Doctoral School of Applied Informatics and Applied Mathematics

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Generalization of Tensor Product Model Based Control Analysis and Synthesis

Ph.D. Thesis Summary

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Budapest
2018

Preliminaries

Robust stability and optimality are central paradigms of model based controller design. It is essential to ensure the fast, low-cost operation of practical applications, reserving the robustness against disturbances, signal noises and parameter uncertainties. For purely linear systems, the LQ, H_2, H_∞ control criteria and pole placement method are essential for system analysis and synthesis.

Parameter dependency and nonlinearity are often inevitable in practical systems. The LPV/qLPV modelling methodology provides an approach to handle these systems similarly as linear systems.

Linear Parameter Varying (LPV) modelling

There is a wide class of systems that can be described with a parametrized linear models as

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{y}(t) \\ \mathbf{z}(t) \end{bmatrix} = \mathbf{S}(\mathbf{p}(t)) \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \\ \mathbf{w}(t) \end{bmatrix},$$

where the state variables are denoted by $\mathbf{x}(t)$, the input signals by $\mathbf{u}(t)$, the noise or disturbances by $\mathbf{w}(t)$, the measured output $\mathbf{y}(t)$, the performance output $\mathbf{z}(t)$, and the $\mathbf{S}(\mathbf{p}(t))$ system matrix can be partitioned to $\mathbf{A}(\mathbf{p}(t))$, $\mathbf{B}(\mathbf{p}(t))$, $\mathbf{C}(\mathbf{p}(t))$, $\mathbf{D}(\mathbf{p}(t))$ etc. system matrices, and is defined over a hyper-rectangular parameter domain:

$$\mathbf{p} \in \Omega = \left[\underline{p}_1, \overline{p}_1\right] \times \cdots \times \left[\underline{p}_N, \overline{p}_N\right] \subset \mathbb{R}^N.$$

qLPV realizations of parameter-varying nonlinear system

Nonlinear systems can also be described as ”quasi-LPV” (qLPV) systems. The concept is the clear extension of local linearization of the state space, where only the $\mathbf{x}(t) \approx \mathbf{x}_d(t)$ case is modelled for control analysis or synthesis. The disadvantages of the concept are the conservatism and the non-uniqueness of the realizations.

LMI based controller design

In the modern model based control, Lyapunov's second, so called direct method plays a key role. It applies positive definite functions for stability verification of autonomous and controlled systems called later Lyapunov functions.

This way, by considering quadratic Lyapunov functions as $V(\mathbf{x}) = \mathbf{x}^T\mathbf{P}\mathbf{x}$, the stability criteria for linear systems leads to definite conditions on matrices. For example, considering an autonomous linear system

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t),$$

it is stable if and only if there exist $\mathbf{P}$ matrix such that,

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} \prec 0, \quad \mathbf{P} = \mathbf{P}^T \succ 0.$$

The Ricatti-equations of LQ, H_2, H_∞ criteria can be easily written as LMIs, and the pole placement constraints as LMI region conditions. The power of this approach is the opportunity to apply multiobjective controller design: The criteria (e.g., constraints on the poles, H_∞ disturbance rejection constraints and H_2 optimal constraint) can be applied together.

These definite, semi-definite conditions had hardly been solved until the convex nature of the Linear Matrix Inequalities (LMIs) was discovered. After the ellipsoid method, the interior point methods were the first, practically relevant polynomial solver.

There are two directions of taking into consideration the uncertainties/parameter dependencies of the linear model: The so called norm bounded description and polytopic envelopes. It is important to denote, that in these cases, the – practically absolutely relevant – robust output feedback design is not a convex problem although the robust/optimal state feedback and the full-order dynamic output feedback methods without uncertainties can be solved via LMI optimisation.

LPV/qLPV models with norm bounded uncertainty

The norm bounded uncertainty description is a popular way to take into account a parameter varying $\mathbf{S}(\mathbf{p})$ matrix or a connected uncertain subsystem with bounded H_∞ norm via the well-known Linear Fractional Transformation. Then the LMI based controller design can be applied or the popular DK, DGK iteration based controller design method for robust output feedback controller design.

