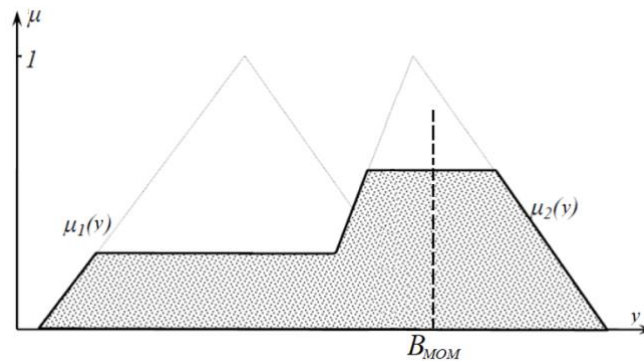


Figure 2.17 Mean of Maximum Defuzzification method

3 FUZZY HIERARCHICAL FAILURE MODE AND EFFECT ANALYSIS

Quality control is an excellent area of production management. However, the impact of used quality control methods depends on the achievement of the measurement and preliminary risk assessment systems [70]. The goal is to analyze the result of measurement failures in the designed system and propose a solution to treat measurement uncertainty.

Today's significant trend is the continuous development of product quality to increase customer pleasure and lead to more effective cost reduction management. Effective quality management also improves production efficiency by addressing the increased quantity of products made and minimizing repairs of non-conforming workpieces. For example, in their paper, Lucie Krejci et al. deal with one of the essential tools for ensuring quality in the production process using the FMEA method used in roller bearings for the automobile industry [72]. Moreover, Potkány et al. [73] have aimed to describe the current state of using selected quality management systems in Slovakian manufacturing enterprises. In their study, the authors focused on applying practices chosen for quality management, ensuring that consumers' demands are addressed and thus contribute to the business's increased performance.

The FMEA method is an improvement model. In other words, when the parameters related to the concepts covered in FMEA change, the analyzes are also revised. Thus, the procedure can turn into an improvement mechanism that constantly analyzes the current situation. Moreover, the fuzzy rule-based approach to the FMEA method provides flexibility, making it possible to obtain more efficient results.

Hierarchical safety management divides complex interrelated problems into smaller ones, solving the problem from the most petite and providing a reliable solution. The minor issues are solved separately, and the results are recombined to answer the main question. The hierarchy is made up of connected subsystems, i.e., it has its own subsystems, etc. [71]. This approach exemplifies a problem-solving strategy that takes a broad view of the problem and focuses on the relationships between the various components of the failure. Solving an issue teaches us how to tackle any problem by considering the cause of the loss, the failure impacts of the internal and external components, and the relationships between parts.

A hierarchically structured FMEA can increase the efficiency of the assessment and the flexibility of the model. Moreover, this FMEA model can be optimized using a fuzzy rule base. Therefore, to prove this, I propose Fuzzy Hierarchical Failure Mode and Effect Analysis (FH-FMEA) [74].

3.1 General description of Hierarchical Failure Mode and Effect Analysis

Different FMEA worksheets such as system, design, process, or service can be designed according to current standards and considered in a form under a hierarchical structure together. Thus, the reusability of system components, the traceability of modifications, and the availability of the required formats for each discipline can be better ensured.

In 2018, Ványi and Pokorádi [75] proposed the hierarchical version of the conventional FMEA method, and successfully examined this approach dealing with the Wheel Speed Sensor (WSS). The WSS sensor is a component of the vehicle's Anti-Block System (ABS) and determines the condition of a certain tire. It is composed of two primary components (see figure 3.1). The magnetic sensor generates periodic signals in response to the rotational speed of the cogwheel-plate attached to each vehicle tire.

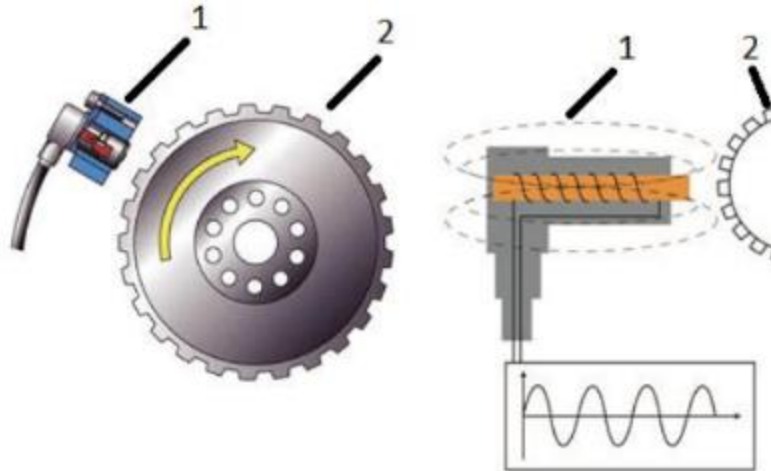


Figure 3.1 Basic description of WSS concept and measurement

(1 – Magnetic sensor; 2 – Cogwheel plate) [76]

In a hierarchical approach, the authors examined two levels that were extensively assessed (system and design) and two levels that were only partially evaluated (effect and cause).

3.1.1 Effect level

The first FMEA worksheet addresses the failure effects that occur from the overall system. The effect level enlists the functions of the product that can be perceived by the user. Failure modes, effects, and causes are identified for each function. These failures are reviewed as a fixed derived severity value for all worksheets. It means that the lower levels will treat unavoidable errors and the severity numbers associated with the function's failure. The analysts have referred to the top of the hierarchy as the Effect Level (EL), shown in table 3.1. This level does not incorporate risk that has been thoroughly analyzed; rather, it focuses on prospective failures, their impacts, and the outcomes of failure numbers.

No	Function	Pot.failure	Pot. effect	S	Cause	O	Prev. /Det. Action	D
EL1	Determine the wheel speed	Signal has not been provided	Velocity cannot be determined	10	No signal provided			
EL2			Wrong value of velocity	7	Periodic signal differs from wheel speed			
EL3	Detecting if wheel has been blocked	Blocking wheel not detected	Vehicle became instable	9	Status of wheel has been detected as rolling instead of blocked			
EL4		Blocking wheel detected instead of rolling wheel	Wrong value of blocking status	8	Wheel blocking has not been detected			
EL5				8	Status of wheel has been detected as blocking instead of rolling			

Table 3.1 Effect Level [75]

3.1.2 System Level

The System Level (SL) is shown with the calculation of RPN (see eq. 2.1) and the Relative RPN [%] (see eq. 2.2) in Table 3.2. It is responsible for analyzing the functions on the system that assist in determining the root cause of failures following the hierarchical relationship of failure effects. The functional potential failure effects must be traceable at the SL and carry the same report throughout the evaluation. This traceability must be done based on the hierarchical connection verified by the FMEA authoring software.

No	Function	Potential Failure	Pot. effect	S	Cause	O	Prev./Det. Action	D	RPN	Relative RPN [%]
SL1	Provide periodic signal according to wheel speed	Periodic signal different from wheel speed	Wrong value of velocity	5	Sensor detects metals continuously	4	D: Check the cable binding P: Use water resist technology	2	40	15.8
SL2		Wrong value of velocity	5	Space between cog is not equal	2	D: Crosscheck from another sensor P: Declare periodical check of cogwheel	3	30	11.9	
SL3		No signal provided	Velocity cannot be determined	10	Sensor does not detect metals	3	D: Check plausible values P: Ensure fixture of sensor is sufficient	2	60	23.8
SL4		Wheel clocking has not been detected	Wrong value of blocking status	8	Space between cogs is not equal	2	D: Audit production P: ensure by EoL measurement	2	32	12.6

SL5	Provider of periodic signal according to wheel status	Status of wheel has been detected as blocking instead of rolling	Wrong value of blocking status	8	Sensor does not detect metals	2	D: Check engine status, too P:Use cross check from other wheel	3	32	12.6
SL6					Sensor detects metals continuously	2	D: Aperiodic signal presenting P: Ensure sensor fixture	3	32	12.6
SL7		Status of wheel has been detected as rolling instead of blocked	Vehicle became instable	9	Sensor detects metals continuously	1	D: Compare stats to another wheel P: Ensure sensor fixture	2	27	10.7

Table 3.2 System Level [75]

3.1.3 Design and Cause Levels

In table 3.3, evaluation of the Design Level (DL) is shown for the next level. The components are analyzed parallel on separated FMEA worksheets, allowing for a simple track of changes and new materials introduction. DL is generally associated with mechanical features, electronic hardware, and software base elements (driver interface to hardware) and functions. Because these software items are attached to an interface to actuate, software functions including calculations, influence, and high-level operations are logically assessed in SL.

No	Function	Potential Failure	Potential Effect	S	Cause	O	Prev./Det. Action	D	RPN	Relative RPN[%]	
DL1	Cogwheel	Space between cogs is not equal	Wrong value of blocking status	5	Dust on surface of cog	2	P: Add notice Assembly instruction D: Check other wheels	3	30	25	
DL2			Wrong value of blocking status	5	Too wide space between cogs	1	P:Production instruction D:Cable protection	2	10	8.3	
DL3	Inductive sensor	Space between cogs are not sufficient	Velocity cannot be determined	10	Cable cut	2	P: Assembly instruction D: Cable protection	2	40	33.3	
DL4			Sensor detects metals continuously	Velocity cannot be determined	10	Cable shorting	2	P: Assembly instruction D: Cable protection	2	40	33.3
DL5											

Table 3.3 Design Level [75]

Finally, in table 3.4, the last level is portrayed as the Cause Level (CL), an essential aspect at the bottom to provide catalogs disciplines in hierarchical built FMEA. Also, high levels are united here in case of failure and effects. The CL identifies potential mechanical design failures linked to hardware and software parts. As a result, the development process, development toolchain, and designer's systemic failures may all be investigated in the case of software errors.

No	Function	Potential Failure	Potential Effect	S	Cause	O	Preventive / Detective Action	D
CL1	Failures of cog wheel	Dust on surface on cog	Wrong value of blocking status	8				
CL2		Too wide space between cogs	Wrong value of blocking status	8				
CL3	Failures of inductive sensor	Cable cut	Velocity cannot be determined	10				
CL4		Cable shorting	Velocity cannot be determined	10				

Table 3.4 Cause Level [75]

In Figure 3.2, the provision is shown by indicating "velocity cannot be determined," as reviewed in Table 3.1 and Table 3.2 of the potential effect column. This effect has a severity ranking number of 10, inherited from EL down to SL, DL, and CL.

Following the provision example outlined at SL, via DL down to CL, verify that these received failures have the exact determination of risk ranking in the overall system. Furthermore, in terms of system evaluation, the function holds 'provide periodic signal according to wheel speed,' which demonstrates potential failure as 'no signal provided,' those effects of velocity cannot be determined' upon the system.

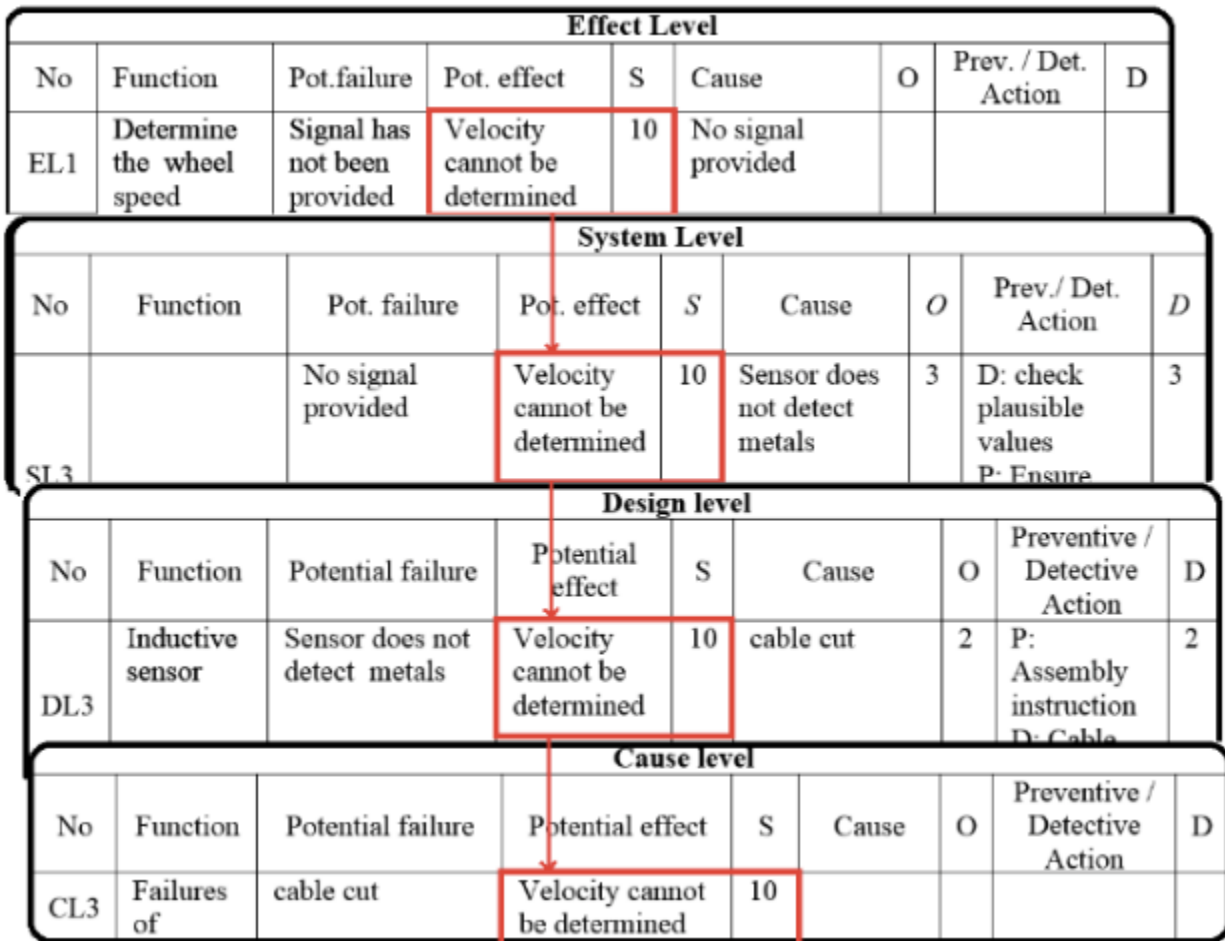


Figure 3.2 Effect linking from top to bottom [75]

Alternatively, Figure 3.2 represents using a domain-specific failure catalog, which refers to the failure cause on EL attached directly to SL, determining the possible failure cause of the specified failure in the under sub-systems. Another benefit of this approach is a shared catalog of typical design failures supporting failure cause evaluation since DL usually must face related causes. Thus, these related meanings of causes are collected one level below and attached up from DL to SL with a common purpose behind.