LPV/qLPV models with polytopic uncertainty

The variety of the system matrices can also be handled by constructing an enclosing polytope around it. For example, considering a parameter varying autonomous LPV model, it can be given with polytopic uncertainty as

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{p}(t))\mathbf{x}(t), \quad \text{where } \mathbf{A}(\mathbf{p}) \in \text{Co}(\mathbf{A}_1, ..., \mathbf{A}_J).$$

Then its quadratic stability can be verified by considering the $V(\mathbf{x}) = \mathbf{x}^T\mathbf{P}\mathbf{x}$ Lyapunov-function candidate. Its derivative is negative definite for all
$\mathbf{A} \in \text{Co}(\mathbf{A}_1, ..., \mathbf{A}_J)$ if and only if

$$\mathbf{P}\mathbf{A}_j + \mathbf{A}_j^T\mathbf{P} \prec 0 \qquad \forall j = 1, ..., J.$$

The polytopic model based control design is syntactically equivalent to control design for the popular Takagi-Sugeno systems. In the past two decadess plenty of methods were published considering a wide range of control goals and possible relaxations of conservatism, that comes from the presence of irrelevant systems within the polytope and from application of simple quadratic Lyapunov functions.

Polytopic LPV/qLPV models based controller design

In polytopic LPV/qLPV models, it is exploited that the actual $\mathbf{S}(\mathbf{p}(t))$ linear dynamics is known within the $\text{Co}(\mathbf{S}_1, \mathbf{S}_2, ..., \mathbf{S}_J)$ polytopic domain and it can be described as

$$\mathbf{S}(\mathbf{p}) = \sum_{j=1}^{J} h_j(\mathbf{p})\mathbf{S}_j,$$

where the $h_j(\mathbf{p})$ membership functions denote convex combinations. By applying parameter dependant matrices in the controller candidate with the same membership functions, we get the so called Parallel Distributed Compensation (PDC).

As an example for more complex control criteria, let us recall the Bounded Real Lemma for H_∞ state feedback design. The following form allows being the $\mathbf{M}(\mathbf{p})$, $\mathbf{X}(\mathbf{p})$ variables on higher summation, as well.

Lemma 1 (Bounded Real Lemma for LPV models) *For the system with the controller* $\mathbf{u}(t) = \mathbf{M}(\mathbf{p}(t))\mathbf{X}^{-1}(\mathbf{p}(t))\mathbf{x}(t)$*, the* H_∞ *condition*

$$||\mathbf{T}_{\mathbf{z},\mathbf{v}}||_\infty < \gamma$$

holds for all $\mathbf{p}(t) \in \Omega$ *trajectories, if for functions* $\mathbf{X}(\mathbf{p})$*,* $\mathbf{M}(\mathbf{p})$*, the following conditions hold for all* $\mathbf{p} \in \Omega$*:*

$$\begin{aligned}
&\mathbf{X}(\mathbf{p}) \succ 0, \\
&\begin{bmatrix} -\mathrm{Sym}(\mathbf{A}(\mathbf{p})\mathbf{X}(\mathbf{p}) + \mathbf{B}_u(\mathbf{p})\mathbf{M}(\mathbf{p})) & * & * \\ \mathbf{B}_v^T(\mathbf{p}) & \gamma\mathbf{I} & * \\ \mathbf{C}(\mathbf{p})\mathbf{X}(\mathbf{p}) + \mathbf{D}_u(\mathbf{p})\mathbf{M}(\mathbf{p}) & \mathbf{D}_v(\mathbf{p}) & \gamma\mathbf{I} \end{bmatrix} \succ 0, \\
&\dot{\mathbf{X}}(\mathbf{p}) = 0.
\end{aligned}$$

By applying variables on higher polytopic summations, the derived criteria are also on higher polytopic summation, as well.