In CL, only the severity analysis is performed for DL by originated design analysis. In figure 3.3, the 'sensor does not detect metals' the failure effect is indicated most crucial of severity 10. The catalog of the used standard is utilized to evaluate occurrence and detection, such as SAEJ1739 or VDA, etc.

Effect Level								
No	Function	Pot.failure	Pot. effect	S	Cause	O	Prev. / Det. Action	D
EL1	Determine the wheel speed	Signal has not been provided	Velocity cannot be determined	10	No signal provided			

System Level								
No	Function	Pot. failure	Pot. effect	S	Cause	O	Prev./ Det. Action	D
	Provide periodic signal	No signal provided	Velocity cannot be determined	10	Sensor does not detect metals	3	D: check plausible values	3

Design level								
No	Function	Potential failure	Potential effect	S	Cause	O	Preventive / Detective Action	D
DL3	Inductive sensor	Sensor does not detect metals	Velocity cannot be determined	10	cable cut	2	P: Assembly instruction	2

Cause level								
No	Function	Potential failure	Potential effect	S	Cause	O	Preventive / Detective Action	D
CL3	Failures of	cable cut	Velocity cannot be determined	10				

Figure 3.3 Cause linking from top to bottom [75]

3.2 Proposed Fuzzy Hierarch Failure Mode and Effect Analysis

In the proposed Fuzzy Hierarchical Failure Mode and Effect Analysis (FH-FMEA) method, the Mamdani-type inference is applied practically at two levels - System Level (see Table 3.2) and Design Level (see Table 3.3).

It is essential to choose membership functions for the fuzzy logic process. The fuzzification first sub-process converts identified input and output variables into linguistic expressions. The suggested FH-FMEA method manages three data classified for fuzzy input and output parameters.

Next, given linguistic terms determine membership degrees to the specified input and output variables. Following that, the output variable's membership degrees take place with the settled rule base. In the final sub-process, the defuzzification method transforms the fuzzy results into a crisp.

In table 3.5., the Severity (*S*) expresses the severity of the failure, and the membership functions variables are linguistically defined by Low (*S1*), Medium (*S2*), and High (*S3*). The Occurrence (*O*) considers the failure occurrence and the membership function variables linguistically described by Improbable (*O1*), Occasional(*O2*), and Probable (*O3*). Finally, the Detectability (*D*) declares the detectability of failure and the membership function variables explained by Detectable easy (*D1*), Detectable (*D2*), and Detectable with Difficulty (*D3*). The input factors that create the risk priority coefficient is determined by choosing a value interval [0,10] based on the failure type selected to be used in the *RPN* calculations in the FMEA worksheet created.

<i>S</i>	<i>S1</i>	Low
	<i>S2</i>	Medium
	<i>S3</i>	High
<i>O</i>	<i>O1</i>	Improbable
	<i>O2</i>	Occasional
	<i>O3</i>	Probable
<i>D</i>	<i>D1</i>	Detectable Easily
	<i>D2</i>	Detectable
	<i>D3</i>	Detectable with Difficulty

Table 3.5 Input Membership Functions

During a fuzzy risk analysis, the *RPN* membership function is described linguistically given in Table 3.6 by Action unnecessary (*R1*), Action Suggested (*R2*), Action Needful (*R3*), Action is Very Needful (*R4*).

<i>RPN</i>	<i>R1</i>	Action is Unnecessary
	<i>R2</i>	Action is Suggested
	<i>R3</i>	Action is Needful
	<i>R4</i>	Action is Very Needful

Table 3.6 Output Membership Functions

The designed FH-FMEA worksheet should be rule-based on experiences after the fuzzification sub-process. Each entry is organized according to the linguistically expressive regions in this sub-

process. The obtained values provide an output based on the fuzzy inference process by the created rule base.

3.3 Case Study

The same categories are applied for the hierarchical levels throughout the preliminary risk assessment. In the fuzzy method, ordinary human thinking commonly utilizes the interval [0, 10]. It differs from the frequently used traditional FMEA boundary numbers that ranged [1, 10], which computes the *RPN* by multiplication (see equation 2.1).

In both cases (SL and DL), the inputs are *S*, *O*, and *D*, which define the range of system failures, and the *RPN* is the predicted output. The trapezoidal membership function (see equation 2.8) is suitable for these parameters, with the linguistic description shown in Table 3.7.

<i>S</i>	<i>C1</i>	Low	$\mu_{Low} = f: (0,0,1,3)$
	<i>C2</i>	Medium	$\mu_{Med} = f: (1,3,4,6)$
	<i>C3</i>	High	$\mu_H = f: (4,6,10,10)$
<i>O</i>	<i>O1</i>	Improbable	$\mu_{Imp} = f: (0,0,1,3)$
	<i>O2</i>	Occasional	$\mu_{Occ} = f: (1,3,4,6)$
	<i>O3</i>	Probable	$\mu_{Pro} = f: (4,6,10,10)$
<i>D</i>	<i>D1</i>	Detectable Easily	$\mu_E = f: (0,0,1,3)$
	<i>D2</i>	Detectable	$\mu_{Det} = f: (1,3,4,6)$
	<i>D3</i>	Detectable with Difficulty	$\mu_{Dif} = f: (4,6,10,10)$
<i>RPN</i>	<i>R1</i>	Action is Unnecessary	$\mu_{Unn} = f: (0,0,1,3)$
	<i>R2</i>	Action is Suggested	$\mu_{Sug} = f: (1,3,4,6)$
	<i>R3</i>	Action is Needful	$\mu_N = f: (4,6,7,9)$
	<i>R4</i>	Action is Very Needful	$\mu_{VN} = f: (7,9,10,10)$

Table 3.7 Membership Functions of F-FMEA

In Table 3.8, the rule base is given, which was created based on experimental work. Eleven rule lines are considered properly in the fuzzy inference process.

No	Rule
{1}	$S1 \cap O1 \cap D1 \Leftrightarrow R1$
{2}	$S1 \cap O2 \cap D1 \Leftrightarrow R2$
{3}	$S1 \cap O1 \cap D2 \Leftrightarrow R2$
{4}	$S1 \cap O2 \cap D2 \Leftrightarrow R3$
{5}	$S2 \cap O1 \cap D1 \Leftrightarrow R2$
{6}	$S2 \cap O2 \cap D1 \Leftrightarrow R3$
{7}	$S2 \cap O1 \cap D2 \Leftrightarrow R3$
{8}	$S2 \cap O2 \cap D2 \Leftrightarrow R3$
{9}	$S3 \Leftrightarrow R4$
{10}	$O3 \Leftrightarrow R4$
{11}	$D3 \Leftrightarrow R4$

Table 3.8 The Rule Base of FMEAs

Finding the definition ranges corresponding to the specified input values is necessary. Then, in any rule line based on *AND* operator used in the system, the operator provides the output as long as the three-input values match, as found in the definition ranges. However, when this situation is based on the *OR* operator, it will be sufficient for a single value to appear from the definition ranges.

In the calculation of *RPN*, when the Action is Very Needful (*R4*) - High (*C3*), Probable (*O3*), and Detectable with Difficulty (*D3*). Therefore, if one of these possibilities occur, the risk is always the highest and evaluated separately in the last rule lines.

Let's take SL1 (see table 3.2) as an example for the FH-FMEAs; the input values taken during the risk determination were determined as

$$\text{Severity: 5, Occurrence: 4, and Detectability: 2} \quad (3.1)$$

Accordingly, *RPN* output values are calculated with the application steps of fuzzy logic as follows.

Table 3.9 shows the rule line of FH-FMEA's; in this composition sub-process, *IF-THEN* structured, to aggregate them, the minimum *AND* operator connections used (see equation 2.28).

The positional information and membership values for *Severity:5*, are given in Figure 3.4.

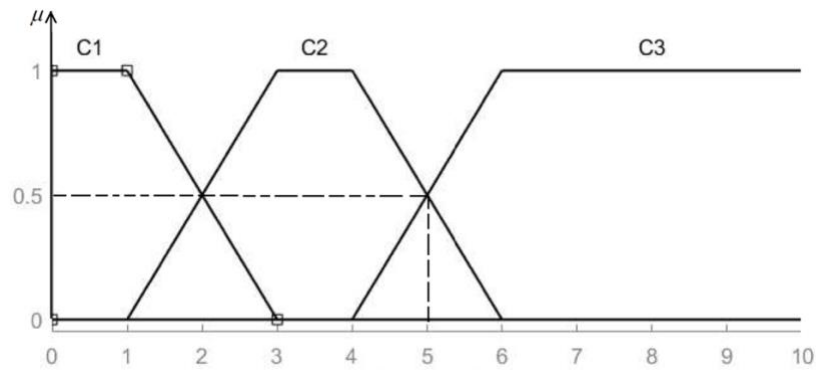


Figure 3.4 Determination of the value of the Severity

The positional information and membership values for *Occurrence:4*, are indicated in Figure 3.5.

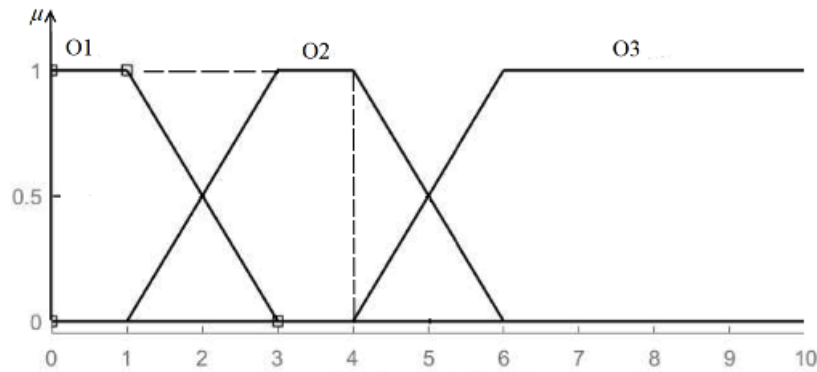


Figure 3.5 Determination of the value of the Occurrence

The positional information and membership values for *Detectability: 2* are shown in Figure 3.6.

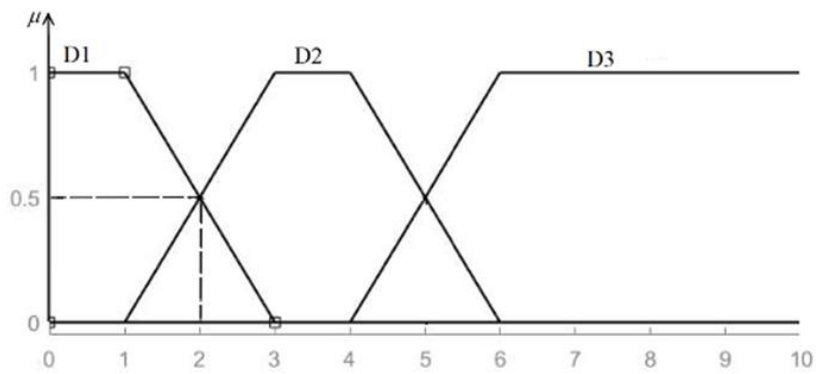


Figure 3.6 Determination of the value of the Detectability

The membership degrees of each input value are found, the rules activated by the input values in the rule table are determined. This state is made up for the composition process and presented in Table 3.9.