The methods can be relaxed by applying membership function-dependant conditions. There are stability criteria for cases $\dot{\mathbf{X}}(\mathbf{p}) \neq 0$ if the $|\dot{w}_j(\mathbf{p})|$ values can be bounded and line-integral type Lyapunov–function candidates, where the derivative condition does not appear. Furthermore, the methods are published for descriptor models as well.

There are approaches, especially for discrete time systems, where the variables of the controller and the Lyapunov function are independent. In these cases, the measurement/estimation opportunities do not constrain the multiplicities in the Lyapunov function candidate. The discrete-time case is relaxed by applying delayed Lyapunov-function and the uncertainty of the model can be taken into account by combining the method with the norm-bound uncertainty description.

To handle systems with time delay, Lyapunov-Krasovski functional or Razumikhin theory based methods can be applied.

It is important to denote, that the non-convex nature of robust output-feedback design appears in this case as well. It results in Bilinear Matrix Inequalities motivating research on approaches to consider convex subsets of the solutions and local optimisation methods on BMIs.

TP model transformation based controller design

Baranyi proposed a polytopic form, the Tensor-Product model where the scalar parameter dependencies are represented in a separated way, and they have their own polytopic structures. The corresponding methodology consists of three main parts.

I. Extension of Lathauwer's tensor algebra for multivariate functions. The concept of tensor algebra is generalized for multivariate functions resulting in the tensor product functions. Here the product of a tensor ($\mathcal{B} \in \mathbb{H}^{I_1 \times \cdots \times I_N}$) with a matrix ($\mathbf{U} \in \mathbb{R}^{M \times I_n}$) along the nth dimension is denoted as $\mathcal{A} = \mathcal{B} \times_n \mathbf{U}$, and it results in the $\mathcal{A}$ tensor of size $I_1 \times \cdots \times I_{n-1} \times M \times I_{n+1} \times \cdots \times I_n$ and for its elements

$$\mathcal{A}_{i_1,\ldots,i_{n-1},m,i_{n+1},\ldots,i_N} = \sum_{i_n=1}^{I_n} \mathcal{B}_{i_1,\ldots,i_N} \mathbf{U}_{m,i_n}.$$

Furthermore, the sequential tensor products are denoted as

$$\mathcal{A} \mathop{\boxtimes}_{i=1}^{3} \mathbf{U}^{(n)} = \mathcal{A} \times_1 \mathbf{U}^{(1)} \times_2 \mathbf{U}^{(2)} \times_3 \mathbf{U}^{(3)}.$$

Now the tensor product function can be defined as:

Definition 2 (Tensor Product (TP) function) *The following form*

$$f(\mathbf{p}) = \mathcal{B} \mathop{\boxtimes}_{n=1}^{N} \mathbf{w}^{(n)}(p_n),$$

of a real $f : \Omega \to \mathbb{R}$ *function is called TP function, where the real* $\mathcal{B}$ *tensor has sizes* $I_1 \times \cdots \times I_N$*, and the* n*-mode weighting functions are* $\mathbf{w}^{(n)} : [\underline{p}_n, \overline{p}_n] \to \mathbb{R}^{I_n}$*, respectively.*

In general, multivariate functions cannot be written into TP form with finite I_n sizes. See function $f(\mathbf{p}) = 1/(p_1 + p_2)$.

Definition 3 (HOSVD based TP form) *The following TP form*

$$f(\mathbf{p}) = \mathcal{S}^{HOSVD} \boxtimes_{n=1}^{N} \mathbf{u}^{(n)}(p_n),$$

where $\mathcal{S}^{HOSVD} \in \mathbb{R}^{I_1 \times \cdots \times I_N}$ *and* $\mathbf{u}^{(n)} : [\underline{p}_n, \overline{p}_n] \to \mathbb{R}^{I_n}$ *is called HOSVD based, if for all* $n = 1..N$*, the scalar weighting functions within* $\mathbf{u}^{(n)}(p_n)$ *are orthonormal, and the subtensors of* $\mathcal{S}^{HOSVD}$ *are orthogonal and ordered by their Frobenius norm.*

The HOSVD-based form is canonical because of its uniqueness properties and it can be generally computed from an arbitrary TP form. Furthermore it allows the reduction of n-mode size with minimal error (in terms of Frobenius-norm).