No	μ_{Si}	μ_{Oi}	μ_{Di}	Rule	μ_{Rj}
{1}	0	0	0.5	$S1 \cap O1 \cap D1 \Rightarrow R1$	0
{2}	0	1	0.5	$S1 \cap O2 \cap D1 \Rightarrow R2$	0
{3}	0	0	0.5	$S1 \cap O1 \cap D2 \Rightarrow R2$	0
{4}	0	1	0.5	$S1 \cap O2 \cap D2 \Rightarrow R3$	0
{5}	0.5	0	0.5	$S2 \cap O1 \cap D1 \Rightarrow R2$	0
{6}	0.5	1	0.5	$S2 \cap O2 \cap D1 \Rightarrow R3$	0.5
{7}	0.5	0	0.5	$S2 \cap O1 \cap D2 \Rightarrow R3$	0
{8}	0.5	1	0.5	$S2 \cap O2 \cap D2 \Rightarrow R3$	0.5
{9}	0.5	–	–	$S3 \Rightarrow R4$	0.5
{10}		0	–	$O3 \Rightarrow R4$	0
{11}			0	$D3 \Rightarrow R4$	0

Table 3.9 F-FMEA Active Rule Schedule

After the membership degrees were found and reflected in each rule base, the maximum operator (see equation 2.30) was used for the aggregation process.

$$\begin{aligned} \mu_{R1} = 0 & \quad ; \quad \mu_{R2} = 0 \\ \mu_{R3} = 0.5 & \quad ; \quad \mu_{R4} = 0.5 \end{aligned} \quad (3.2)$$

The membership function's *RPN* parameters and the defuzzification method's final step are shown in figure 3.7, which we apply here to the Centroid method (see equation 2.31).

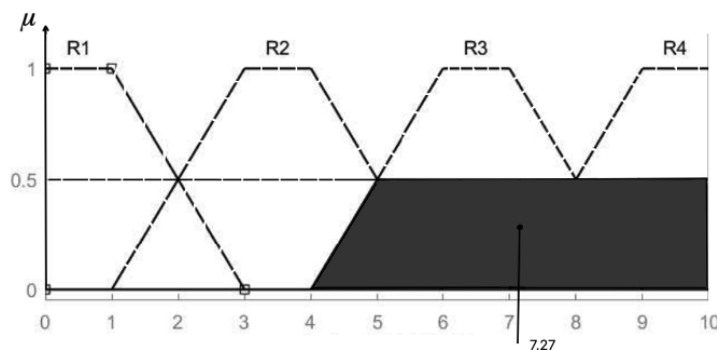


Figure 3.7 Defuzzification result for SL1

Acknowledging the [5] work, FMEA worksheets of wheel speed sensor is treated hierarchically by following them separately as EL, SL, DL, and CL (see Tables 3.1-3.4). The SL and DL findings in Tables 3.10-3.11 are obtained by applying the fuzzy rule-based steps outlined above with the COG defuzzification method.

No	<i>S</i>	<i>O</i>	<i>D</i>	<i>FRPN</i>	<i>Rel_FRPN</i>
SL1	5	4	2	7.27	12.27
SL2	5	2	3	7.27	12.27
SL3	10	3	3	8.94	15.09
SL4	8	2	2	8.94	15.09
SL5	8	2	3	8.94	15.09
SL6		2	3	8.94	15.09
SL7	9	1	2	8.94	15.09

Table 3.10 FH-FMEA of System Level

No	<i>S</i>	<i>O</i>	<i>D</i>	<i>FRPN</i>	<i>Rel_FRPN</i>
DL1	5	2	3	7.27	23.51
DL2	5	1	2	5.77	18.66
DL3	10	2	2	8.94	28.91
DL4	10	2	2	8.94	28.91

Table 3.11 FH-FMEA of Design level

After the data were obtained for the FH-FMEA model, conventional FMEA method results were compared to analyze the difference.

Following the comparison, Figure 3.8 shows that the risk effect is higher at the SL of FH-FMEA in the case of SL2, S4, and SL7. However, it gives a significantly lower value at the SL3 while SL1, SL5, and SL6 generate close results.

The observation of "clear" numerical outcomes confirms an advantage of FH-FMEA. It can also consolidate other quality management requirements commonly used with the fuzzy rule base in the automotive industry. For instance,

$$\text{"IF one of the determinants is greater THAN 5, Action is Very Needful."} \quad (3.3)$$

It can be observed in the instance of SL3, where Severity is 10, but because the other two variables have low values, RPN (hence *Rel_RPN*) of the conventional FMEA is relatively small.

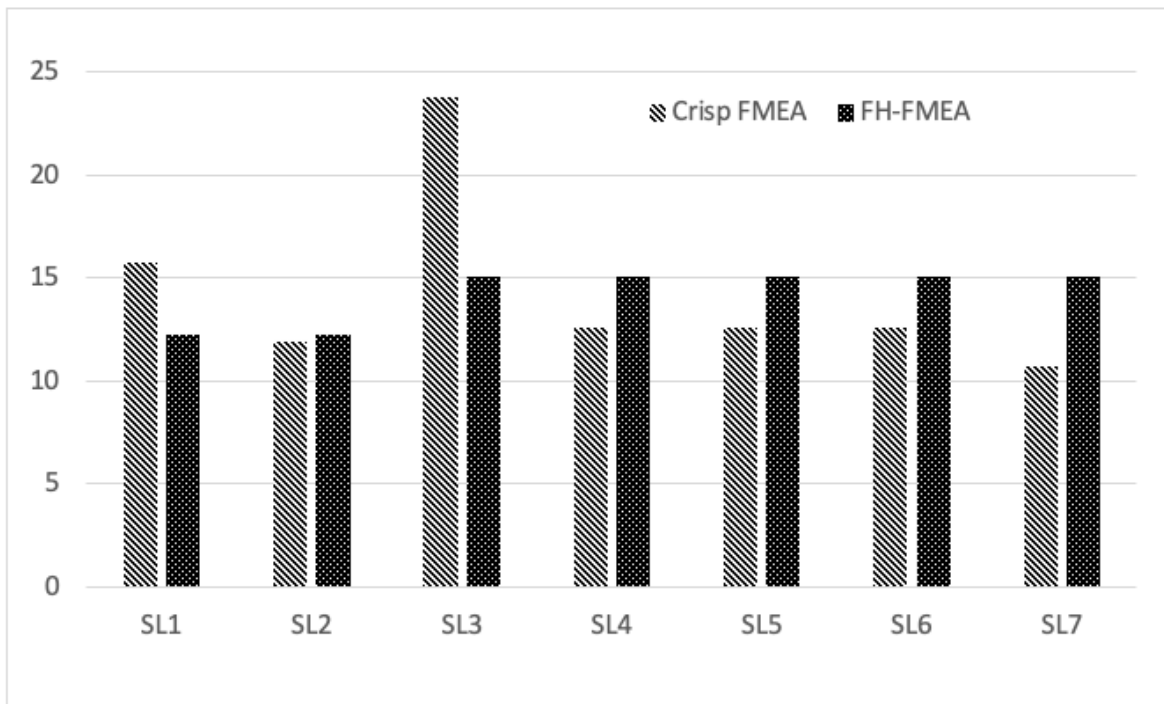


Figure 3.8 Comparison of *Rel_RPNs* of Conventional and FH-FMEA at System Level

The similar result is obtained for the Design level. Thus, for example, FH-FMEA results are usually lower, but a specific case in DL2, where this technique works a higher impact. The severity of DL2 is 5, which is the cutoff value of the criteria mentioned above, but the other two determinants are small. As a result, the fuzzy rule based FMEA can effectively manage this "borderline" issue. The classical FMEA uses subjective, but crisp *S*; *O*; *D* data end determines their multiplication. The fuzzy FMEA can characterize subjectivity of experts by fuzzy membership functions

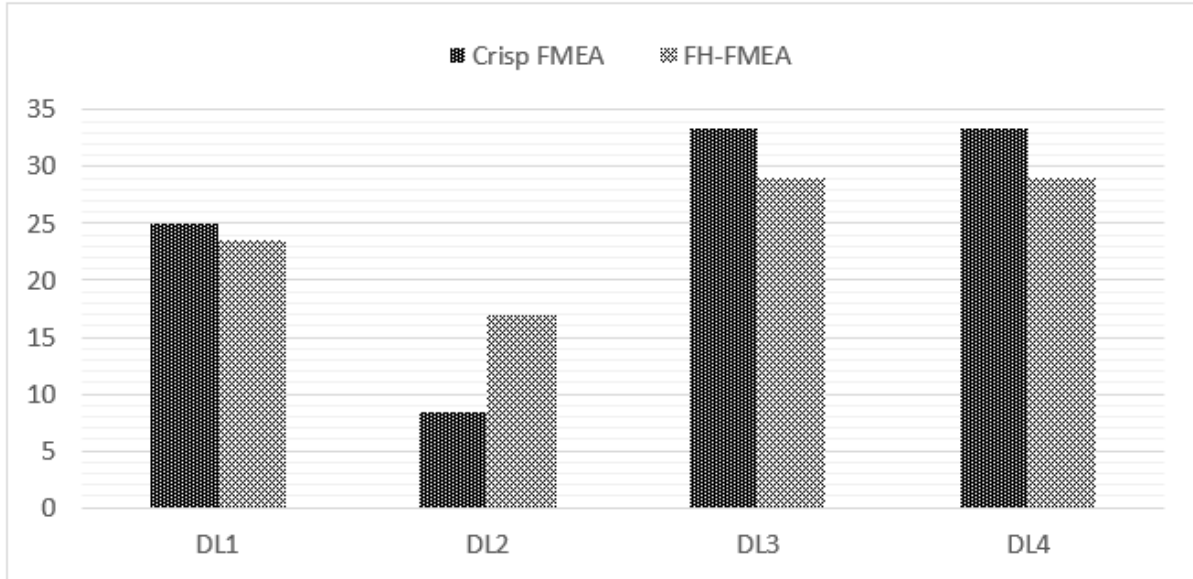


Figure 3.9 Comparison of *Rel_RPNs* of Conventional and FH FMEA at Design Level

3.4 Summary

The fundamental idea in the FMEA distribution is the determination of a Risk Priority Number. Hence, the cognitive evaluations the ranking of the concerned RPN values to submit alternatives for increasing the reliability of the tested system, which should be evaluated routinely.

I proposed an extension of the crisp H-FMEA method, which is used to define, classify, and assess risk factors. The hierarchical structure decreases complexity and, as a result, increases efficiency while also ensuring more transparency and extensibility in the system model. The fuzzy approach improves the model's reliability by allowing the system to operate with subjectivity in the data and evaluation process.

These numerical results clearly show that RPN is advantageous to determine by fuzzy rule inference. In this way, fuzzy sets can model the expert's thought with subjectivity rules and the defuzzification method. Furthermore, having a well-defined rule framework makes it simple to ensure the surface of appropriate quality management requirements.

4 LEVEL-SPECIFIC EVALUATION-BASED FUZZY HIERARCHICAL FAILURE MODE AND EFFECT ANALYSIS

In light of Industry 4.0 and the increasing complexity of engineering goods and planned systems, engineering systems qualified functions have become even more critical than before. As a result, safety and reliability must be considered more thoroughly while designing and developing engineering systems.

The mathematical models have proven how useful they are in engineering systems together with fuzzy sets. For example, FMEA, a well-established concept in reliability analysis, has been developed by adopting fuzzy sets to provide more accurate results.

Fuzzy-based prediction models can be used in manufacturing to substitute time- and cost-consuming measurements. The study by Tóth-Laufer and Horváth shows that predicting the quality elements of the manufactured workpiece is critical [77].

Furthermore, the fuzzy approach can be helpful when making a decision based on the opinion of experts who aren't objective [78]. An excellent example of this is the Lukács's study, which uses fuzzy rules to evaluate data on airborne noise [79]. The author's goal was to examine acoustic perceptibility in situations where insufficient data is available.

The short reaction time is also a significant concern in real-time and adaptive systems. This requirement can be a limitation of Mamdani-type fuzzy inference due to the high computational conditions of the defuzzification step. To lower the computing requirements in Mamdani-type fuzzy control, Dombi and Tóth-Laufer recommended various reduction techniques [80]. As a result, the authors gave a faster solution in the cases where a short reaction time is needed while maintaining the advantageous features of the Mamdani model.

Ozguney and Burkan worked on a flexible link robot manipulator's rotation tracking control using fuzzy logic. The authors have considered two important parameters: tip deflection and angle of the link. They have used the law of fuzzy logic control to develop a controller to determine control gain parameters in the sliding mode control [81]. Consequently, applying the fuzzy logic controller is proven effective even in the system's external disorders presence and parametric uncertainties.

The previous study successfully emphasizes FH-FMEA, which worked out first in the paper [6], where the membership functions of levels are the same.

This investigation aims to present a methodological approach for implementing FH-FMEA for level-specific membership functions, where the membership functions remodel depending on the level - as a development of the previous study and paper [82]. Moreover, to ensure advanced reliability of the system, the data between the levels is transferred at the condition of fuzzy numbers.

4.1 Proposed Level-Specific Evaluation-based FH-FMEA

In the original model of FH-FMEA, the corresponding membership functions are adopted at each level to analyze the evaluation. The proposed risk analysis method's novelty is applying different membership functions at different levels; furthermore, the parameters are transferred as a fuzzy number instead of a crisp value from the SL to the DL.

The fuzzy inference process must be completed for an assessment of the criticality level. The first is to determine the categories of S ; D ; O and RPN , their membership functions, and the rule base for evaluating the RPN value by the analysis team. Next, based on the specific technical considerations, the analysts discuss the case under study and collaboratively define categories, membership functions, and logical rules.

4.1.1 System Level

In the interval [0, 10], the same input and output membership functions are used here, as in Chapter 3 and our paper [83]. However, different parameters were selected for the S fuzzy set to adapt the proposed method in this Chapter.