The goal of TP model transformation is to transform a given function $f(\mathbf{p})$ into HOSVD based TP form in the investigated Ω parameter space. To achieve this, an equidistant grid is defined on Ω and the function is first described in its points and then it is reconstructed between the points.

Method 4 (TP model transformation)

Step 4.1 (Discretisation) *A tensor is constructed that contains the value of the function on an equidistant grid over the investigated parameter domain.*

Step 4.2 (Extracting the discretised TP function) *A discretised TP function is obtained by applying HOSVD on the discretised tensor.*

Step 4.3 (Reconstruction of the continuous TP function) *The best approximation of the weighting functions between the gridpoints (in terms of Frobenius-norm) is determined. In practice, usually linear interpolation is applied instead.*

In some cases, the function can exactly be described only via a TP function with sizes up to – theoretically – infinity. Their exact numerical reconstruction is not possible with finite M_n gridsizes, only approximating descriptions can be achieved.

The resulted core tensor always holds the orthogonality and ordering properties. But the reconstructed weighting functions do not hold orthonormality. The error of orthogonality converges to zero as M_n values are increased to infinity, but their norms converges to zero in the meantime.

II. Algorithms to derive Polytopic Tensor Product form from the HOSVD-based one.

The primary motivation of the methodology was its control oriented application: to derive polytopic models for LPV/qLPV models as

$$\mathbf{S}(\mathbf{p}) = \mathcal{S} \boxtimes_{n=1}^{N} \mathbf{w}^{(n)}(p_n),$$

where $\mathcal{S} \in \mathbb{R}^{I_1 \times \cdots \times I_N \times O \times I}$ and the values of $\mathbf{w}^{(n)} : [\underline{p}_n, \overline{p}_n] \to \mathbb{R}^{I_n}$ functions denote convex combinations, that is called Polytopic TP model.

It is derived from the HOSVD-based TP form via algebraic manipulation of the weighting matrices as

$$\mathbf{u}^{(n)}(p_n) = \mathbf{w}^{(n)}(p_n)\mathbf{T}^{(n)},$$

where $\mathbf{T}^{(n)}$ is called transformation matrix.

By performing the operation for all $n = 1..N$ the polytopic TP form can be obtained as

$$f(\mathbf{p}) = \mathcal{S}^{HOSVD} \boxtimes_{n=1}^{N} \left(\mathbf{w}^{(n)}(p_n)\mathbf{T}^{(n)}\right) = \\ = \underbrace{\left(\mathcal{S}^{HOSVD} \boxtimes_{n=1}^{N} \mathbf{T}^{(n)}\right)}_{\mathcal{S}} \boxtimes_{n=1}^{N} \mathbf{w}^{(n)}(p_n).$$

There are methods to derive the HOSVD based form without constructing the (typically large) discretised tensor and to decrease the computational load by performing truncations in sequences of HOSVD.

Identification is used in discretisation step for time-delayed systems in the so called TP$^{\tau}$ model transformation and for other description forms in the so called Multi TP model transformation. However if different state variables are used in the gridpoints, the LMI based control analysis and synthesis does not guarantee the stability and the performance.

III. Polytopic model based controller design methods. By exploiting that the Polytopic TP form is a special polytopic model, it can be rewritten as

$$\mathbf{S}(\mathbf{p}) = \mathcal{S} \underset{n=1}{\overset{N}{\boxtimes}} \mathbf{w}^{(n)}(p_n) = \sum_{r=1}^{R} h_r(\mathbf{p})\mathbf{S}_r,$$

and the methods elaborated for polytopic/Takagi-Sugeno models can be immediately applied.

Motivation of the Theses

As the previous part has showed, the TP model transformation based controller design was born by the marriage of HOSVD for functions, algorithms to derive Polytopic TP model from the HOSVD-based TP form and the polytopic model based control analysis and synthesis methods – inheriting their properties.

During my master's diploma research and period of the Young Research Award, I have applied it on different non-linear and time-delay feedback problems:

1. A haptic telemanipulator arm with time-delayed communication and unknown remote environment, where the effect of time-delay must be decreased because it influences the sensed properties of the remote environment and may cause unstable behaviour. [KJ-10, KJ-11, KJ-12, KJ-13]

2. The prototype of so called dual-excenter actuator. That is a special vibration actuator that is able to work at independently chosen frequency and amplitude in contrast to the widely used vibration devices. Its control is challenging because of its highly non-linear behaviour and the unknown working environment. [KJ-2]

3. Academic systems as inverted pendulum and translational oscillator with a rotational actuator (TORA). [KJ-6, KJ-7, KJ-9]

The development and the corresponding theoretical and numerical supervisions dredged up more practical demands and sources of conservatism. The following questions and problems had arisen:

– The separation of parameter dependencies (defined in the TP function) is not exploited during the controller design, but it can blow up the complexity of the model (measured here as the n-mode sizes of the HOSVD-based form) although the property is used only to construct more simple representation of the form.

– Previous methods to obtain polytopic forms are rarely optimised and they derive – geometrically – simplex polytopes. It is not investigated how their properties influence the achievable performance and how it could be made better in general, and how the specialities of the actual control problem could be taken into account.

– After complexity reduction, the omitted part should be also taken into consideration in control design, otherwise the applied analysis or synthesis method cannot provide guarantee for stability and performance.

– The benefits of HOSVD-based form are its compactness and possibility for complexity reduction with minimal error - although this error does not characterize the obtained polytopic form if pseudo inverse was applied. Furthermore, centralisation and reSVD is necessary for deriving polytopic form. These reasons question the necessity of the HOSVD-based form and suggest that an other intermediate form should be defined that directly leads to the Polytopic TP forms.

Furthermore, the following computational and theoretical reasons:

– The formalism is problematic for non-scalar functions..

– The error of numeric reconstruction converges to zero only as the density of discretisation tends to infinity. There is always bias in the approximation.

– The generalization from discretised to continuous weighting functions is laborious.

Motivated by these reflections and by benefits of the former structure, the theses build up the methodology from scratch to construct a theoretically substantiated solid framework with plenty of tools for various practical problems avoiding the unnecessary conservatism.

The dissertation first presents that the derivation of polytopic forms, that leads to determination of enclosing polytopes for point sets on a usually higher dimensional affine subspace and it concludes the specialities of simplex and non-simplex enclosing polytopes.

Then it extends the definition of Polytopic TP model to avoid the unnecessary separations in such a way, that it can depends on arbitrarily chosen parameter sets. For its derivation a multi-affine form is defined and then only enclosing polytopes must be determined. The new definition allows using parameter dependencies with higher multiplicities to apply it for constructing LMI criteria, and to handle the error of necessary complexity reductions caused by separation.

It describes how the vertices of polytopic models influence the achievable performance, furthermore, it introduces new methods to derive (near) Minimal Volume Simplex and Non-Simplex enclosing polytopes and gives methods to take into account the control properties of the considered LPV/qLPV models.

Finally, it establishes the Polytopic TP model based control analysis and synthesis by introducing that the variables can be also in Polytopic TP form on the same multi-polytopic structure, but with arbitrary multiplicities. This way, special controller and Lyapunov-function candidates can be used, that does not depend on certain parameter sets, and with higher multiplicities on other ones. The introduced TP algebra can be used to describe the derived definite conditions, and a recursive method is provided to extract them into LMIs/BMIs.

The proposed control design workflow and the corresponding methods are illustrated in Fig. 1.