S	S1	Low	$\mu_{Low} = f: (0,0,2,4)$
	S2	Medium	$\mu_{Med} = f: (2,4,7,9)$
	S3	High	$\mu_H = f: (7,9,10,10)$
O	O1	Improbable	$\mu_{Imp} = f: (0,0,1,3)$
	O2	Occasional	$\mu_{Occ} = f: (1,3,4,6)$
	O3	Probable	$\mu_{Pro} = f: (4,6,10,10)$
D	D1	Detectable Easily	$\mu_E = f: (0,0,1,3)$
	D2	Detectable	$\mu_{Det} = f: (1,3,4,6)$
	D3	Detectable with Difficulty	$\mu_{Dif} = f: (4,6,10,10)$
RPN	R1	Action is Unnecessary	$\mu_{Unn} = f: (0,0,1,3)$
	R2	Action is Suggested	$\mu_{Sug} = f: (1,3,4,6)$
	R3	Action is Needful	$\mu_N = f: (4,6,7,9)$
	R4	Action is Very Needful	$\mu_{VN} = f: (7,9,10,10)$

Table 4.1 Membership Function for System Level

After identified membership functions, in Figures 4.1-4.3, trapezoidal membership functions are also applied here, as they are simpler to present in risk assessment. The categories name that have also been used as in section 3, show a system of fuzzy sets for input and output with trapezoidal with different parameters.

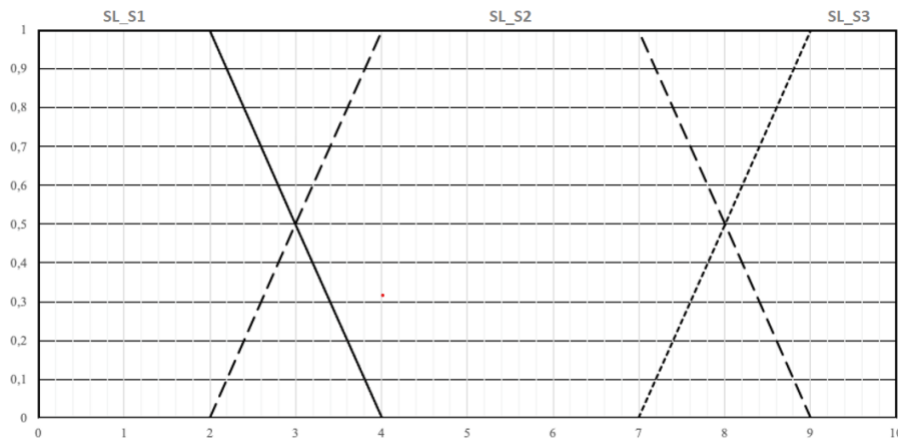


Figure 4.1 Membership Functions “Severity” Categories (System Level)

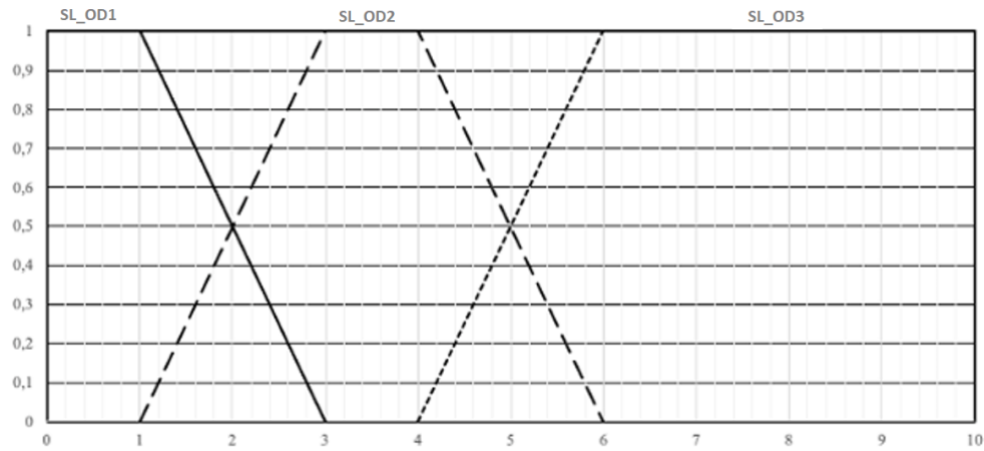


Figure 4.2 Membership Functions “Occasional and Detectability” Categories (System Level)

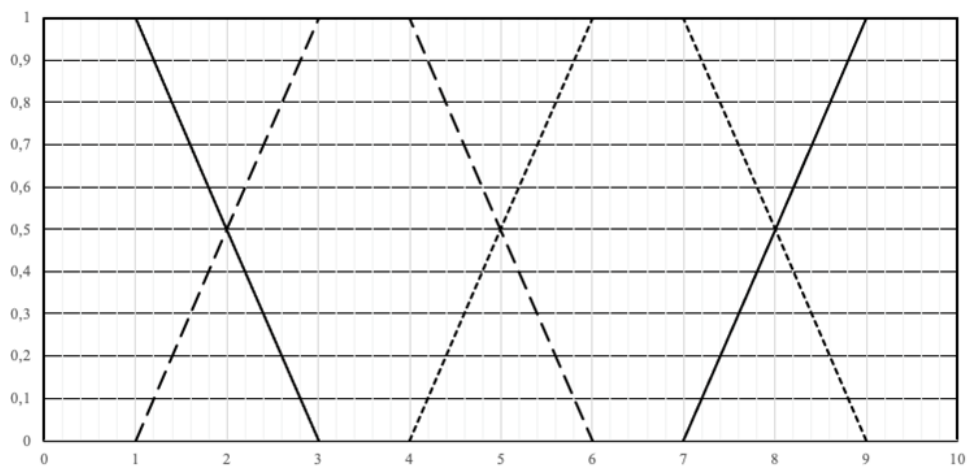


Figure 4.3 Membership Functions “F_RPN” Categories

As can be seen from the shape of the membership functions in the above figures, different parameters have been adopted here for System Level.

As an example, let's use SL4 of WSS's Level-specified Fuzzy Hierarchical Failure Mode and Effect Analysis (LsFH-FMEA),

$$\text{Severity: 8, Occurrence: 2, and Detectability: 2.} \quad (4.1)$$

No	μ_{Si}	μ_{Oi}	μ_{Di}	Rule	μ_{Rj}
{1}	0	0.5	0.5	$S1 \cap O1 \cap D1 \Rightarrow R1$	0
{2}	0	0.5	0.5	$S1 \cap O2 \cap D1 \Rightarrow R2$	0
{3}	0	0.5	0.5	$S1 \cap O1 \cap D2 \Rightarrow R2$	0
{4}	0	0.5	0.5	$S1 \cap O2 \cap D2 \Rightarrow R3$	0
{5}	0.5	0.5	0.5	$S2 \cap O1 \cap D1 \Rightarrow R2$	0.5
{6}	0.5	0.5	0.5	$S2 \cap O2 \cap D1 \Rightarrow R3$	0.5
{7}	0.5	0.5	0.5	$S2 \cap O1 \cap D2 \Rightarrow R3$	0.5
{8}	0.5	0.5	0.5	$S2 \cap O2 \cap D2 \Rightarrow R3$	0.5
{9}	0.5	–	–	$S3 \Rightarrow R4$	0.5
{10}		0	–	$O3 \Rightarrow R4$	0
{11}			0	$D3 \Rightarrow R4$	0

Table 4.2 Calculation of F-FMEA (System Level)

As can be seen, the membership values are aggregated by Max operator (see equation 2.30) as follows:

$$\begin{aligned} \mu_{R1} &= 0.0 & ; & & \mu_{R2} &= 0.5 \\ \mu_{R3} &= 0.5 & ; & & \mu_{R4} &= 0.5 \end{aligned} \quad (4.2)$$

The last subprocess is the defuzzification using the Centroid method:

$$F_RPN = 5.77 \quad (4.3)$$

4.1.2 Design Level

Different membership functions enable the representation of level technicalities, while the fuzzy method allows the mathematical modeling of the estimator's subjective opinions. In addition, communicating input parameters as a fuzzy number can send uncertain information between levels, having a more significant effect on F_RPN .

In table 4.3., the DL of the membership function of S fuzzy sets is considered. Here, only the S membership functions designed for SL are different. In addition, note that DL's membership functions and fuzzy sets are the same as in Chapter 3.

S	S1	Low	$\mu_{Low} = f: (0, 0, 1, 3)$
	S2	Medium	$\mu_{Med} = f: (1, 3, 4, 6)$
	S3	High	$\mu_H = f: (4, 6, 10, 10)$

Table 4.3 Categorized Membership Function of Severity (Design Level)

SL to DL, the parameters are conducted as a fuzzy number rather than a crisp integer when using different membership functions. The fuzzified input values obtained from the SL of S are connected to the DL of S . For each impacted SL membership function, fuzzy numbers to be transferred to the DL can be set by (equation 2.24). An example of figures 4.4-4.6 shows the severity change from the SL to the DL as a fuzzy number.

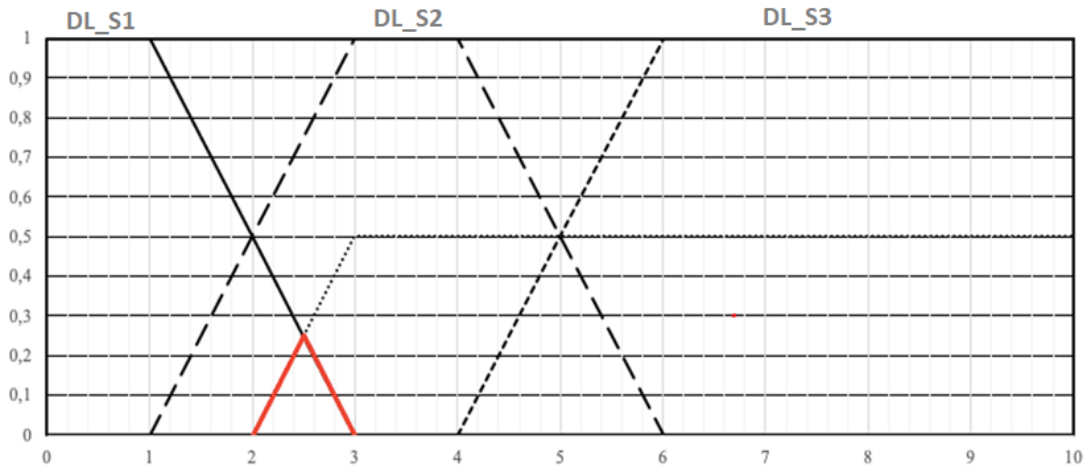


Figure 4.4 Determination of μ_{S1} (Design Level)

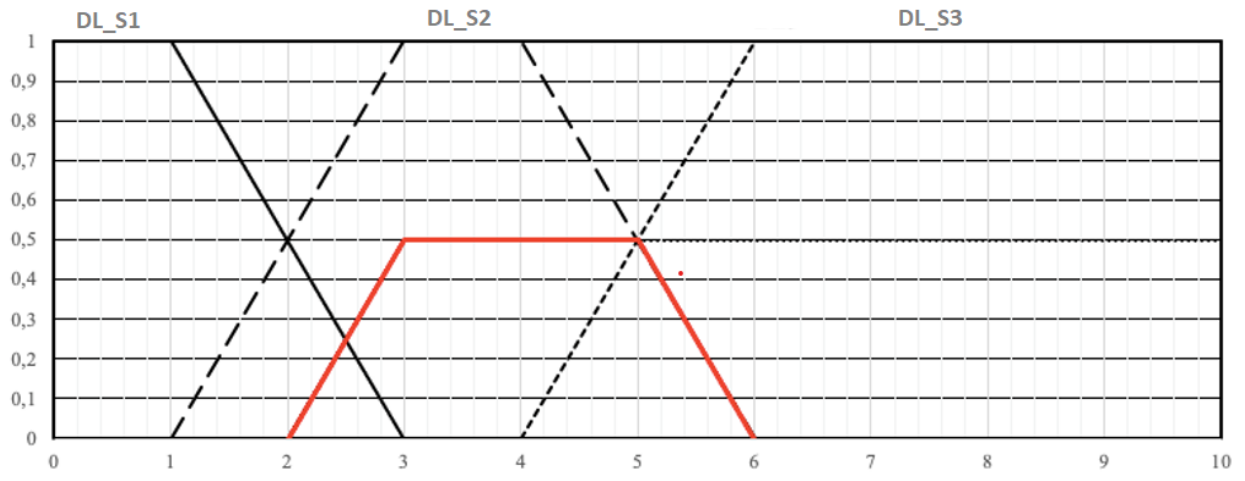


Figure 4.5 Determination of μ_{S2} (Design Level)

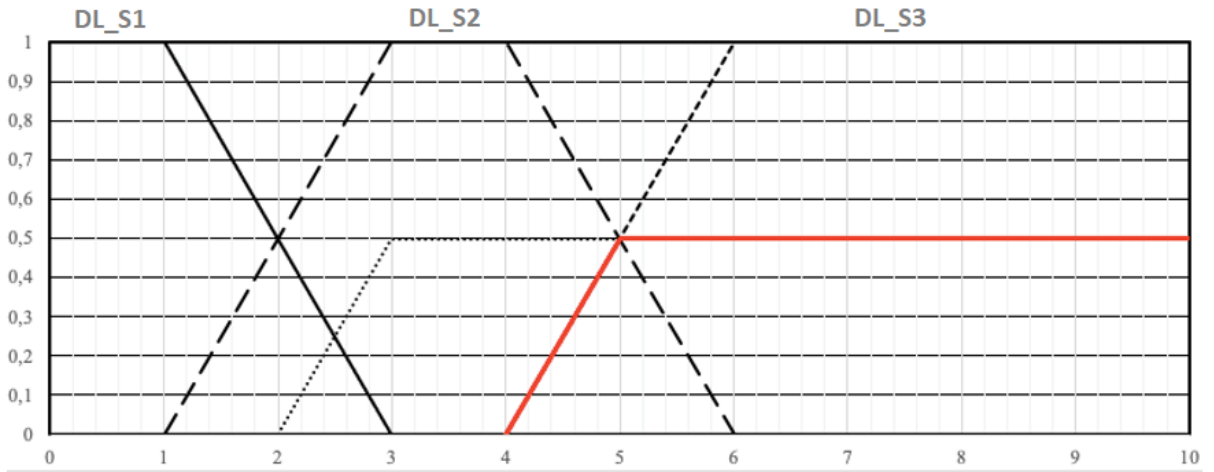


Figure 4.6 Determination of μ_{S3} (Design Level)

Table 4.4 depicts the Design level's calculation based on the system Level's fuzzy rule-base result and the inputs listed above.

No	μ_{Si}	μ_{Oi}	μ_{Di}	Rule	μ_{Rj}
{1}	0.25	0.5	0.5	$S1 \cap O1 \cap D1 \Rightarrow R1$	0.25
{2}	0.25	0.5	0.5	$S1 \cap O2 \cap D1 \Rightarrow R2$	0.25
{3}	0.25	0.5	0.5	$S1 \cap O1 \cap D2 \Rightarrow R2$	0.25
{4}	0.25	0.5	0.5	$S1 \cap O2 \cap D2 \Rightarrow R3$	0.25
{5}	0.5	0.5	0.5	$S2 \cap O1 \cap D1 \Rightarrow R2$	0.5
{6}	0.5	0.5	0.5	$S2 \cap O2 \cap D1 \Rightarrow R3$	0.5
{7}	0.5	0.5	0.5	$S2 \cap O1 \cap D2 \Rightarrow R3$	0.5
{8}	0.5	0.5	0.5	$S2 \cap O2 \cap D2 \Rightarrow R3$	0.5
{9}	0.5	–	–	$S3 \Rightarrow R4$	0.5
{10}		0	–	$O3 \Rightarrow R4$	0
{11}			0	$D3 \Rightarrow R4$	0

Table 4.4 Composition process of LsFH-FMEA

As can be seen, the membership values are obtained as follows,

$$\begin{aligned} \mu_{R1} = 0.25 & \quad ; \quad \mu_{R2} = 0.5 \\ \mu_{R3} = 0.5 & \quad ; \quad \mu_{R4} = 0.5 \end{aligned} \tag{4.4}$$

and demonstrated in figure 4.7.

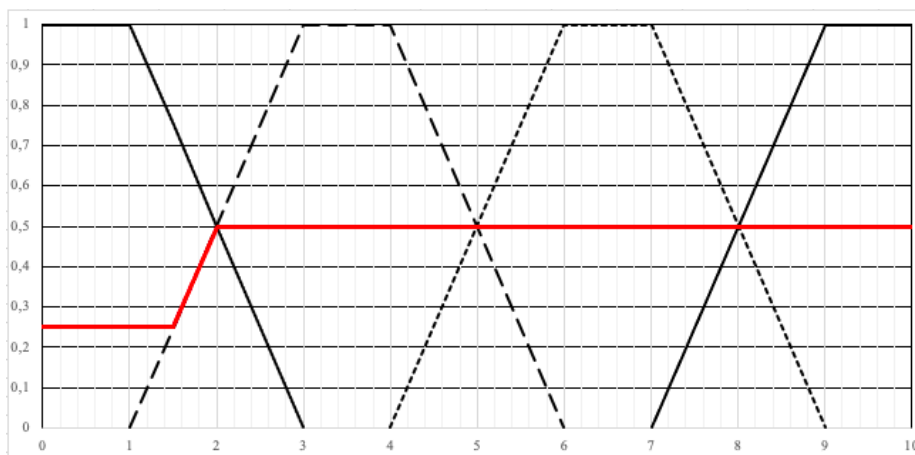


Figure 4.7 Aggregation result

Finally, defuzzification via the Center of Gravity method is performed:

$$F_RPN_{DL} = 5.40 \quad (4.5)$$

The Level-specific Hierarchical Fuzzy FMEA result demonstrates the distinction between the technicalities of SL and DL.

No	Case 1: $\mu_{Si}:5, \mu_{Oi}:4, \mu_{Di}:2$							Case 2: $\mu_{Si}:5, \mu_{Oi}:2, \mu_{Di}:3$							Case 3: $\mu_{Si}:10, \mu_{Oi}:3, \mu_{Di}:3$							
	SL			DL				SL			DL				SL			DL				
	μ_{Si}	μ_{Oi}	μ_{Di}	μ_{Si}	μ_{Oi}	μ_{Di}	μ_{Rj}	μ_{Si}	μ_{Oi}	μ_{Di}	μ_{Si}	μ_{Oi}	μ_{Di}	μ_{Rj}	μ_{Si}	μ_{Oi}	μ_{Di}	μ_{Si}	μ_{Oi}	μ_{Di}	μ_{Rj}	
1	0	0	0.5	0.25	0	0.5	0	0	0.5	0	0.25	0.5	0	0	0	0	0	0	0	0	0	0
2	0	1	0.5	0.25	1	0.5	0.25	0	0.5	0	0.25	0.5	0	0	0	1	1	0	1	1	0	0
3	0	0	0.5	0.25	0	0.5	0	0	0.5	1	0.25	0.5	1	0.25	0	0	0	0	0	0	0	0
4	0	1	0.5	0.25	1	0.5	0.25	0	0.5	1	0.25	0.5	1	0.25	0	1	1	0	1	1	0	0
5	1	0	0.5	1	0	0.5	0	1	0.5	0	1	0.5	0	0	0	0	0	0	0	0	0	0
6	1	1	0.5	1	1	0.5	0.5	1	0.5	0	1	0.5	0	0	0	1	1	0	1	1	0	0
7	1	0	0.5	1	0	0.5	0	1	0.5	1	1	0.5	1	0.5	0	0	0	0	0	0	0	0
8	1	1	0.5	1	1	0.5	0.5	1	0.5	1	1	0.5	1	0.5	0	1	1	0	1	1	0	0
9	0	-	-	1	-	-	1	0	-	-	1	-	-	1	1	-	-	1	-	-	1	1
10		0	-		0	-	0		0	-		0	-	0		0	-		0	-	0	0
11			0			0	0			0			0	0			0			0	0	0
<i>Aggregation</i>			$\mu_{R1}=0; \mu_{R2}=0.25$ $\mu_{R3}=0.5; \mu_{R4}=1$				$\mu_{R1}=0; \mu_{R2}=0.25$ $\mu_{R3}=0.5; \mu_{R4}=1$				$\mu_{R1}=0; \mu_{R2}=0$ $\mu_{R3}=0; \mu_{R4}=1$											
<i>Defuzzification</i>			6.84				6.84				8.94											

Table 4.5 Calculation of LsFH-FMEA

The case 4, SL to DL sample was presented in detail before the case study. The table 4.6 shows S5, SL6, and SL7.

No	Case 5: $\mu_{Si}:5, \mu_{Oi}:4, \mu_{Di}:2$							Case 6: $\mu_{Si}:5, \mu_{Oi}:2, \mu_{Di}:3$							Case 7: $\mu_{Si}:10, \mu_{Oi}:3, \mu_{Di}:3$						
	SL			DL				SL			DL				SL			DL			
	μ_{Si}	μ_{Oi}	μ_{Di}	μ_{Si}	μ_{Oi}	μ_{Di}	μ_{Rj}	μ_{Si}	μ_{Oi}	μ_{Di}	μ_{Si}	μ_{Oi}	μ_{Di}	μ_{Rj}	μ_{Si}	μ_{Oi}	μ_{Di}	μ_{Si}	μ_{Oi}	μ_{Di}	μ_{Rj}
1	0	0.5	0	0.25	0.5	0	0	0	0.5	0	0.25	0.5	0	0	0	1	0.5	μ_{Si}	μ_{Oi}	μ_{Di}	0
2	0	0.5	0	0.25	0.5	0	0	0	0.5	0	0.25	0.5	0	0	0	0	0.5	0	1	0.5	0
3	0	0.5	1	0.25	0.5	1	0.25	0	0.5	1	0.25	0.5	1	0.25	0	1	0.5	0	0	0.5	0
4	0	0.5	1	0.25	0.5	1	0.25	0	0.5	1	0.25	0.5	1	0.25	0	0	0.5	0	1	0.5	0
5	0.5	0.5	0	0.5	0.5	0	0	0.5	0.5	0	0.5	0.5	0	0	0	1	0.5	0	0	0.5	0
6	0.5	0.5	0	0.5	0.5	0	0	0.5	0.5	0	0.5	0.5	0	0	0	0	0.5	0	1	0.5	0
7	0.5	0.5	1	0.5	0.5	1	0.5	0.5	0.5	1	0.5	0.5	1	0.5	0	1	0.5	0	0	0.5	0
8	0.5	0.5	1	0.5	0.5	1	0.5	0.5	0.5	1	0.5	0.5	1	0.5	0	0	0.5	0	1	0.5	0
9	0.5	-	-	0.5	-	-	0.5	0.5	-	-	0.5	-	-	0.5	1	-	-	0	0	0.5	1
10		0	-		0	-	0		0	-		0	-	0		0	-	1	-	-	0
11			0			0	0			0			0	0			0		0	-	0
<i>Aggregation</i>			$\mu_{R1}=0; \mu_{R2}=0.25$ $\mu_{R3}=0.5; \mu_{R4}=0.5$				$\mu_{R1}=0; \mu_{R2}=0.25$ $\mu_{R3}=0.5; \mu_{R4}=0.5$				$\mu_{R1}=0; \mu_{R2}=0$ $\mu_{R3}=0; \mu_{R4}=1$										
<i>Defuzzification</i>			6.31				6.31				8.94										

Table 4.6 Calculation of LsFH-FMEA

The F-FMEA model is finalized to adapt the level-specified hierarchical model appropriately. Severity's membership functions are modified in DL, and fuzzy input numbers are obtained and transferred to the SL failures.

In Table 4., the original FH-FMEA that is obtained in 3 Chapter, and LsFH-FMEA results are shown.

No	S	O	D	FH_RPN	LFH_RPN	Relative FH_RPN	Relative LFH_RPN
SL1	5	4	2	7.27	6.84	12.77	13.80
SL2	5	2	3	7.27	6.84	12.77	13.80
SL3	10	3	3	8.94	8.94	15.09	18.03
SL4	8	2	2	8.94	5.40	15.09	10.89
SL5	8	2	3	8.94	6.31	15.09	12.73
SL6		2	3	8.94	6.31	15.09	12.73
SL7	9	1	2	8.94	8.94	15.09	17,83

Table 4.7 System Levels of FH-HMEA, and LsFH-FMEA

Figure 4.8 illustrates the failure of System-Level, LsFH-FMEA is higher relative RPN than FH-FMEA in most cases; however, for SL4, it generates a significantly lower RPN value. Consequently, it is observed that LsFH-FMEA makes more solid results when the relative percentages are considered.

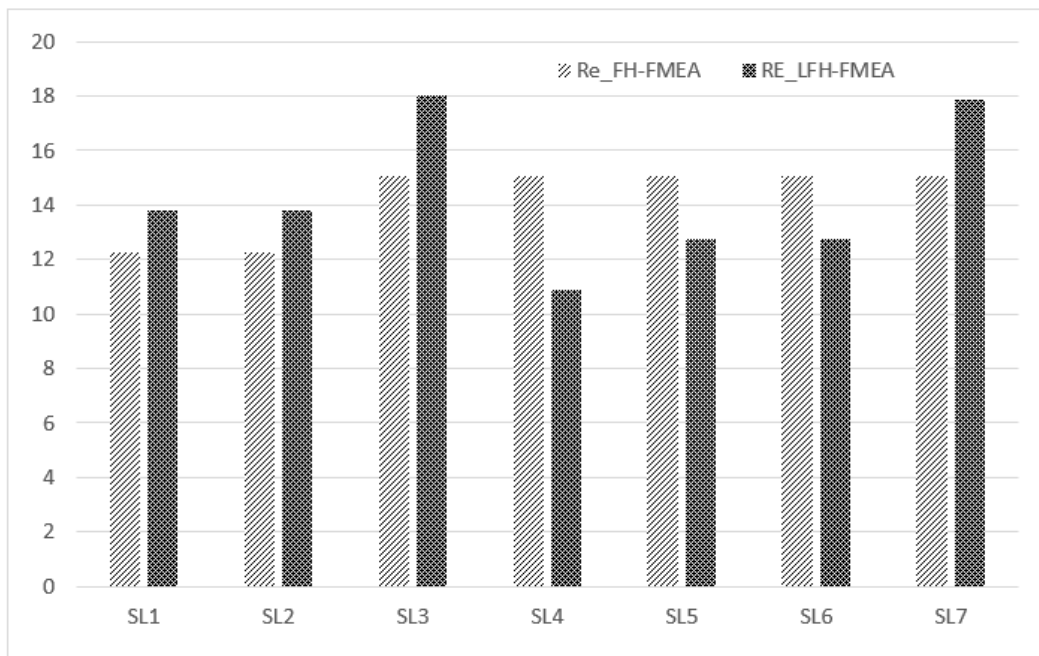


Figure 4.8 Comparison relative RPN result

4.2 Conclusion

Safety and reliability are critical aspects in the design and development of engineering systems. However, a fuzzy approach is required to achieve a strong result due to subjectivity and uncertainty.