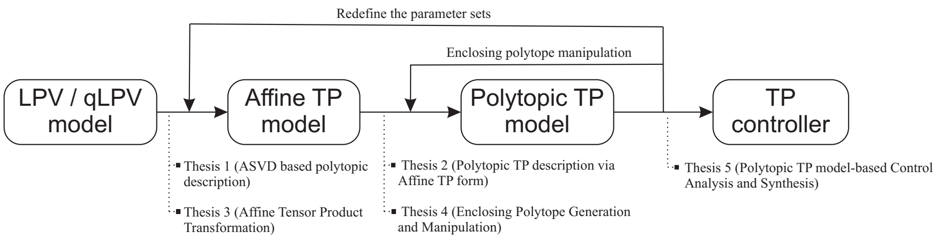

Figure 1: The proposed control design workflow and the corresponding theses

New Scientific Results

Thesis 1. ASVD based polytopic description

The Affine Singular Value Decomposition of multivariate functions

$$\mathfrak{f}(\mathbf{x}) = \sum_{d=1}^{D+1} \mathfrak{a}_d v_d(\mathbf{x})$$

provides a factorization with the following properties:

- *It represents the affine structure of the image set by describing the affine hull via an the offset ($\mathfrak{a}_{D+1}$) and an orthogonal, ordered basis ($\mathfrak{a}_1, ..., \mathfrak{a}_D$) in such a way, that the related homogeneous coordinate ($\mathbf{v}(\mathbf{x})$) are orthonormal functions as in the SVD.*

- *Through ASVD the derivation of polytopic description can be transformed into a geometrical problem: Determination of enclosing polytope for a D dimensional point set (image of $\mathbf{v}(\mathbf{x})$).*

- *It is a canonical representation, because it shows uniqueness in terms of similar as the SVD.*

- *The ASVD form is suitable for complexity reduction with minimal error (in terms of Frobenius norm) by omitting the last basis directions, where the complexity is understood as the D dimension of the affine hull.*

The proposed method is suitable for numerical reconstruction through analytical (exact) or discretisation based (approximating) initial forms.

Corresponding publications: [KJ-1, KJ-3, KJ-7].

Thesis 2. Polytopic TP description via Affine TP form

The former definition of Polytopic TP form can be relaxed along the following properties:

- *arbitrary parameter sets can be used instead of complete separation of the scalar parameter dependencies,*
- *parameter dependencies can be used with arbitrarily high multiplicities to serve the further optimisation structures directly,*
- *the formalism is extended to Hilbert-spaces in general, by defining matrix-tensor product to Hilbert spaces.*

Consider the function $\mathfrak{f}(\mathbf{p}) : \Omega \to H$. *According to the chosen parameter sets (denoted as* $\mathbf{p}_1, \mathbf{p}_2, \dots \subseteq \mathbf{p}$ *and their domains accordingly as* $\Omega_1, \Omega_2, \dots$*), the TP form*

$$\mathfrak{f}(\mathbf{p}) = \mathcal{F} \times_1 \mathbf{w}^{(1)}(\mathbf{p}^{(1)}) \times_2 \mathbf{w}^{(2)}(\mathbf{p}^{(2)}) \dots$$
$$where \quad \mathbf{w}^{(k)} : \Omega_k \to \mathbb{R}^{J_k}, \quad and \quad \mathcal{F} \in H^{J_1 \times J_2 \times \dots}$$

is called a Polytopic TP form if

$$\mathbf{w}^{(k)}(\mathbf{p}^{(k)}) \mathbf{1}^{J_k \times 1} = 1, \quad \mathbf{w}^{(k)}(\mathbf{p}^{(k)}) \geq \mathbf{0} \quad \forall \mathbf{p}^{(k)} \in \Omega_k.$$

The extended polytopic TP form can be derived through Affine TP form

$$\mathbf{f}(\mathbf{p}) = \mathcal{F}^{aff} \times_1 \mathbf{v}^{(1)}(\mathbf{p}^{(1)}) \times_2 \mathbf{v}^{(2)}(\mathbf{p}^{(2)}) \dots,$$

in which the dependencies on the parameter sets show ASVD structures. The derivation requires enclosing polytopes for the weighting functions in the euclidean space with the given dimension.