I proposed a new level-specific evaluation-based hierarchical fuzzy failure mode and effect analysis model in which different membership functions can be applied at different levels. At the same time, from the SL to DL, the parameters are transmitted as a fuzzy number. In this way, different technicalities of levels can be presented. Furthermore, transmitting inputs as fuzzy numbers helps to transmit uncertain information between levels.

The proposed system has the following advantages:

- different membership functions enable the representation of level technicalities
- fuzzy technique enables qualitative and quantitative modeling of the estimator's subjective opinions
- uncertain information can be transmitted between levels with a higher effect on F_RPN by transferring input parameters as a fuzzy number.

As a result, during preliminary risk assessment, the proposed level-specific, fuzzy rule-based technique can be performed in the situation of different technicalities of hierarchical levels and subjectivities of examiners.

5 FUZZY RULE-BASED FAILURE MODE AND EFFECT ANALYSIS WITH SUMMATIVE DEFUZZIFICATION METHODS

Today, due to rising quality standards, it is now more important than ever to optimize the production process. Technological advancements have made it feasible to increase the quality and reliability of the manufactured workpiece as well as the manufacturing process. On the other hand, assessing all probable settings and parameters that influence quality standards is expensive, time-consuming, and impossible in some situations. Some problem-solving methodologies, including Lean Manufacturing, Six Sigma Statistical Engineering, are available to address this issue [84]. The FMEA, a widely used engineering risk assessment technique, also achieves this objective [85]. This method provides a systematic quality improvement to prevent possible failures in the system, process, design, or services.

Since the mid-1990s, the Fuzzy Failure Modes and Effect Analysis (F-FMEA) hybridized methodology has been employed to extend the classic FMEA method. Bowles and Peláez have developed the first Fuzzy logic-based Failure Mode, Effects, and Criticality Analysis (FMECA) used to characterize the prioritizing of failures in terms of corrective actions. The scientist proposed two techniques to estimate criticality that are different but connected. The first relies on the traditional RPN calculation, which uses numerical rankings and crisp inputs collected by the user or obtained by reliability analysis. Second, they consider qualitative factors early in the design process, where less detailed data is available, allowing fuzzy inputs and demonstrating the straightforward application of the linguistic rankings specified for RPN calculations [86].

In the literature, numerous case studies have been effectively presented the use of F-FMEA until now. Jakula Balaraju et al.'s recent research proposed valuable obtainments to the Fuzzy-FMEA risk evaluation approach for the Load-Haul-Dumber (LHD) machine [87]. The authors investigated the behavior of LHD's failure in potential failure modes that gave data on different factors, such as the failure modes on the equipment performance, the occurrence reasons for failure modes, current operating machines state, reliable life, etc. Furthermore, assessments of essential management practices or control measures, such as possible design modifications and component replacement, were examined in these investigations to assure the desired level of usability.

Additionally, Kelvin Pun et al. have used F-FMEA to examine new product development in the flexible electronics industry [88]. The purpose of the article was to reduce the risks of producing new goods for high-tech companies in a short amount of time. As a result, the authors created a one-of-a-kind methodology for making the product trustworthy for an extended period by prioritizing failure modes with F-FMEA.

Many different effects are exposed during the use of machinery, tools, and devices, such as wear, corrosion, fatigue, temperature, aging, etc. The importance of maintenance is indisputable to fully perform the process at the time of operation. Bearings are one of the essential elements used in machines. They are critical pieces utilized in the machine-building sector and used to move and spin. Before, bearings were damaged due to material, design, or manufacturing faults. Today, most bearings are damaged due to improper lubrication, contamination, misalignment, assembly error, mis bearing, overloading, and electrical erosion. As a result of these effects, the technical system cannot fulfill the function determined at the design stage, either entirely or partially. They expire earlier than the values calculated during the design phase [89] [90] [91].

This chapter aims to produce a summative-defuzzification method with the F-FMEA model as a case study of the bearing manufacturing process. That is succeeded first by comparing some conventional defuzzification methods, and I propose several novel summative-defuzzification methods, with different combined subprocess results in the fuzzy inference process.

5.1 Conventional Fuzzy Rule Based F-FMEA

Mamdani type inference systems are well suited for system modeling, as they have more intuitive and relatively easy rule bases that experts can create. According to Pokoradi, a general Mamdani type fuzzy inference system FMEA also is modeled to obtain the output data by calculating the input data in four stages: fuzzification, inference (firing strength calculation, and implication), composition, and defuzzification (see figure 5.1) [92].

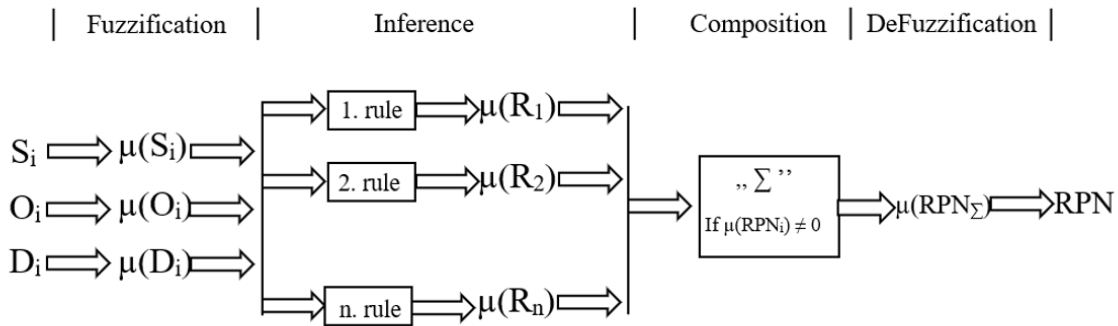


Figure 5.1 Flow chart of Fuzzy logic inference process based on FMEA

A fuzzification process is a unit that converts the crisp values to fuzzy values (remember, it was between 0 and 1) by using the membership function. In other words, we can say that it calculates the membership degree of each input value to the fuzzy set(s).

A fuzzification process is a unit that converts the crisp values to fuzzy values (remember, it was between 0 and 1) by using the membership function. In other words, we can say that it calculates the membership degree of each input value to the fuzzy set(s). The expert's opinion is represented by the specified fuzzy membership functions for the input parameters. The obtained fuzzified values are used for computations in the next step.

The inference process determines the output value by logic rules after the input and output values definition and making the fuzzification process. Here, the rules base is performed by calculating the firing strength and implication generated subprocesses. The membership functions of different input parameters are combined using a conjunction or disjunction operator in firing strength calculation, representing each rule's crisp value of the rule antecedent. Following the evaluation of

the conditioning aspect of the rule, the outcome should be addressed in the implication phase, employing a conjunction or product operator.

After the inference process, the composition process aggregates all obtained fuzzy rule results, resulting in a single complex shape. As a result, the conclusion of each rule is combined here by max, sum, or probabilistic operator.

The final process is defuzzification, generating the crisp value to determine the consequence of the aggregated fuzzy set.

5.2 F-FMEA with Summative Defuzzification

To increase the reliability of the analysis used in risk assessments, taking more than one opinion into account makes the result more reliable. Therefore, it is critical to look at the risk assessment method from various angles to come up with a more accurate conclusion. Nevertheless, when experience-based results produce conflicting reports, an aggregation algorithm may provide an optimum resolution. This hypothesis is the same in the conventional fuzzy inference system when analyzing data based on a single expert study.

The traditional fuzzy rule based FMEA using defuzzification methods can also be effective when the assessment is completed based on a single expert analysis. Therefore, when this method is considered with more than one view, it becomes more reliable.

Figure 5.2 represents the Summative Defuzzification Fuzzy rule-based Failure Mode and Effect Analysis (SDF-FMEA) inference process flow chart. The conventional process model is updated depending on two aggregations of data points.

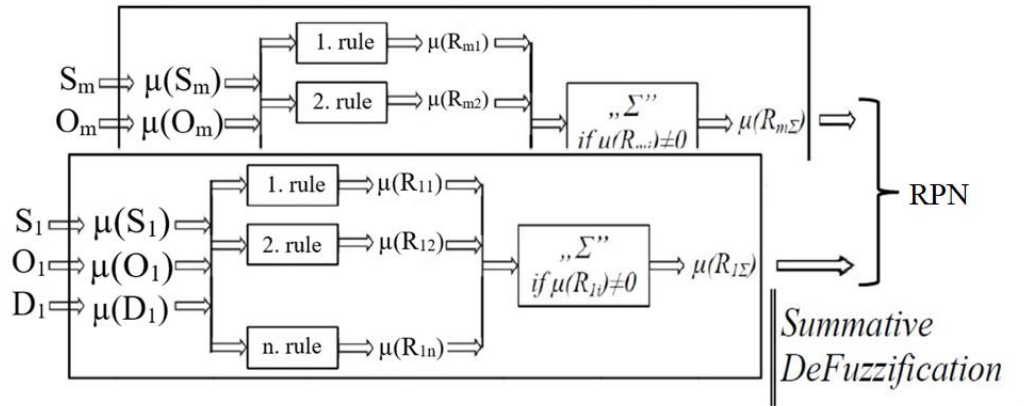


Figure 5.2 Summative Defuzzification Process

5.3 Summative Defuzzification Methods

As concerned as previous studies, the defuzzification subprocess is the final result in the fuzzy inference process in which the given fuzzy input sets are converted to a precise value. The found defuzzified crisp value represents the action in a process control obtained from the fuzzy inference mechanism. The appropriate defuzzification method can be determined according to task to get more efficient results.

Based on multiple opinions of case studies, where the Mamdani type of fuzzy inference can be used, the following proposed methods can analyze by combining two outcomes in a fuzzy inference process to find an average value.

5.3.1 Summative Traditional Center of Gravity (STCoG) with Defuzzification Method

The SCOG technique combines each experience scenario by using the traditional CoG and the fuzzy crisp's CoG. First, the aggregated fuzzified sets should be determined, taking into account overlapping areas more than once for the defuzzification method of CoG. Then, the classical CoG method is executed on the fuzzy sets generated in the different cases of sub-conclusions.

$$R_{SCoG} = \frac{\sum_{j=1}^m \sum_{i=1}^n \int_{-\infty}^{\infty} \mu_{ji}(y) y dy}{\sum_{j=1}^m \sum_{i=1}^n \int_{-\infty}^{\infty} \mu_{ji}(y) dy} \quad (5.1)$$

where μ_{ji} is the truth value of the i^{th} sub-conclusion in the case of the j^{th} experience, n is the number of sub-conclusions, m is the number of opinions.

For example, Figure 5.3 gives two sub-conclusions that must be determined individually with the traditional COG method.

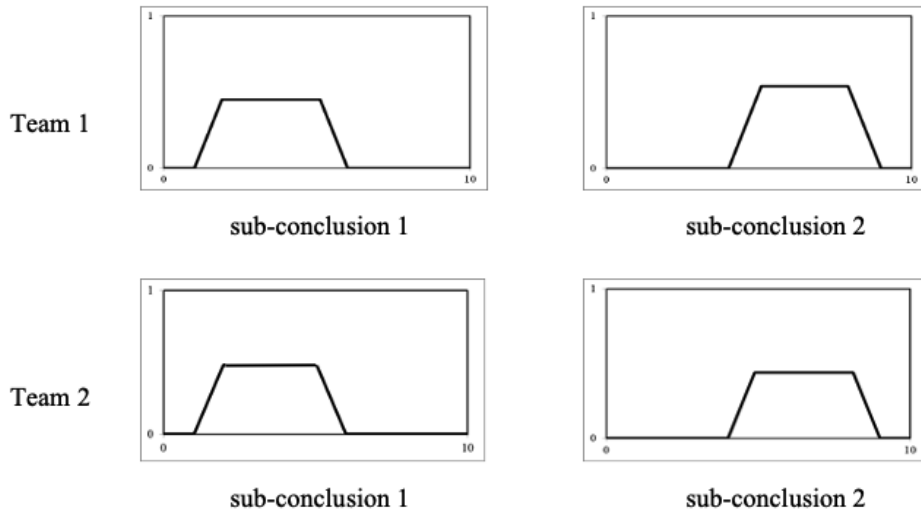


Figure 5.3 Sub-conclusions of the Fuzzy Failure of Teams

5.3.2 Summative Combined CoA and CoG (SCoAG) Defuzzification

Method

The SCoAG method hybridizes CoA and CoG defuzzification methods to achieve an average result [93].

Here, the expert opinion should first be determined by the aggregated fuzzy set of the composition process to determine the conclusion sets of different views. After that, the COA and COG methods are evaluated separately from obtained fuzzy sets and combined.

$$R_{SCoAG} = \frac{\sum_{j=1}^m \int_{-\infty}^{\infty} \mu_{\Sigma_j}(y) y dy}{\sum_{j=1}^m \int_{-\infty}^{\infty} \mu_{\Sigma_j}(y) dy} \quad (5.2)$$

where μ_{Σ} is the maximum height of the conjunct set of sub-conclusions.

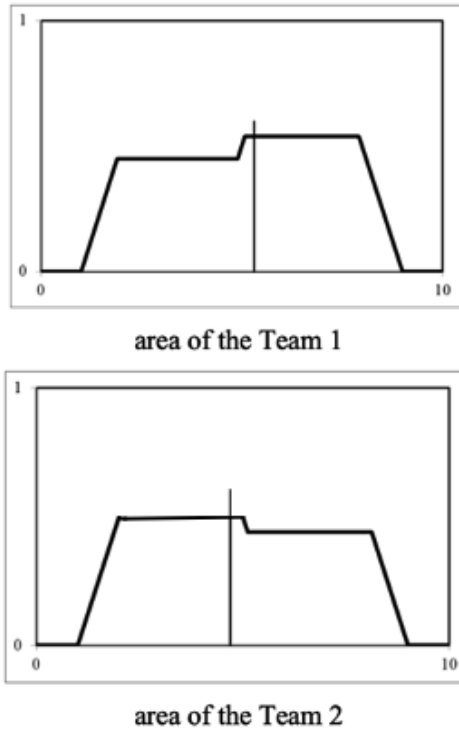


Figure 5.4. Areas of the Opinions of the Failure of Teams

5.3.3 Summative Center of Area (SCoA) Defuzzification Method

In the Summative Center of Area (SCoA), two separate studies are directly aggregated with the Zadeh's max operator in the composition process to create a single fuzzy output.

Individually, the process of fuzzification and inference are considered, and their reflection is aggregated in the composition process.

Finally, COA is used in the defuzzification process see the equation following,

$$R_{CoA} = \frac{\int_{-\infty}^{\infty} \mu_{\Sigma}(z)zdz}{\int_{-\infty}^{\infty} \mu_{\Sigma}(z)dz} \quad (5.3)$$

where μ_{Σ} is the maximum height of the conjunct set of sub-conclusions of Figure 5.5 is the combination of max conclusions of Figure 5.4.

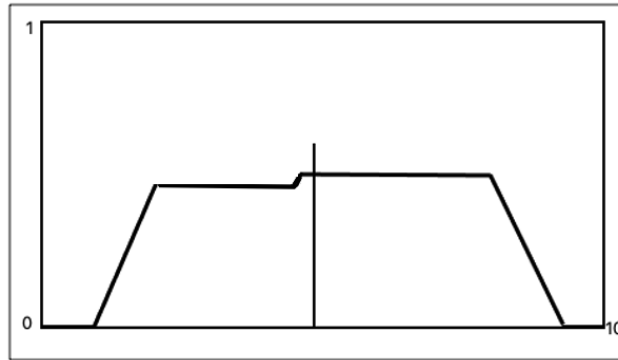


Figure 5.5 Combined result of the Failure of Teams

5.4 Case Study

In this case study, I investigated the bearing manufacturing processes F-FMEA supported using summative defuzzification methods. Moreover, in my publication [94], this study has been presented with traditional defuzzification methods, and here results are compared operating the same inputs in the adjusted system. Importantly, although two different expert group opinions on average in the original analyzed system, the Summative-defuzzification-assisted fuzzy rule-based FMEA considers the group views separately.

In Table 5.1, the failure codes are listed, and Table 5.2 indicates the input values of Team 1 and Team 2, including both individual and average. Trapezoidal fuzzy membership functions are used during the evaluation as in previous studies.

Function	Failure Mode	Failure Effect	Code	Occasion (occurrence)
Outer diameter of bearing	Big	Installation issue, short fatigue life	B1	adjusting of a machine
			B2	omission of finishing
	Small	slack-running fit, early failure	A1	breakage of cone belt
			A2	improper emulsion concentration
			A3	continuity of charging is improper
			A4	congestion before finishing

Table 5.1 Failure codes

	Opinion	B1	B2	A1	A2	A3
S	Team 1	4.00	4.00	4.00	4.00	4.00
O		3.10	9.00	2.10	3.20	2.70
D		2.00	2.00	1.00	1.90	2.00
S	Team 2	3.66	3.66	3.33	3.33	3.33
O		2.50	9.00	1.90	2.80	3.30
D		2.33	2.00	1.66	1.76	2.66
S	Average	3.83	3.83	3.67	3.67	3.67
O		2.8	9.00	3.00	3.00	3.00
D		2.17	2.00	1.33	1.83	2.33

Table 5.2 Two different input data of FMEA

In the third section, the F-FMEA of WSS that structuring hierarchical studied, the input and output values of Membership Functions and same rule base for the F-FMEA that I presented in the case study were also evaluated in this section (see Table 3.7).

Let's take Teams of A1 (see Table 5.2) as an example for the F-FMEAs of the bearing manufacturing process. The input values of Team 1 are taken during the risk evaluation were determined as,

$$\text{Severity: } 4.00, \text{ Occurrence: } 2.10, \text{ and Detectability: } 1.00 \quad (5.4)$$

and the input values of Team 2 as,

$$\text{Severity: } 3.33, \text{ Occurrence: } 1.90, \text{ and Detectability: } 1.66 \quad (5.5)$$

Accordingly, *RPN* output values are calculated below using the outlined fuzzy logic inference process.

In Figure 5.6., the indication of positional information and membership values for opinions of the first *Severity: 4.00* and the Second *Severity: 3.33*.

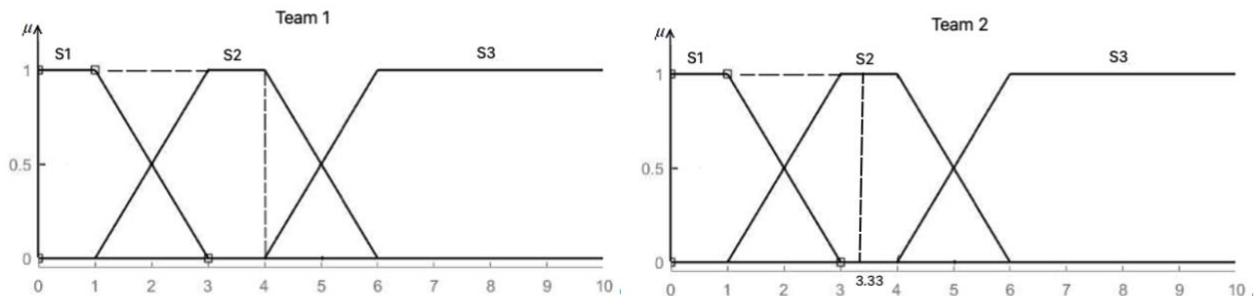


Figure 5.6 Determination of Severity values

In Figure 5.7., the indication of positional information and membership values for first opinion *Occurrence: 2.10* and the Second opinion *Occurrence: 1.90*.

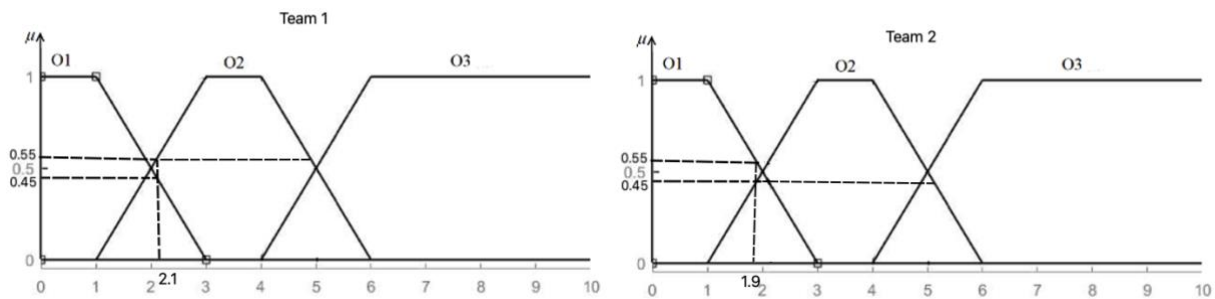


Figure 5.7 Determination of Occurrence values

In Figure 5.8, the indication of positional information and membership values for first opinion *Occurrence: 1.00* and the Second opinion *Occurrence: 1.66*.

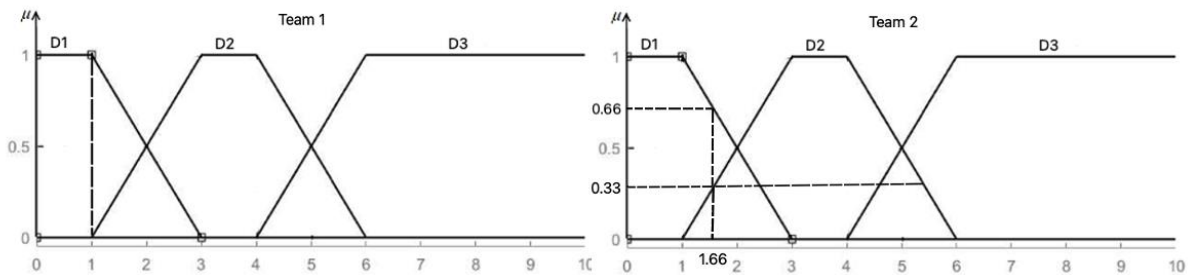


Figure 5.8 Determination of Detectability values

Table 5.3 demonstrates the rule line of F-FMEA's in both cases; *IF-THEN* structured, the minimum *AND* operators used to connect in firing strength calculation (2.28), and each rule is reflected in the implication process (2.29). This state has been created for the following step of composition process.

NO	Team 1			Team 2			Rules	Team 1	Team 2
	μ_{Si}	μ_{Oi}	μ_{Di}	μ_{Si}	μ_{Oi}	μ_{Di}		μ_{Rj}	μ_{Rj}
{1}	0	0.45	1	0	0.55	0.66	$S1 \cap O1 \cap D1 \Rightarrow R1$	0	0
{2}	0	0.55	1	0	0.45	0.66	$S1 \cap O2 \cap D1 \Rightarrow R2$	0	0
{3}	0	0.45	0	0	0.55	0.33	$S1 \cap O1 \cap D2 \Rightarrow R2$	0	0
{4}	0	0.55	0	0	0.45	0.33	$S1 \cap O2 \cap D2 \Rightarrow R3$	0	0
{5}	1	0.45	1	1	0.55	0.66	$S2 \cap O1 \cap D1 \Rightarrow R2$	0.45	0.55
{6}	1	0.55	1	1	0.45	0.66	$S2 \cap O2 \cap D1 \Rightarrow R3$	0.55	0.45
{7}	1	0.45	0	1	0.55	0.33	$S2 \cap O1 \cap D2 \Rightarrow R3$	0	0.33
{8}	1	0.55	0	1	0.45	0.33	$S2 \cap O2 \cap D2 \Rightarrow R3$	0	0.33
{9}	0	–	–	0	–	–	$S3 \Rightarrow R4$	0	0
{10}		0	–	–	0	–	$O3 \Rightarrow R4$	0	0
{11}			0	–	–	0	$D3 \Rightarrow R4$	0	0

Table 5.3 F-FMEA Active Rule Schedule

The maximal operator (see eq. 2.30) was employed for the aggregation procedure once the membership degrees were identified and reflected in each rule base. The first Team 1 of A1 composition results is provided following as,

$$\begin{aligned} \mu_{R1} &= 0 & ; & \mu_{R2} = 0.45 \\ \mu_{R3} &= 0.55 & ; & \mu_{R4} = 0 \end{aligned} \quad (5.6)$$

and the first Team 2 of A1 composition results is provided following as,

$$\begin{aligned} \mu_{R1} &= 0 & ; & \mu_{R2} = 0.55 \\ \mu_{R3} &= 0.45 & ; & \mu_{R4} = 0 \end{aligned} \quad (5.7)$$

Figure 5.9 illustrates the RPN parameters of the membership function and the final step of the defuzzification method, which we apply to the CoG and CoA here.

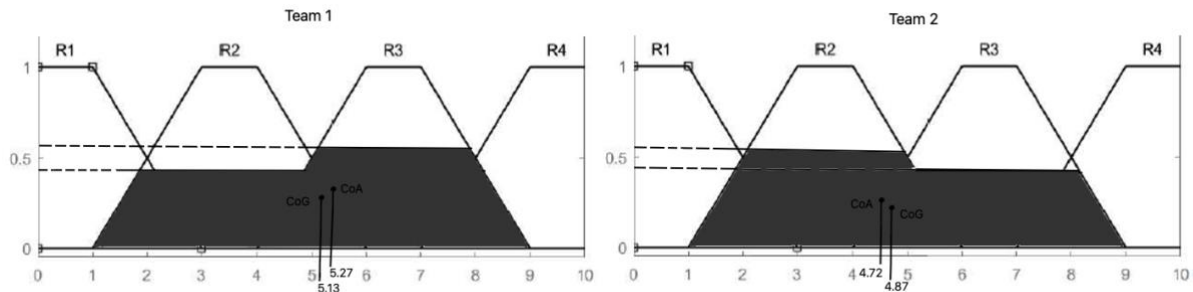


Figure 5.9 Output variables “Risk Priority Number”

Table 5.4 confirms the results of the original CoG, CoA defuzzification methods and modified systems. The actual outcomes, in which evaluations are based on an average of expert judgments (see table 5.2), are displayed in the CoG and CoA columns with their F-RPN and Relative F-RPN values. The columns of STCoG, SCoAG, and SCoA show the proposed F-FMEA achievement using the summative approaches where the expert group opinions are assessed separately.