If the parameter sets are disjoint, the following statements hold:

- *The complexity (geometric dimension) of the dependencies on the parameter sets can be reduced with minimal error (in terms of Frobenius-norm).*
- *The representation is canonical, since it inherits the uniqueness properties of ASVD.*

Corresponding publications: [KJ-1, KJ-4].

Thesis 3. Affine Tensor Product Transformation

The proposed Affine Tensor Product Transformation provides numerical methods to reconstruct the Affine TP form for multivariate functions considering arbitrary parameter sets:

- *Exact TP form can be obtained if the dependencies from the parameter sets can be separated analytically.*
- *In other cases, approximating TP form can be obtained by constructing the initial TP form*

$$\hat{\mathfrak{f}}(\mathbf{p}) = \mathcal{D} \boxtimes_{k=1}^{K} \boldsymbol{\alpha}^{(k)}(\mathbf{p}^{(k)}) = \sum_{m_1=1}^{M_1} \cdots \sum_{m_K=1}^{M_K} \mathfrak{d}_{m_1,\ldots,m_K} \prod_{k=1}^{K} \alpha_{m_k}^{(k)}(\mathbf{p}^{(k)})$$

via discretisation.

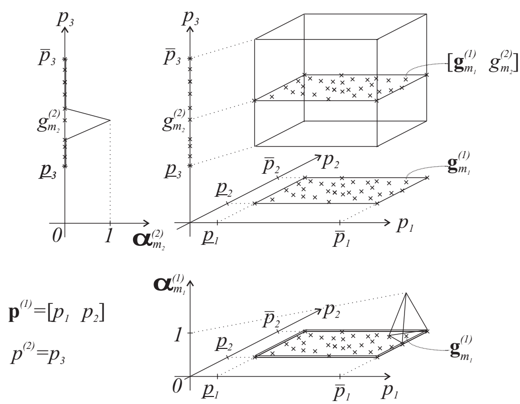

Figure 2: Illustration of discretisation via multivariate interpolatory functions

Furthermore, the exactness of the derived (discretised or complexity reduced) Affine TP form

$$\hat{\mathfrak{f}}(\mathbf{p}) = \hat{\mathcal{F}} \mathop{\boxtimes}_{k=1}^{K} \mathbf{v}^{(k)}(\mathbf{p}^{(k)})$$

can be restored by determining the ASVD of its error (without parameter separation)

$$\mathfrak{f}(\mathbf{p}) - \hat{\mathfrak{f}}(\mathbf{p}) = \tilde{\mathcal{F}} \times_{K+1} \mathbf{v}^{(K+1)}(\mathbf{p})$$

and inserting it in the Affine TP form as a new parameter dependency:

$$\mathfrak{f}(\mathbf{p}) = \underbrace{\left(\hat{\mathcal{F}} \times_{K+1} \mathbf{1}^{(D_{K+1})} + \tilde{\mathcal{F}} \mathop{\boxtimes}_{k=1}^{K} \mathbf{1}^{(D_k)} \right)}_{\mathcal{F}} \mathop{\boxtimes}_{k=1}^{K} \mathbf{v}^{(k)}(\mathbf{p}^{(k)}) \times_{K+1} \mathbf{v}^{(K+1)}(\mathbf{p}),$$

where $\mathbf{1}^{(D)} = \begin{bmatrix} \mathbf{0}^{1\times D} & 1 \end{bmatrix}^T$.

Corresponding publications: [KJ-1, KJ-4, KJ-8].

Thesis 4. Enclosing Polytope Generation and Manipulation

The envelope of polytopic models usually includes a larger set of LTI systems than the LPV/qLPV models highly increasing the conservativeness of the controller design. It is essential to avoid or at least minimize their presence of additional systems without significantly increasing the number of vertices.