Failure	F-RPN					Relative FRPN [%]				
	CoG	CoA	STCoG	SCoAG	SCoA	CoG	CoA	STCoG	SCoAG	SCoA
B1	6.04	6.2	5.89	6.11	6.5	18.44	18.96	18.33	18.64	19.40
B2	8.94	9.00	8.94	9.00	9	27.30	27.52	27.81	27.46	26.86
A1	4.77	4.5	4.99	4.96	5	14.56	13.76	15.54	15.15	14.92
A2	6.5	6.5	6.21	6.38	6.5	19.85	19.88	19.32	19.48	19.40
A3	6.5	6.5	6.11	6.31	6.5	19.85	19.88	19.01	19.26	19.40

Table 5.4 Result of comparison with published results [85]

The following conclusions can be inferred from the aforementioned case study:

- failure of the B2 has the highest values of F-RPN and Relative F-RPN;
- failure of the A1 has the smallest values of F-RPN and Relative F-RPN;
- failures of B1, A2, and A3 have the close values of F-RPN and Relative F-RPN
- summative defuzzification methods have more descriptive results than traditional methods seen in A2 and A3 failures. Traditional methods produced the same results for F-RPN and Relative F-RPN, while summative methods have differentiated them.
- failure of B2 has the same result in the case CoG and STCoG, just like CoA and SCoAG, SCoA methods. This predicament develops as a result of the two expert groups evaluating this failure equally.

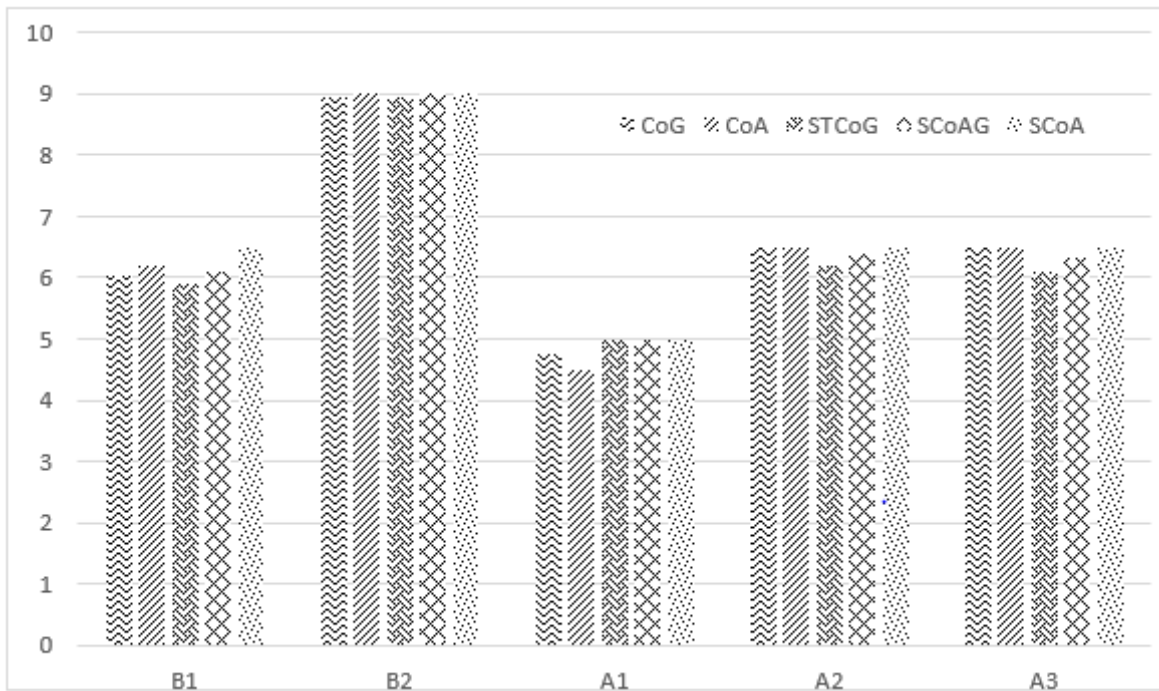


Figure 5.10 Comparison of F-RPN Results

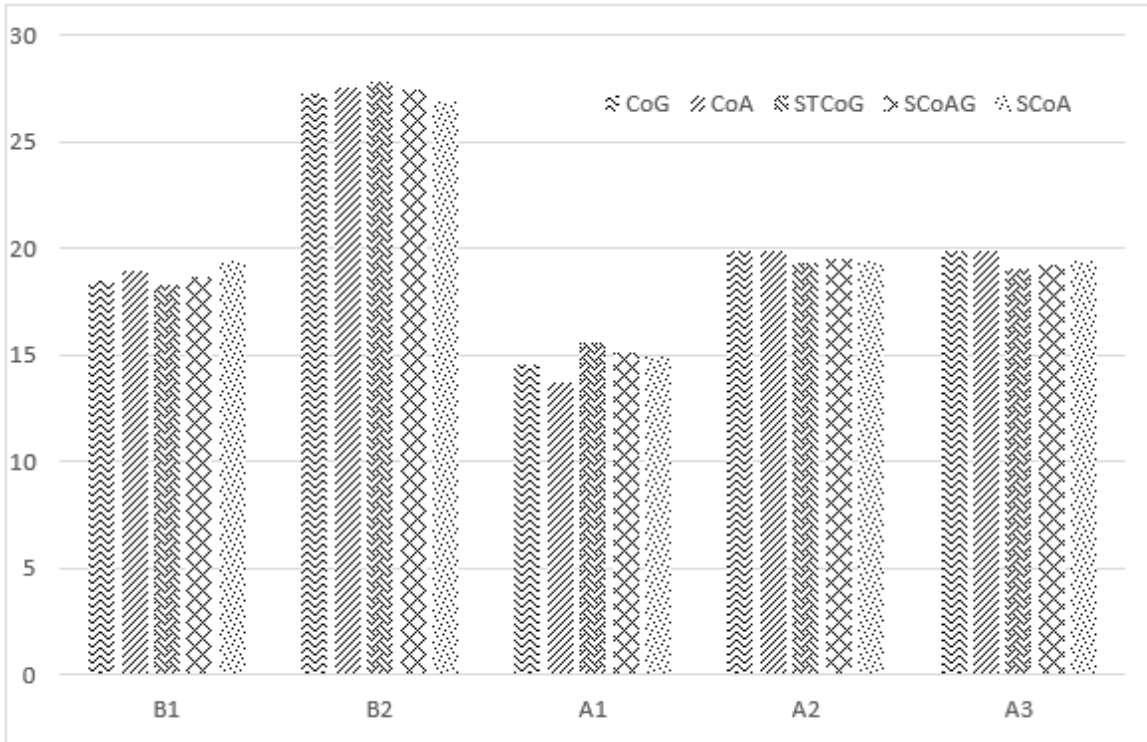


Figure 5.11 Comparison of Relative F-RPN Results

5.5 Conclusion

Optimizing the manufacturing process has been crucial as quality requirements have risen in recent years. Furthermore, assessing all conceivable settings and parameters that influence quality criteria is both time-consuming and costly demanding, and in some situations, impossible. Failure Modes and Effects Analysis (FMEA), a commonly used engineering technique, is a great method of preventing any potential faults in a system, process, design, or service. However, the method's usefulness is limited because both quantitative and qualitative parameters might be seen among the risk variables, which must be carefully addressed to produce a realistic result. The Fuzzy-FMEA approach extends the classic FMEA method to handle the problem as outlined above.

I have emphasized in my previous studies that F-FMEA is used more effectively when the analysis is based on a single expert assessment. However, if many perspectives from different specialists are available, a modified procedure is required to ensure a more trustworthy analysis. In order to overcome this situation during the bearing manufacturing process, I propose a novel F-FMEA approach presenting Summative defuzzification (SCoG, SCoAG, and SCoA) methods. In this

situation, the perspectives of different experts are examined separately, and the overall result is formed based on the aggregated results using defuzzification methods.

Following the theoretical basics, I proposed using an F-FMEA approach supported by summative defuzzification methodologies to analyze the possible failures in the bearing manufacturing process. Moreover, the obtained results were compared to the results of the original F-FMEA model.

According to our findings, Summative-defuzzification- supported F-FMEA inference produces more significance based on multiple expert viewpoints.

6 SCIENTIFIC RESULTS

In this research, I aimed to develop mathematical modeling of the Mamdani-type Fuzzy Inference Process (MFIP) to minimize the possible failures in engineering components required for automobiles. I have presented the improved MFIP model by examining two different Failure Mode and Effect Analysis (FMEA) models that have studied before. I handled the developed MFIP concept with FMEA, minimizing overall losses and identifying the risk context and acceptability, ensuring success.

As a result of the thesis, my scientific developments are as follows:

Thesis 1 (T1): I have proposed a new fuzzy rule-based extension of the crisp Hierarchical Failure Mode and Effect Analysis named Fuzzy Hierarchical Failure Mode and Effect Analysis (FH-FMEA), which is used to define, classify, and assess risk factors [S8].

Thesis 2 (T2): I have proposed a new Level-specific Evaluation-based Fuzzy Hierarchical Failure Mode and Effect Analysis model (LsFH-FMEA) in which different membership functions can be applied at different levels. At the same time, from the System Level (SL) to Design Level (DL), the parameters are transmitted as a fuzzy number. In this way, different technicalities of levels can be presented. Furthermore, transmitting inputs as fuzzy numbers helps to transmit uncertain information between levels [S9].

Thesis 3 (T3): I have worked out a new evaluation structure, based on the conventional Fuzzy FMEA (F-FMEA) model, called Summative Defuzzification. I have proven their possibilities of use in case of fuzzy rule-based risk assessment. Its great advantage the ability of taking into account different expert opinions in the same model.

Thesis 3a (T3a): I have worked out Summative Center of Gravity (SCoG) Defuzzification Method, where the sub-conclusions, and the aggregated fuzzy sets are evaluated by the CoG method as well [S7].

Thesis 3b (T3b): I have worked out Summative Center of Area (SCoA) defuzzification Method, where the sub-conclusions, and the aggregated fuzzy sets are evaluated by the center of Area (CoA) method as well.

Thesis 3c (T3c): I have worked out the Summative Combined CoA and CoG (SCoAG) Defuzzification Method, where the sub-conclusions are evaluated by CoA, then aggregated fuzzy sets are evaluated by the Center of Gravity (CoG) method [S7] [S12].

Thesis 3d (T3d): I have proven their possibilities of use in Fuzzy rule-based FMEA [S7] [S12].

6.1 Recommendations for future usage

Involving the Failure Mode and Effects Analysis (FMEA) projects at the production facilities can bring more advantages by observing System, Design, and Process failures. In my future research, I will focus on the risk assessment of electric vehicle subsystems and their components, such as Traction battery pack, Charge Port, Electric motor, etc.

Fuzzy rule-based hierarchical FMEA models, which I have developed in this dissertation, will bring more effective results. Moreover, summative defuzzification methods can improve the system based on different outcomes. In addition, new rule bases can be created according to the system's structure, and the Mamdani-type fuzzy inference process mathematically can be improved.

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7.3 Publications of the candidate not related to the dissertation

- [S13] S. Koçak, “Industry: Safety in Human-Robot Collaboration,” *Management, Enterprise and Benchmarking in the 21st Century*, pp. 180–188, 2018.
- [S14] H. Altaleb and S. Koçak, “The Risk of Using Biometrics,” in *Proceedings of FIKUSZ Symposium for Young Researchers*, 2018, pp. 32–42.

LIST OF ABBREVIATIONS

ABS:	Anti-lock Braking System
AIAG:	Automotive Industry Action Group
AP:	Action Priority
ASIL:	Automotive Safety Integrity Level
ASQC:	American Society for Quality Control
COA:	Center of Area
COG:	Center of Gravity
CL:	Cause Level
D:	Detectability
DL:	Design Level
EoL	End of Line (test)
EL:	Effect Level
F-FMEA:	Fuzzy rule-based FMEA
FH-FMEA:	Fuzzy Hierarchical Failure Mode and Effect Analysis
FMEA:	Failure Mode and Effect Analysis (FMEA)
FMECA:	Effect, and Criticality Analysis
FSR:	Functional Safety Requirement
H:	High
HARA:	Hazard Analysis and Risk Assessment
IEC:	International Electrotechnical Commission
IoT:	Internet of Things

ISO:	International Organization for Standardization
L:	Low
LOM:	Largest of Maximum
LsFH-FMEA:	Level-specified Fuzzy Hierarchical Failure Mode and Effect Analysis
M:	Medium
MF:	Membership Function
MFIP:	Mamdani-type Fuzzy Inference
MOM:	Mean of Maximum
NASA:	National Aeronautics and Space Administration
O:	Occurrence
RPN:	Risk Priority Number
S:	Severity
SAE:	Society of Automotive Engineers
SCoA:	Summative Center of Area
SCoAG:	Summative Traditional Center of Gravity
SDF-FMEA:	Summative Defuzzification Fuzzy rule-based Failure Mode and Effect Analysis
SL:	System Level
SOM:	Smallest of Maximum
STCoG:	Summative Traditional Center of Gravity
TSR:	Technical Safety Requirement
VDA:	German Association of the Automotive Industry
WSS:	Wheel Speed Sensor

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STATEMENT

For the independence of work, literature resources of the quote in an appropriate way

As the author, **Mr. Sinan Koçak** takes an official statement as follows:

- a) Firstly, this “**Advanced Fuzzy Rule-based Failure Mode and Effect Analysis**” doctoral dissertation is my own fully exclusive work;
- b) Secondly, both the used sources and references are marked and listed correctly. All parts that I have taken from another source, either verbatim or in the same content but reworded are clearly marked with the source;
- c) Finally, all my scientific results and theses of my resource process are the results of my own work.

Budapest, 11.04.2022

written by:

.....
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