This aspect can be taken into consideration through a two phase approach in order to maximize the achievable performance with the polytopic model:

1. *First, the polytopic model is generated by determining the enclosing polytopes based on simple geometric aspects.*

2. *By analysing the actual polytopic model, geometric manipulations are performed on the enclosing polytopes to achieve satisfying control performance.*

According to this concept, the following enclosing polytope generation and manipulation methods are proposed:

- *Generation of Minimal Volume Simplex (MVS).*

- *Manipulation of the MVS by applying constraints to close some of the vertices to the convex hull.*

- *Deriving Non-Simplex enclosing polytopes by cutting regions off from the polytope by one or more halfspaces.*

- *Local Minimization of volume of Enclosing Non-Simplex polytopes.*

The methods are elaborated for higher dimensional spaces in general and the minimal volume has only approximating meaning because the volume minimization problem is highly non-convex.

Corresponding publications: [KJ-3, KJ-6, KJ-7, KJ-9].

Thesis 5. Polytopic TP model-based Control Analysis and Synthesis

The concept of Polytopic Tensor Product (TP) Model based control analysis and synthesis has been revisited and renewed by proposing the use of TP-structured variables in the definite conditions derived from the applied control criteria. For TP forms that can depend on multivariate parameter sets (optionally with two times or higher multiplicities)

$$\mathbf{X}(\mathbf{p}) = \mathcal{X} \underbrace{\times_1 \mathbf{w}^{(1)}(\mathbf{p}^{(1)}) \times_2 \mathbf{w}^{(1)}(\mathbf{p}^{(1)}) \cdots \times_{M_1} \mathbf{w}^{(1)}(\mathbf{p}^{(1)})}_{M_1} \underbrace{\times_{M_1+1} \mathbf{w}^{(2)}(\mathbf{p}^{(2)}) \ldots}_{M_2} \ldots,$$

a compact TP formalism was proposed

$$\mathbf{X}(\mathbf{p}) = \mathcal{X} \mathop{\boxtimes}_{k=1}^{K(\mathbf{M})} \mathbf{w}^{(l(k,\mathbf{M}))}(\mathbf{p}^{(l(k,\mathbf{M}))}), K(\mathbf{M}) = \sum_i M_i,$$

where

$$l(k, \mathbf{M}) = i \ \text{ if } \sum_{a=1}^{i-1} M_a < k < \sum_{a=1}^{i} M_a,$$

and the $\mathbf{M}$ *multiplicity vector describes the structure. By setting the multiplicities, the parameter dependencies can be neglected or considered with arbitrary high complexity in the variables of controller-candidate, Lyapunov-function candidate and slack variables as well.*

The definite conditions on the structures of these variables e.g.,

$$\mathbf{A}(\mathbf{p})\mathbf{X}(\mathbf{p}) + \mathbf{X}(\mathbf{p})\mathbf{A}^T(\mathbf{p}) \prec 0$$

or the Bounded Real Lemma result in definite conditions on Polytopic TP forms. They can be handled in general by defining a recursive method to reformulate them into Linear Matrix Inequalities (LMIs) or Bilinear Matrix Inequalities (BMIs), etc. according to the design method in consideration.

Corresponding publication: [KJ-5].

Summary

The theses essentially renew the tensor product model based control design methodology and the corresponding transformation. By allowing using special controller and Lyapunov-function candidates, that can depend on some parameters with arbitrarily high multiplicity holding robustness from the other ones, better solutions an be achieved. This way, the separated parameter dependencies can be the central benefit of the methodology.

Furthermore, the conservatism of the approach is decreased as the parameter dependencies must be separated only to sets to be handled in different ways during control design, respectively. This relaxation highly generalizes the method since former TP model definition and the polytopic models without separation are its special cases. Furthermore, the transformation method allows to obtain exact models even if complexity reduction was applied.

Finally, to minimise the conservatism of polytopic description of parameter dependencies, the theses propose various methods to generate and manipulate them according to the practical demands.

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